Completeness of Earthquake Catalogues and Statistical Forecasting

by

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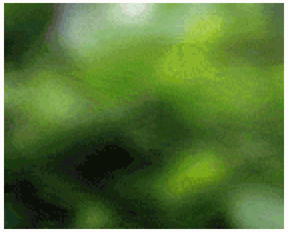
International Conference on “Global Change”
Islamabad, 13-17 November 2006

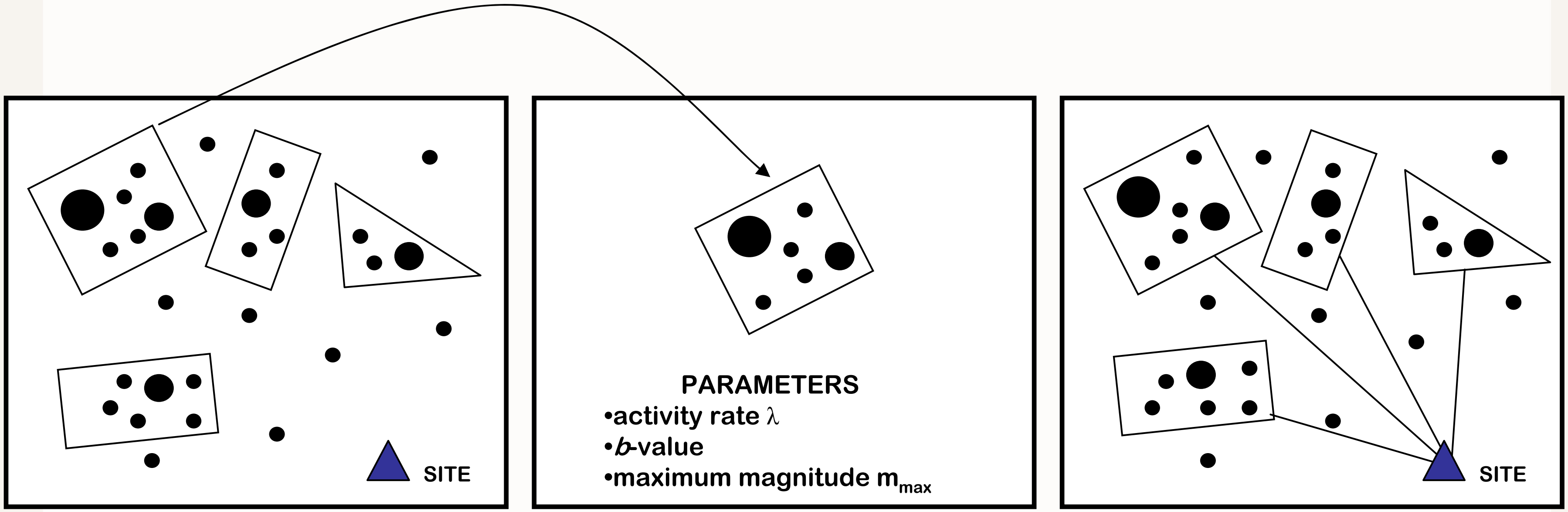
OUTLINE

- Incompleteness of catalogues
- Uncertainties (location, magnitudes and origin times)
- Inadequate model of seismicity
- Seismogenic zones
- Maximum earthquake magnitude m_{max}

Surprise, Surprise!



APPROACH 1: Parametric Procedure (Cornell, 1968)



Advantages:

- Can account for seismic gaps, non-stationary seismicity, faults etc.

Disadvantages:

- Specification of seismogenic zones
- Requires knowledge of seismic hazard parameters (e.g. activity rate, *b*-value, m_{max}) for each zone

APPROACH 2
Non-parametric “Historic” Procedure (Veneziano, et al., 1984)

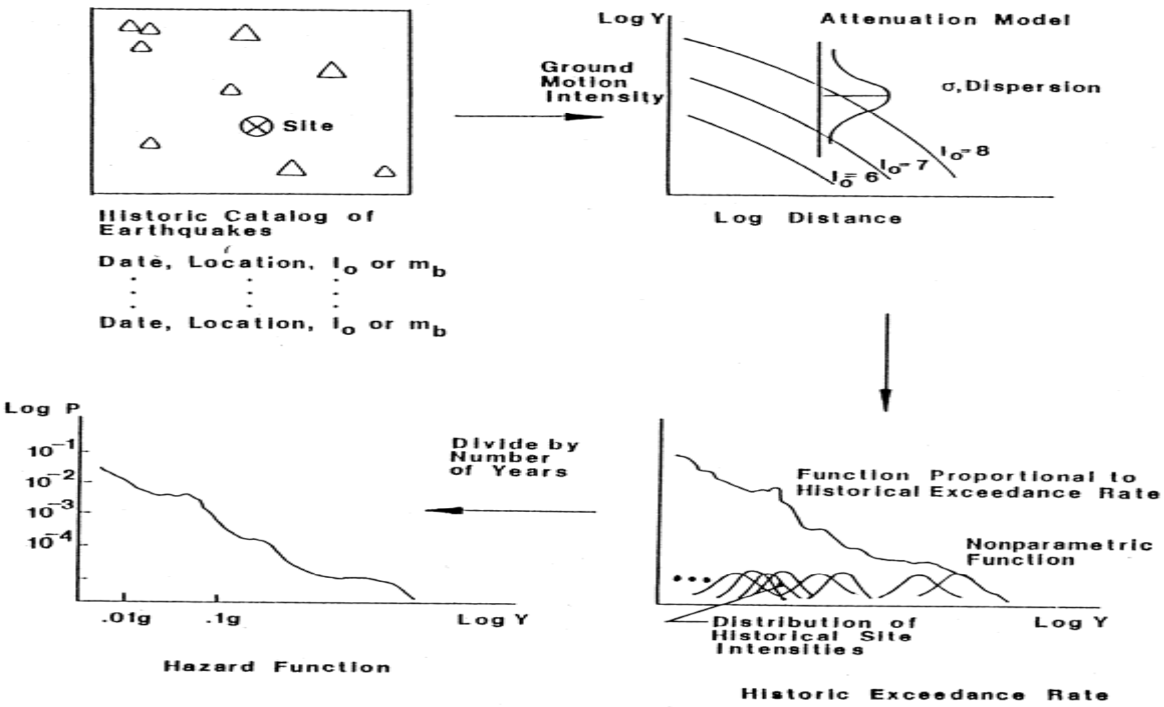


Fig. 11. Steps in non-parametric historic hazard method (Veneziano *et al.*, 1984).

Advantages

- No **division** into seismic zones needed
- Seismic parameters no required

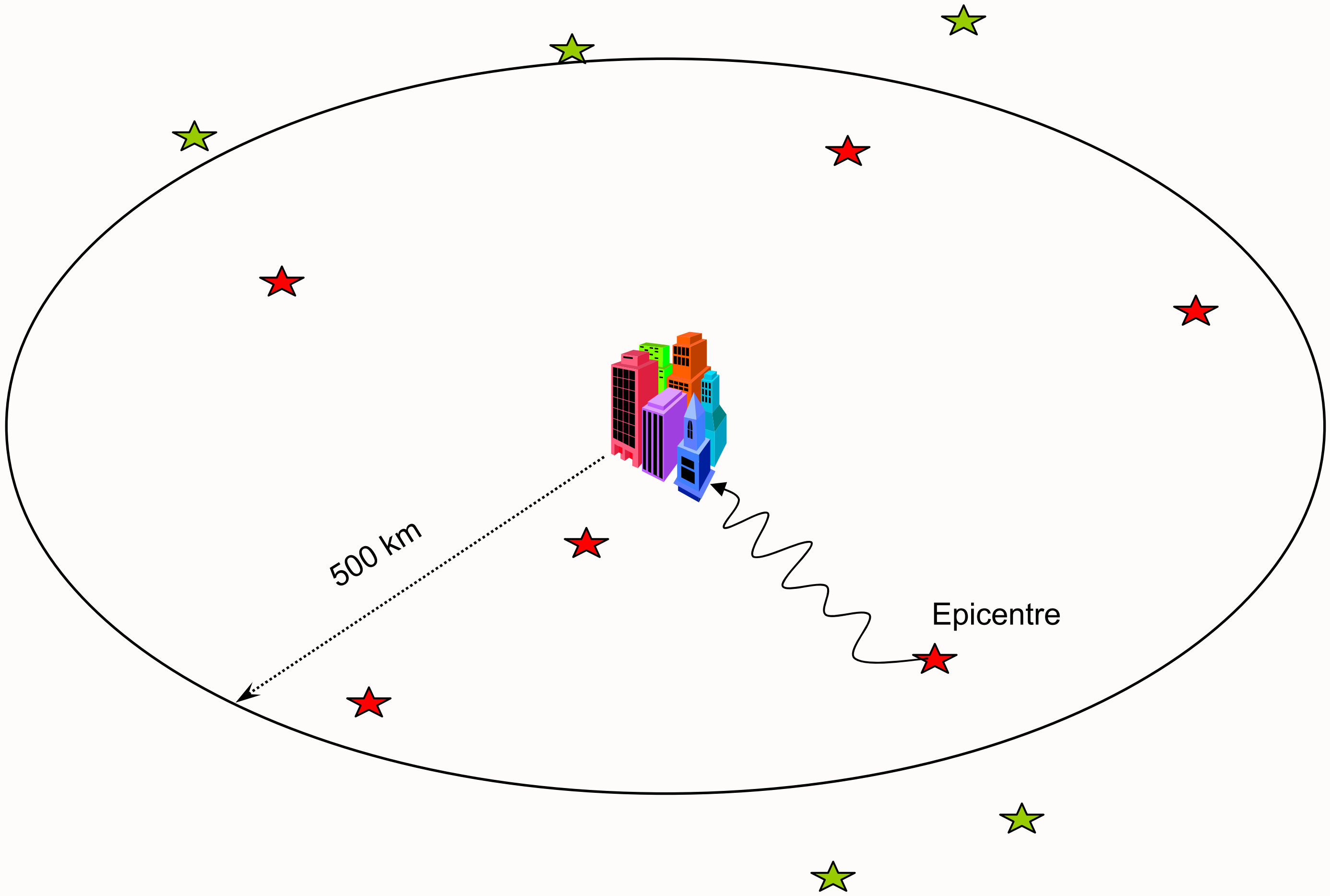
Disadvantages

- Unreliability at low probabilities, or at the areas with low seismicity
- The procedure does not take into account **incompleteness** and **uncertainty** of earthquake catalogues

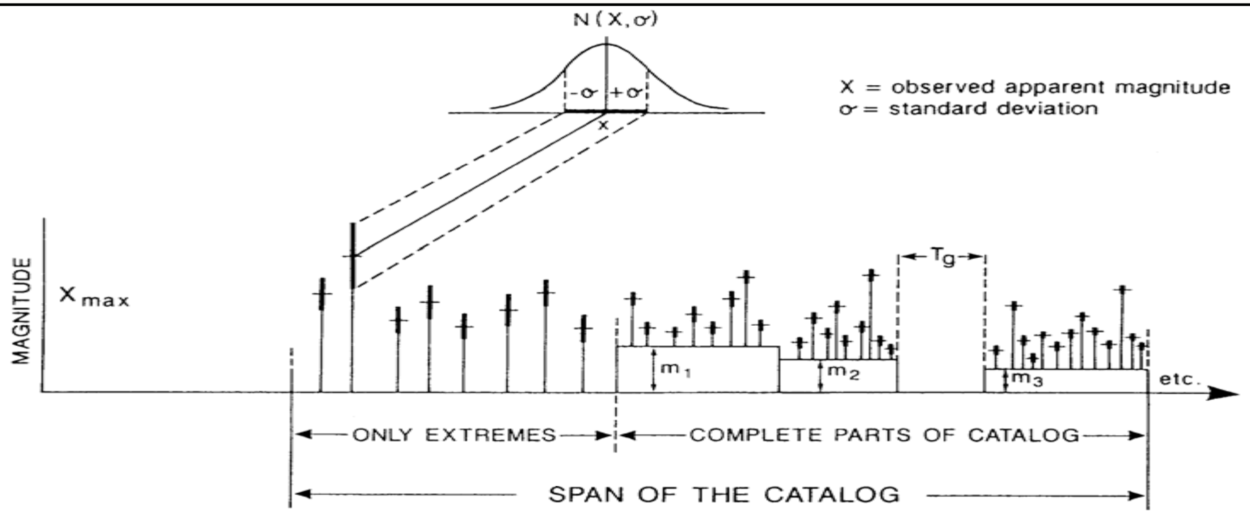
ALTERNATIVE
Combination of both
The 'Parametric - Historic' procedure

- Assessment of basic hazard parameters (e.g. seismic activity rate, *b*-value, m_{max}) for the **area** in the vicinity of the particular site
- Assessment of the distribution function for amplitude of ground motion for a **specified site**.
- If the procedure is applied to all grid points a seismic hazard map can be obtained

PARAMETRIC-HISTORIC PROCEDURE



INCOMPLETENESS AND UNCERTAINTIES OF SEISMIC CATALOGUES

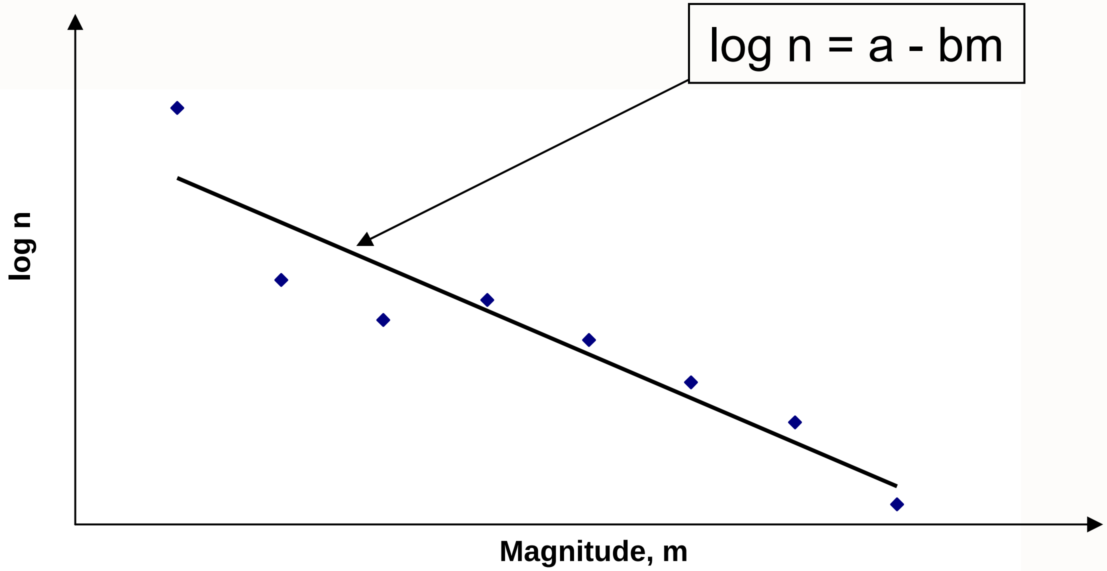


(b)

FIG. 1. An illustration of data that can be used to obtain basic seismic hazard parameters by the proposed procedures. Our approach permits the combination of the largest earthquakes with complete data and variable threshold magnitudes. It makes possible to use the largest known historical earthquake (X_{max}) that occurred before our catalog begins. It also accepts "gaps" (T_g) when records are missing or seismic networks were not in operation. (a) "Hard bounds" model of earthquake magnitude uncertainty. Magnitude of each earthquake is specified by two values: the lower and the upper magnitude limit. It is assumed that such an interval contains the real unknown magnitude. (b) "Soft bounds" model of earthquake magnitude uncertainty. Following Tinti and Mulargia (1985), it is assumed that the observed magnitude is the true magnitude distorted by a random error ϵ. ϵ is free from systematic errors and follows a Gaussian distribution with zero mean and standard deviation σ.

Kijko, A. and Sellevoll, M.A. 1992. Estimation of earthquake hazard parameters from incomplete data files. Part II. Incorporation of magnitude heterogeneity. BSSA, 82(1), 120-134.

MAGNITUDE DISTRIBUTION



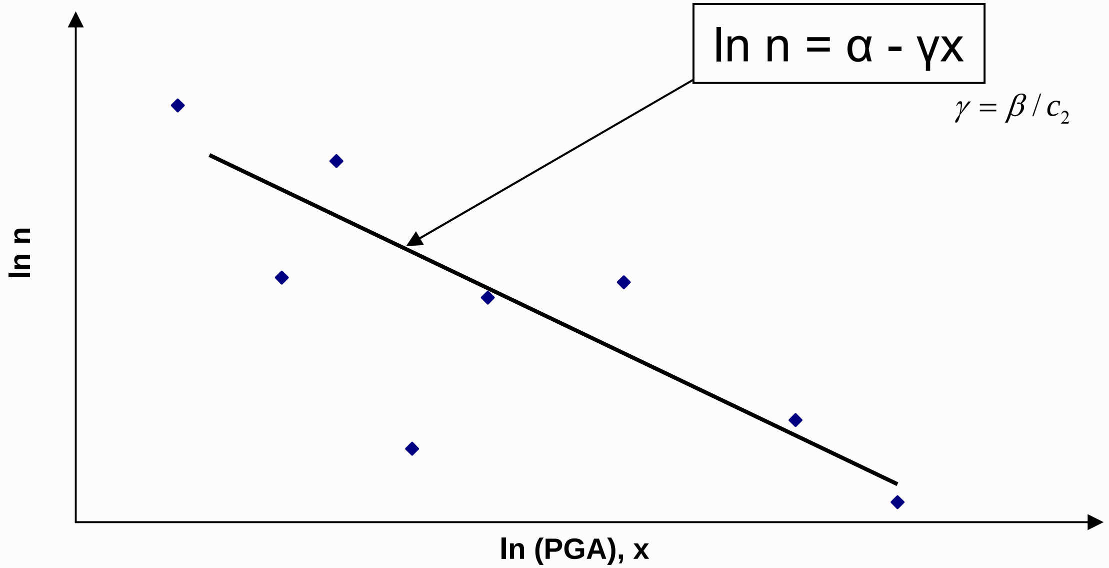
APPLICATION TO THE GUTENBERG-RICHTER MAGNITUDE DISTRIBUTION

$$F_M(m) = \begin{cases} 0 & for\, m < m_{\min}, \\ \dfrac{1-\exp[-\beta(m-m_{\min})]}{1-\exp[-\beta(m_{\max}-m_{\min})]}, & for\, m_{\min} \le m \le m_{\max} \\ 1, & for\, m > m_{\max}. \end{cases}$$

How???

$where\ \beta = b\ln(10)$

PGA DISTRIBUTION



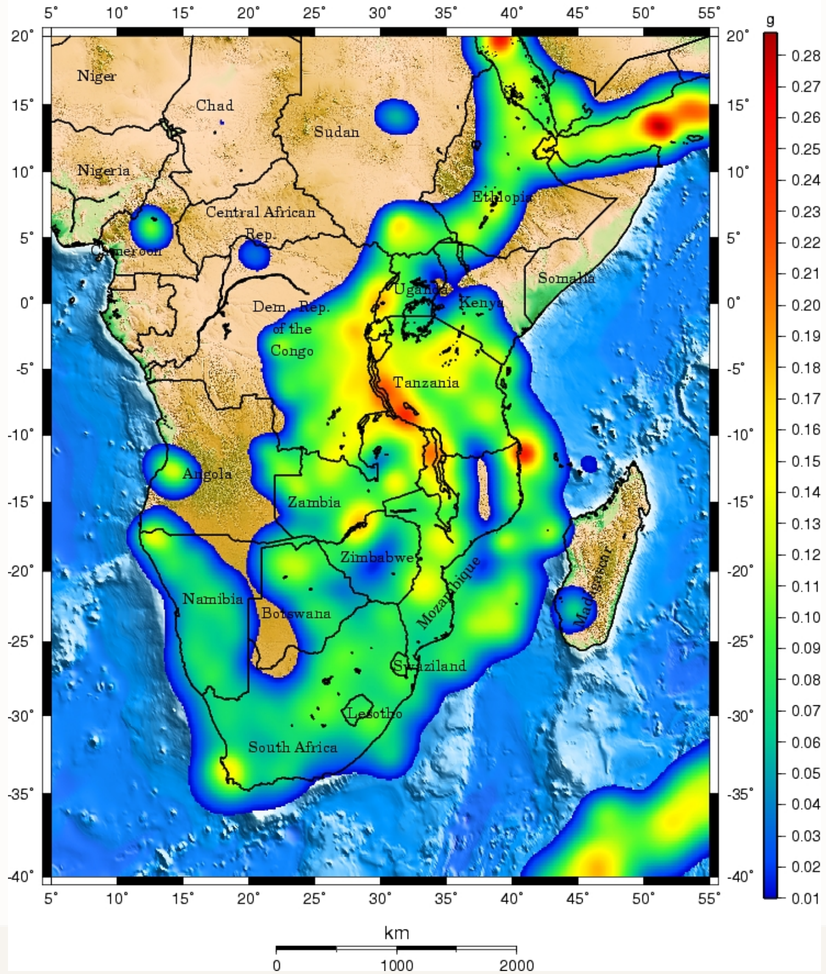
- Please note: $\gamma = \beta / c_2$ where

$$\ln a = c_1 + c_2 \cdot m + c_3 \cdot r + c_4 \cdot \ln r,$$

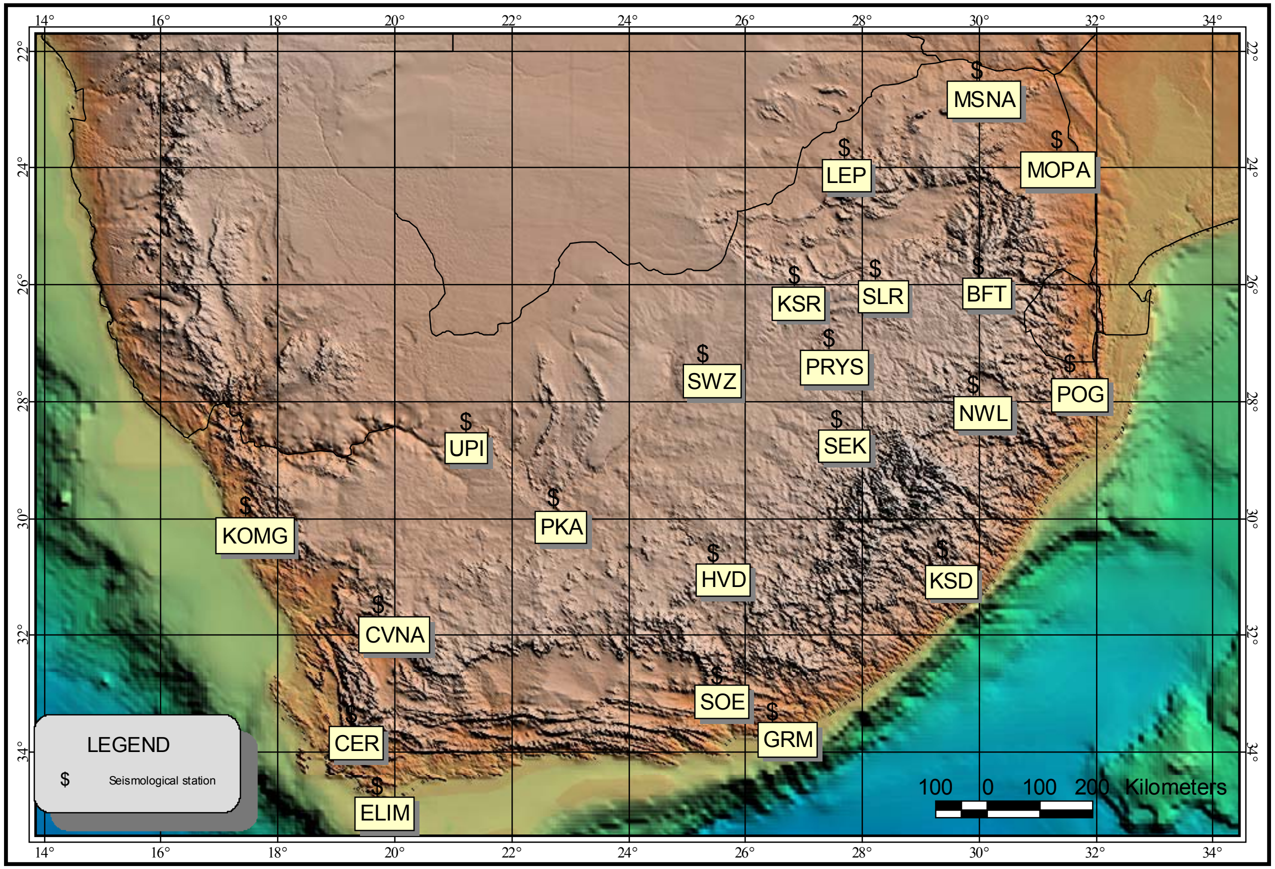
APPLICATIONS

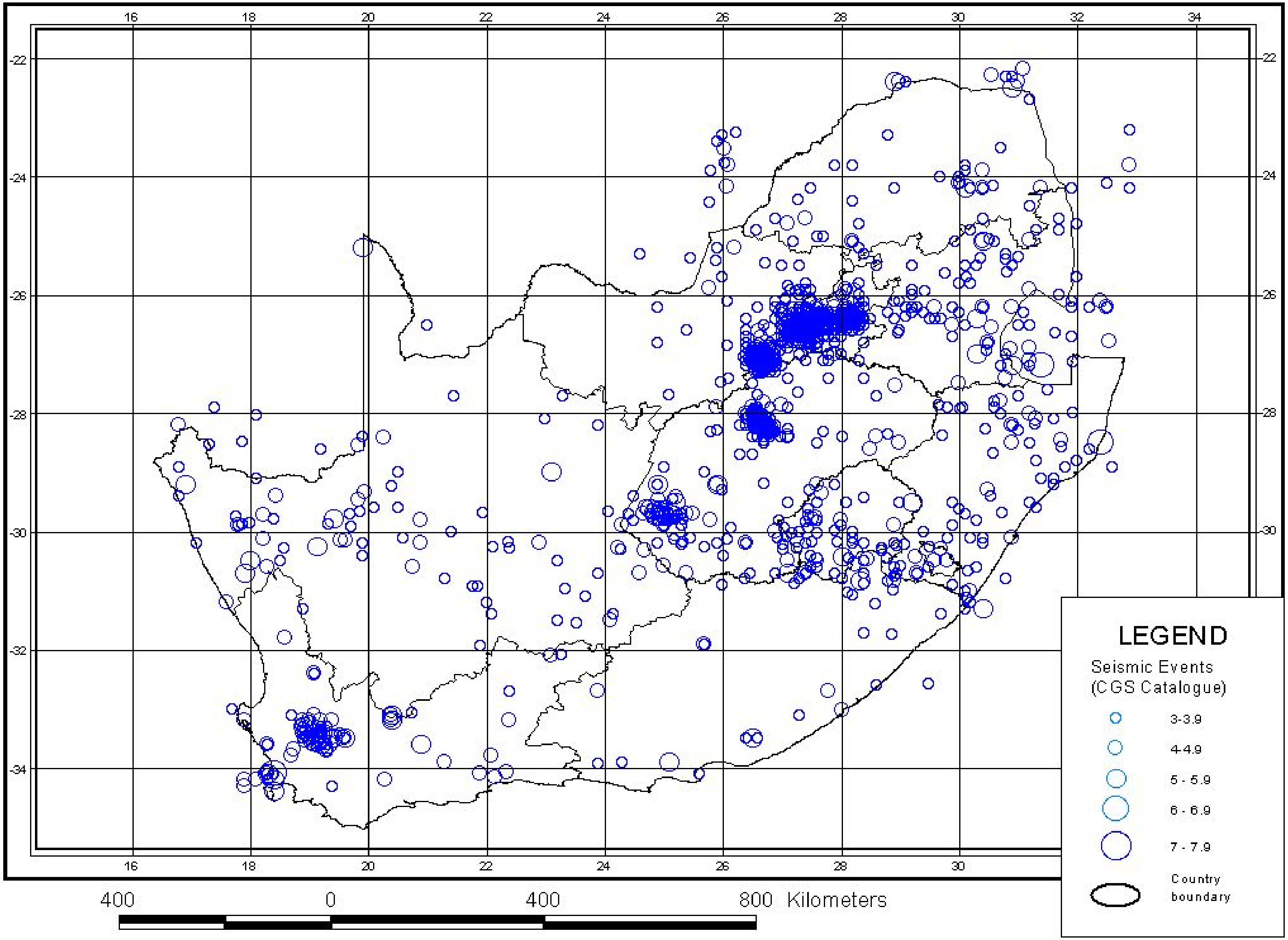
Seismic Hazard Map of Sub-Saharan Africa

10% Probability of exceedance of PGA in 50 years



SOUTH AFRICA
Seismological Monitoring Network





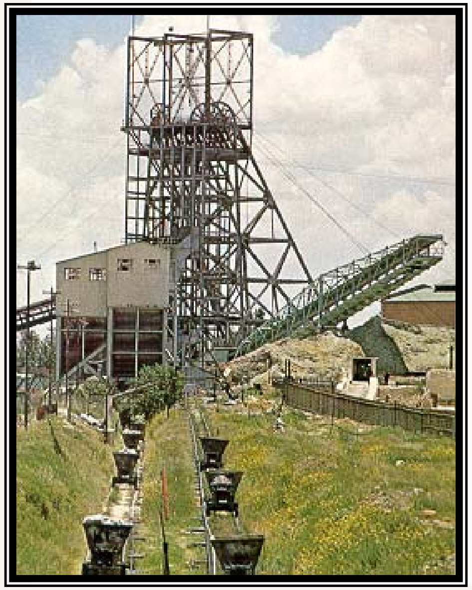
LEGEND
Seismic Events
(CGS Catalogue)
3-3.9
4-4.9
5 - 5.9
6 - 6.9
7 - 7.9
Country
boundary
400
0
400
800 Kilometers

Gold and Diamonds

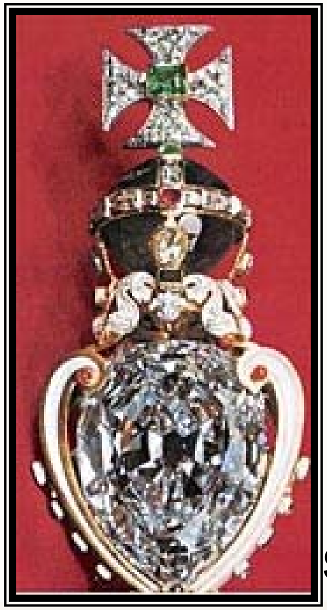
Gold pouring

Krugerrand

Kimberley Big Hole

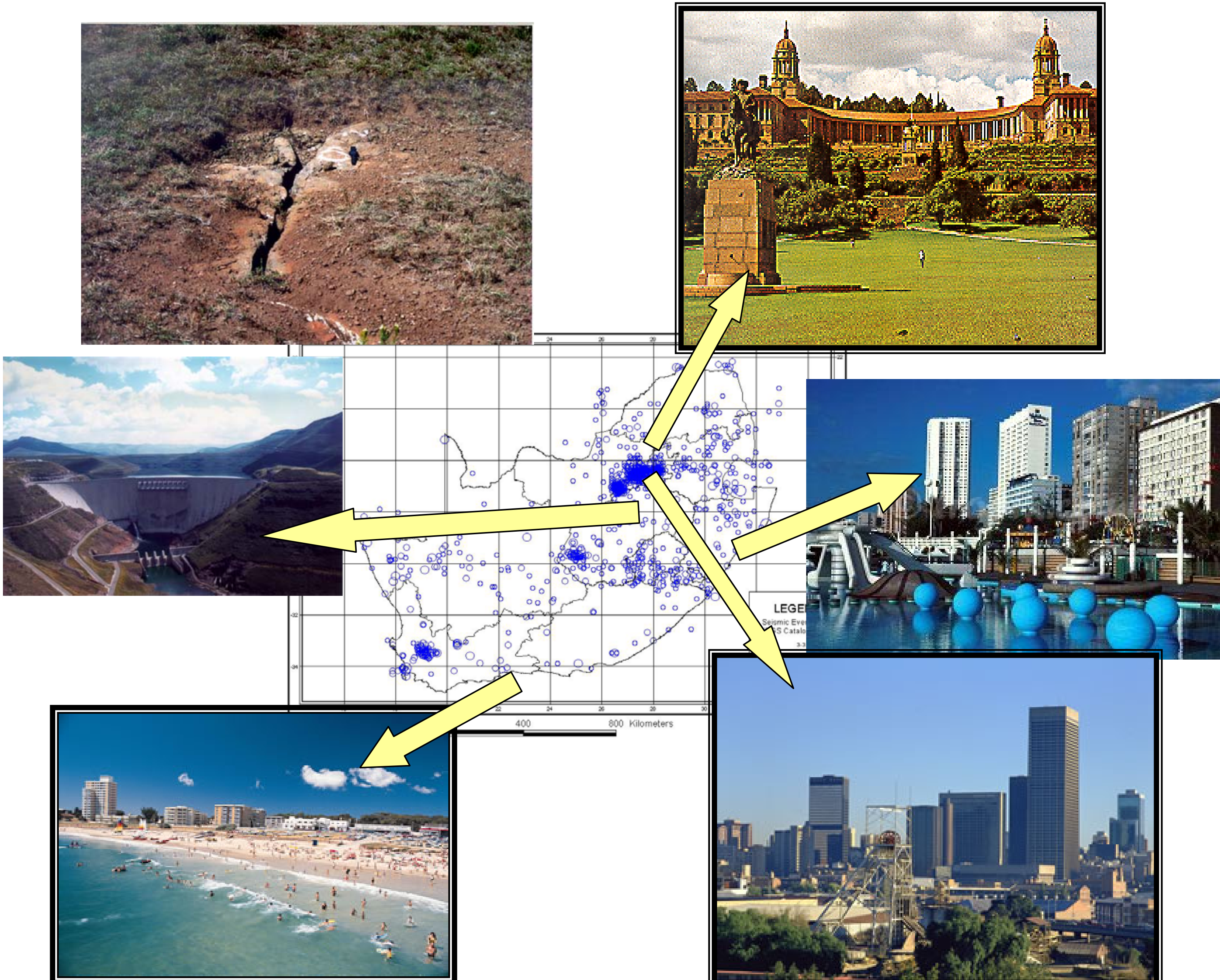


Mine Shaft



Star of Africa

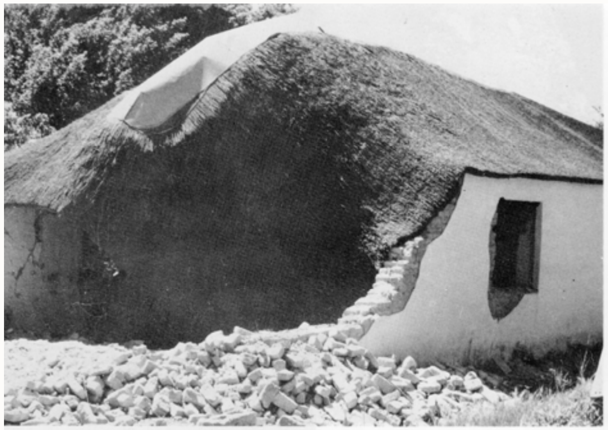
Diamonds


400
800 Kilometers

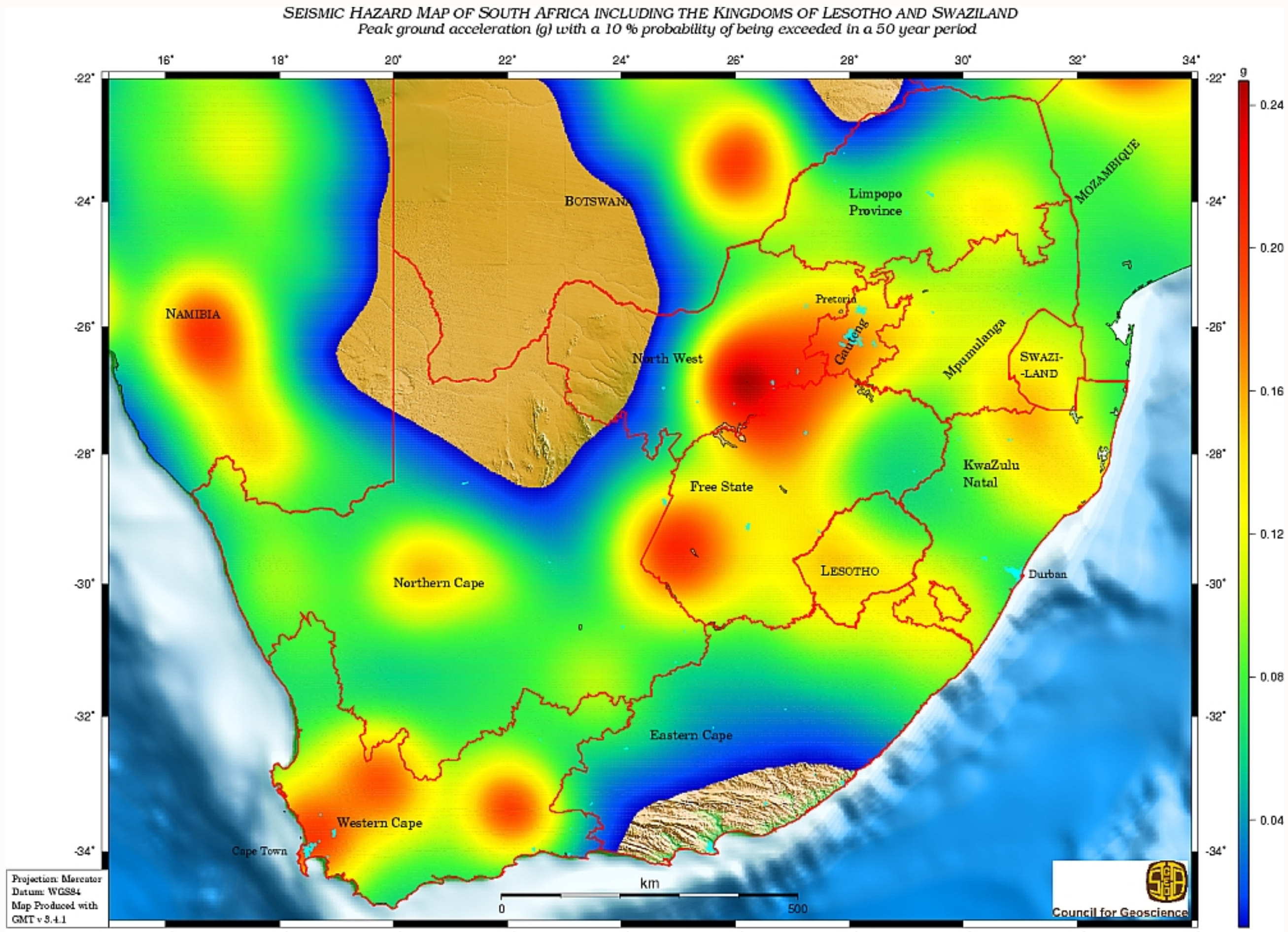
Magnitude = 5.2 at Welkom 1976

Stilfontein, 9 March 2005, M_L = 5.3

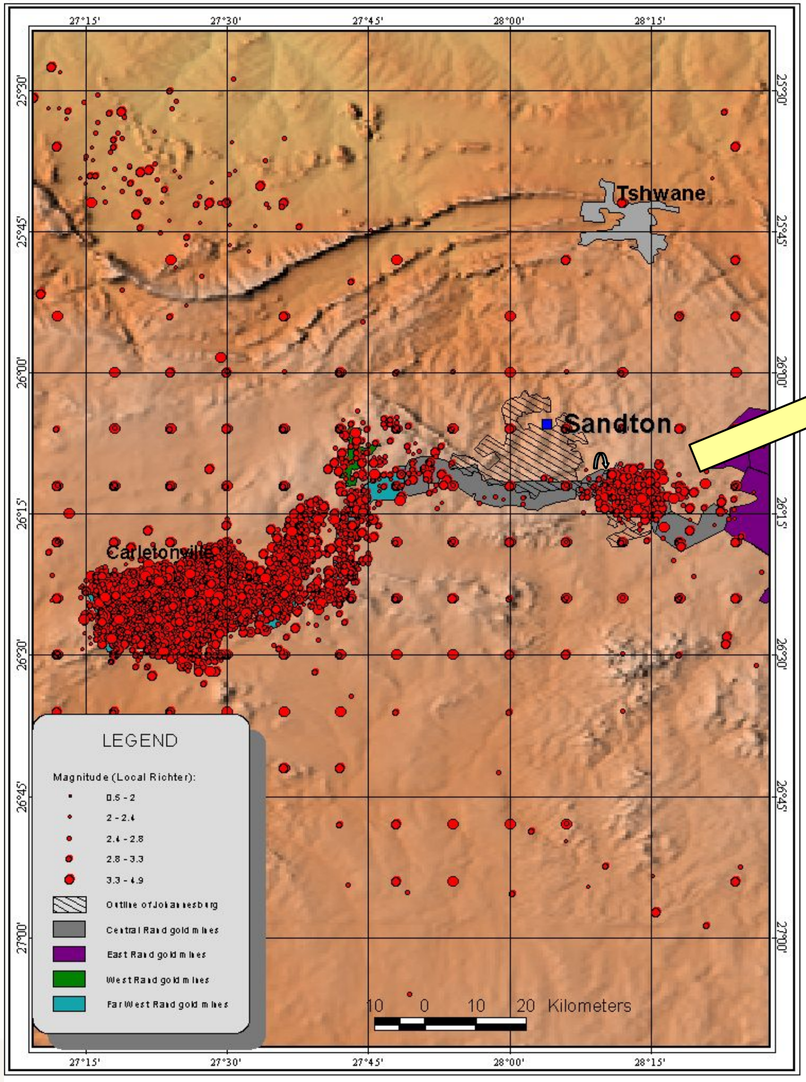
Tulbagh, 29 September 1969, M_L = 6.3



Seismic Hazard Map of South Africa



Maximum Possible Earthquake magnitude in Johannesburg/Pretoria?



Definition of m_{max}

- The maximum regional magnitude, m_{max}, is the upper limit of magnitude for a given region

$$\hat{m}_{\max} = m_{\max}^{obs} + \Delta$$

The Generic Formula for Estimation of the Maximum Regional Magnitude, m_{max}

$$\hat{m}_{\max} = m_{\max}^{obs} + \int_{m_{\min}}^{m_{\max}} \left[F_M(m) \right]^n \mathrm{d}m$$

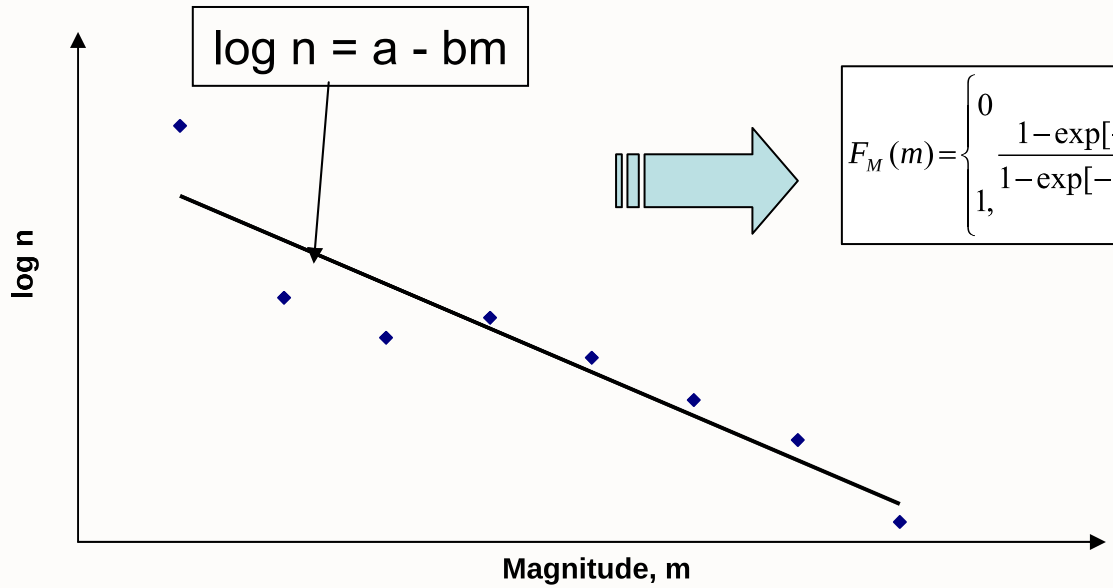
THREE REAL LIFE CASES

1. Earthquake Magnitudes are Distributed according to the Gutenberg-Richter relation

2. Earthquake Magnitude Distribution deviates largely from the Gutenberg-Richter relation

3. No specific model for The Earthquake Magnitude Distribution is assumed

CASE 1: Earthquake Magnitudes are Distributed according to the Gutenberg-Richter relation



$$F_M(m) = \begin{cases} 0 & for\ m < m_{\min}, \\ \dfrac{1-\exp[-\beta(m-m_{\min})]}{1-\exp[-\beta(m_{\max}-m_{\min})]}, & for\ m_{\min} \le m \le m_{\max} \\ 1, & for\ m > m_{\max}. \end{cases}$$

where $\beta = b \ln 10$

Case 1

$$\hat{m}_{\max} = m_{\max}^{obs} + \frac{E_1(n_2) - E_1(n_1)}{\beta \exp(-n_2)} + m_{\min} \exp(-n)$$

where

$$n_1 = \frac{n}{\{1 - \exp[-\beta(m_{\max}^{obs} - m_{\min})]}$$

$$n_2 = n_1 \exp[-\beta(m_{\max}^{obs} - m_{\min})]$$

And $E_1(\bullet)$ denotes an exponential integral function

Case 2: The Earthquake Magnitude Distribution deviates largely from the Gutenberg-Richter relation

distinctly nonlinear recurrence relationships predicted by this model (Figure 9). Examples include subduction zones in Alaska (Utsu 1971; Purcaru, 1975; Lahr and Stephens, 1982; Davison and Scholz, 1984) and Mexico (Singh *et al.*, 1981, 1983), and crustal faults in Turkey, Sweden and Greece (Båth, 1981, 1982, 1983), Japan (Wesnousky *et al.*, 1988), and the New Madrid region of the Central United States (Main and Burton, 1984). Note that the recurrence data shown in Figure 9 are reproduced directly from the original publications. These observational data suggest

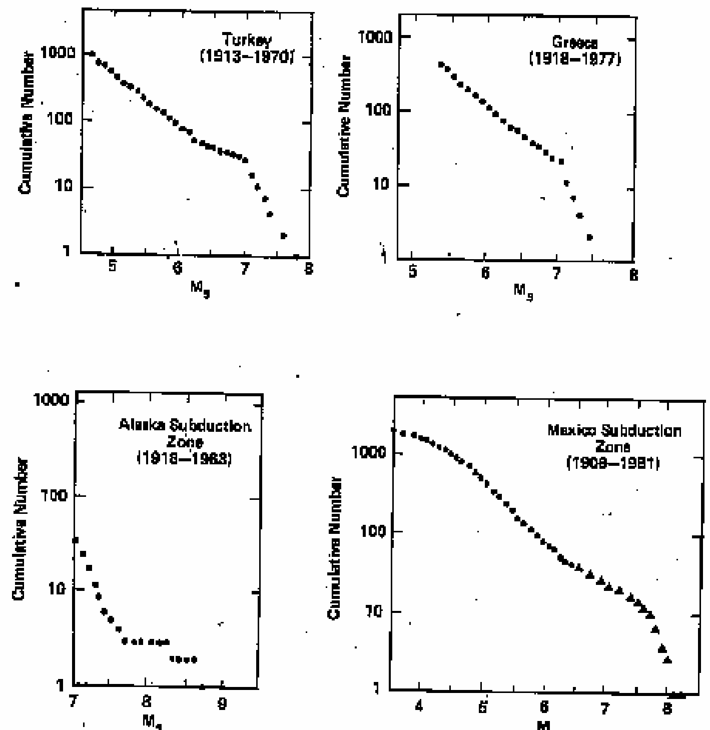


FIG. 9. Frequency magnitude plots based on historical and instrumental seismicity in the Alaska subduction zone (Utsu, 1971), the Mexican subduction zone (Singh *et al.*, 1983), Greece (Båth, 1983), and Turkey (Båth, 1981). On the plot for the Mexican subduction zone, the triangles represent data for the period 1906 to 1981 (75.5 yr); the circles are data from 1963 to 1981 normalized to 75.5 yr. The plots are reproduced directly from the original publications. Note the significant departure from a log-linear relationship.

that the magnitude range or increment of the characteristic earthquake is about one-half magnitude unit, and that the increment between the minimum characteristic magnitude and the portion of the recurrence curve showing exponential behavior at recurrence rates greater than the rate for characteristic events is about one magnitude unit (Figure 9). In other words, the magnitude range showing nonexponential behavior in a cumulative plot is about 1.5 magnitude units. This is in general agreement with the model proposed by Singh *et al.* (1983) whereby they

Case 2: The Earthquake Magnitude Distribution deviates largely from the Gutenberg-Richter relation

Earthquake magnitude distribution follow the Gutenberg-Richter relation with some uncertainty in the β value

$$F_M(m) = \begin{cases} 0 & for\ m < m_{\min} \\ C_{\beta)}\{1 - [p/(p + m - m_{\min})]^q\}, & for\ m_{\min} \le m \le m_{\max} \\ 1 & for^{\beta}\ m > m_{\max} \end{cases}$$

Case 2

$$\hat{m}_{\max} = m_{\max}^{obs} + \frac{\delta^{1/q} \exp[nr^q/(1-r^q)]}{\beta}\left[\Gamma(-1/q, \delta r^q) - \Gamma(-1/q, \delta)\right]$$

This formula can be used where there exists temporal trends, cycles, short-term oscillations and pure random fluctuations in the seismic process

where

$$\delta = nC_\beta, \qquad C_\beta = \frac{1}{1-r^q}, \qquad r = \frac{p}{p + m_{\max} - m_{\min}},$$

$$p = \bar{\beta}/(\sigma_\beta)^2, \qquad q = (\bar{\beta}/\sigma_\beta)^2$$

$\Gamma(\bullet)$ is the incomplete delta function and σ_β is the known standard deviation of β

What does one do when the emperical distribution of earthquake magnitude is of the following form...

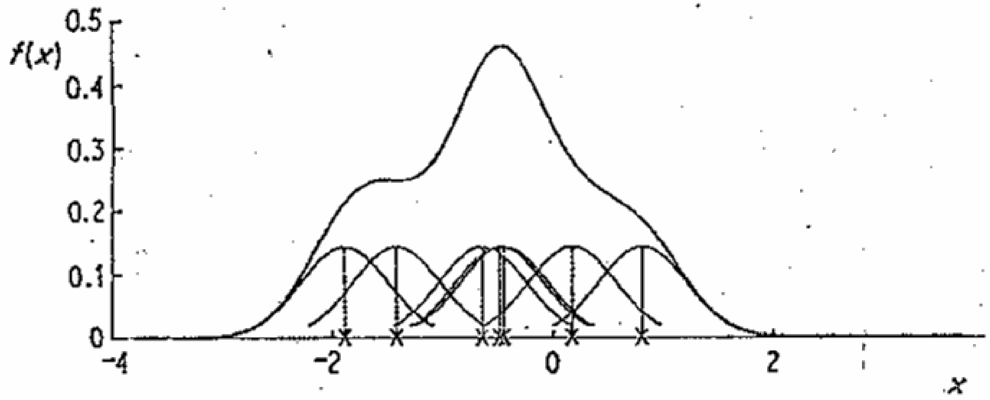
Case 3: No specific model for The Earthquake Magnitude Distribution is assumed

- A non-parametric estimator of an unknown PDFfor sample data *mi* , *i*=1,...*n*, :

$$\hat{f}_M(m) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{m - m_i}{h}\right)$$

- Where h is a smoothing factor and $K(\bullet)$ is a kernel function

Case 3: Continued…



Case 3: Continued...

- From the functional form of the kernel and from the fact that the data comes from a finite interval, one can derive the estimator of the CDF of earthquake magnitude:

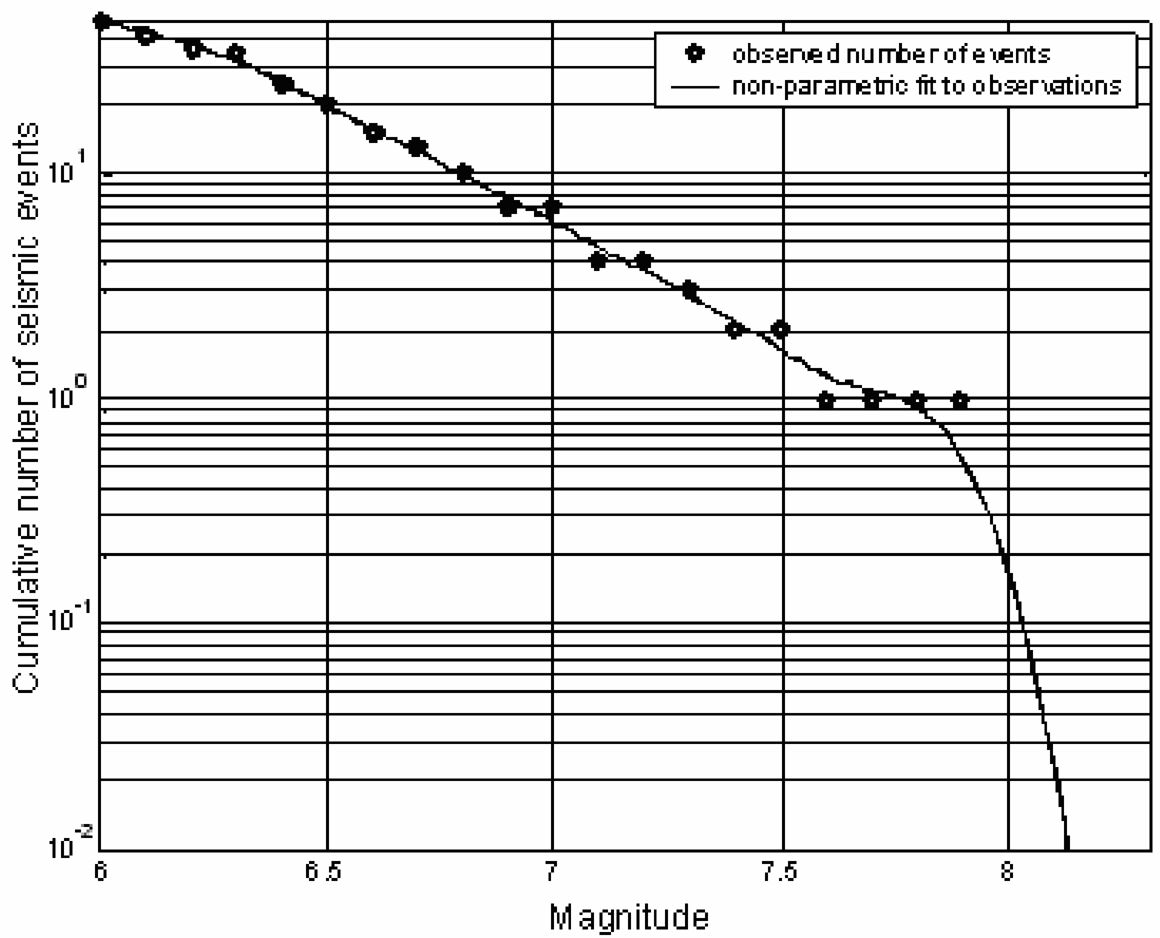
$$\hat{F}_M(m) = \begin{cases} 0, & \text{for } m < m_{\min,} \\ \dfrac{\sum_{i=1}^{n} \phi\left(\dfrac{m - m_i}{h}\right) - \phi\left(\dfrac{m_{\min} - m_i}{h}\right)}{\sum_{i=1}^{n} \phi\left(\dfrac{m_{\max} - m_i}{h}\right) - \phi\left(\dfrac{m_{\min} - m_i}{h}\right)}, & \text{for } m_{\min} \le m \le m_{\max} \\ 1, & \text{for } m > m_{\max} \end{cases}$$

- Where $\phi(\bullet)$ denotes the standard Gaussian CDF

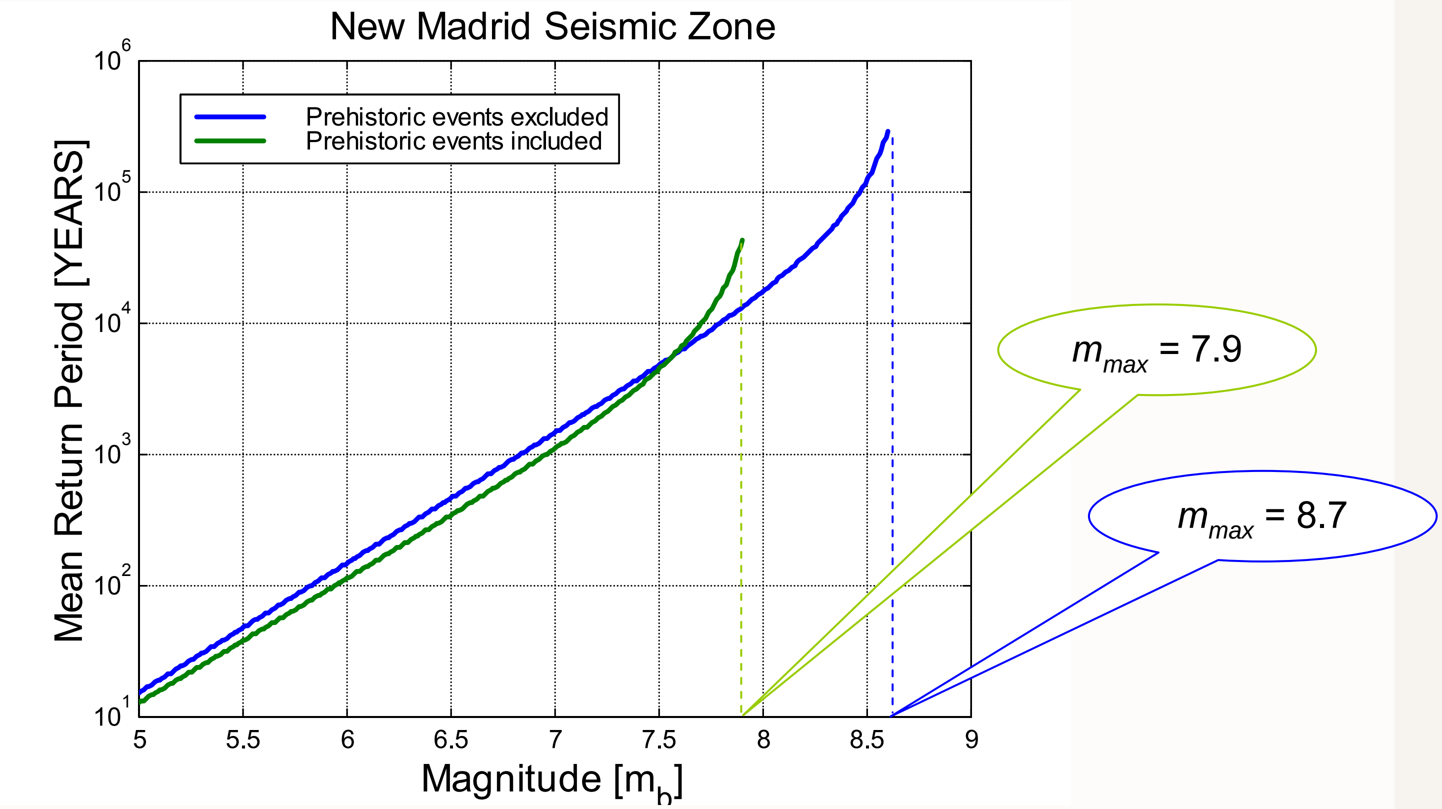
Application 1: Southern California

CASE	Assumptions	$m_{max} \pm SD$
1	Gutenberg-Richter	8.32 ±0.43
2	Gutenberg-Richter+ Uncertainty in b-value	8.31 ±0.42
3	No model for distribution is assumed (Non-parametric procedure)	8.34 ±0.45
	Field et al. 1999	7.99

Fitting of Observations using the Non-Parametric Procedure



Application 2: New Madrid Zone



CONCLUSIONS

- It is possible to develop a lot of useful tools which is capable of taking even the most **diverse** behaviour of seismic activity into account
- A lot needs to be done to improve it.

THE END

THANK YOU