Electrostatic Drift waves Driven Zonal-Flows in Electron-Positron-Ion Plasma

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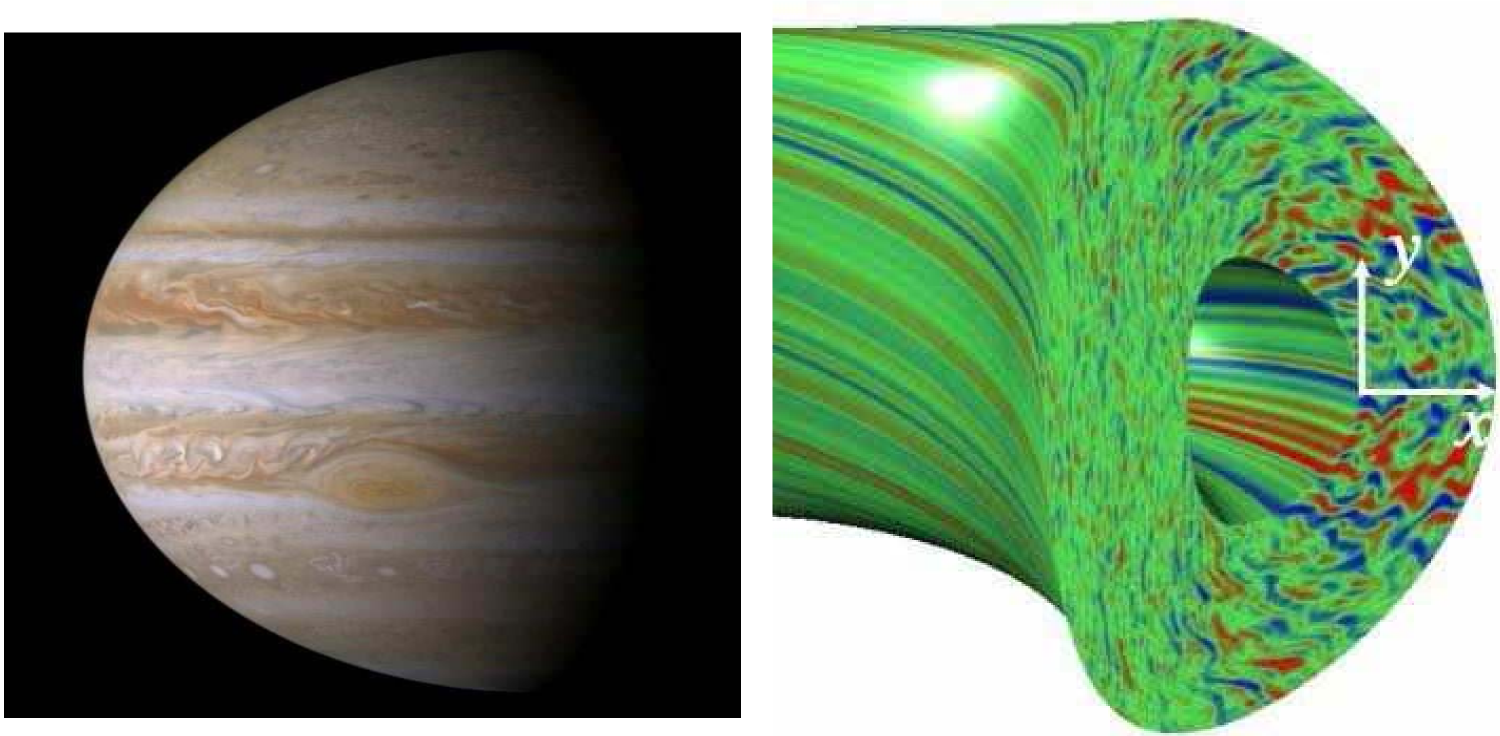

Fig. 1. Left panel shows NASA image PIA04866: Cassini Jupiter Portrait, a mosaic of 27 images taken in December 2000 by the Cassini spacecraft. Right panel shows a simulation of plasma potential fluctuations in a tokamak cut at a fixed toroidal angle as produced by the GYRO code (courtesy Jeff Candy http://fusion.gat.com/theory/pmp/). Note that, in the plasma case, zonal flows are in the y-direction when slab geometry is used.

1. Linear theory of electrostatic drift waves

Let us consider the two dimensional motion of a quasi-neutral EPI plasma. We consider a local perturbation of plasma potential $\varphi(t, x, y)$ and assume that plasma is uniform along the z-axis which is parallel to the external magnetic field $\mathbf{B}_0$.

The unperturbed plasma densities of electrons and positrons are $n_{e0}(x)$ and $n_{p0}(x)$ with corresponding temperatures $T_{eo}(x)$ and $T_{po}(x)$ are inhomogeneous.

The ions are considered to be cold. The quasi-neutrality condition is satisfied

$$n_{e0}(x) = Zn_{i0}(x) + n_{p0}(x)$$

Electrons and positrons follow the Boltzmann equilibrium. The drift wave regime in plasma takes place when $\frac{\omega}{\omega_{ci}} << 1$ **.**

We consider small-scale vortex structures $a/r_s \ll 1$ **where** a **is the perpendicular size of structure and**

$$r_s = (T_e/M)^{1/2}/\omega_{ci}, \qquad \omega_{ci} = ZeB_0/M$$

The linear equation for electrostatic drift waves is

$$-\frac{Ze}{M\omega_{ci}}\left(1-\frac{n_{p0}}{n_{e0}}\right)\frac{\partial\nabla_\perp^2\varphi}{\partial t}+\frac{e\omega_{ci}}{T_e}\left(1+\frac{n_{p0}}{n_{e0}}\frac{T_e}{T_p}\right)\frac{\partial\varphi}{\partial t}-\frac{Ze}{M}\frac{n'_{e0}-n'_{p0}}{n_{e0}}\frac{\partial\varphi}{\partial y}$$

$$-\frac{Ze}{M\omega_{ci}}\frac{n'_{e0}-n'_{p0}}{n_{e0}}\frac{\partial^2\varphi}{\partial t\partial x}=0. \qquad (1)$$

We keep here last small term because it gives new space structure of the electrostatic drift waves and necessary dispersion also. Thus the electrostatic drift waves under consideration now have the following structure and dispersion

$$\varphi \sim e^{\mp \frac{x}{2L}} e^{ik_x x + ik_y y - i\omega_k t}, \qquad (2)$$

and

$$\omega_{\mathbf{k}} = \frac{\mp k_y v_*}{\beta + r_s^2 \left(k_\perp^2 + 1/4L^2\right)}. \qquad (3)$$

$$k_\perp{}^2 = k_x{}^2 + k_y{}^2,\ r_L = \left(T_e / M\omega_{Bi}{}^2\right)^{\frac{1}{2}}$$

Here $v_* = T_e / \mathrm{M}\omega_{ci} L$ **is drift velocity,**

$$\beta = n_{eo}\left(1 + \frac{n_{po}}{n_{eo}}\frac{T_e}{T_p}\right) \Big/ Z\left(n_{eo} - n_{po}\right) \qquad (4)$$

We have introduced the inverse inhomogeneity length

$$L^{-1} = \left|n_{i0}^{'}/n_{i0}\right| = \left|n_{e0}^{'} - n_{p0}^{'}\right| / \left(n_{e0} - n_{p0}\right). \qquad (5)$$

and $\mp$ sign corresponds to positive and negative signs of n_{b0}^{2} / n_{i0}. (The latter can change sign depending on the difference between n_{e0} and n_{p0}).

2. Nonlinear theory of electrostatic drift waves

Nonlinear vortical solitary dipolar structures of the electrostatic drift waves are described by the following classical Hasegawa-Mima equation

$$-\frac{Ze}{M\omega_{ci}}\left(1-\frac{n_{p0}}{n_{e0}}\right)\frac{\partial\nabla_\perp^2\varphi}{\partial t}+\frac{e\omega_{ci}}{T_e}\left(1+\frac{n_{p0}}{n_{e0}}\frac{T_e}{T_p}\right)\frac{\partial\varphi}{\partial t}-\frac{Ze}{M}\frac{n'_{e0}-n'_{p0}}{n_{e0}}\frac{\partial\varphi}{\partial y}$$

$$-\frac{Ze}{M\omega_{ci}}\frac{n'_{e0}-n'_{p0}}{n_{e0}}\frac{\partial^2\varphi}{\partial t\partial x}-\frac{Z^2e^2}{M^2\omega_{ci}^2}\left(1-\frac{n_{p0}}{n_{e0}}\right)J(\varphi,\nabla_\perp^2\varphi)=0. \quad (6)$$

$$\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$$

$$J(a,b) = (\partial a/\partial x)(\partial b/\partial y) - (\partial a/\partial y)(\partial b/\partial x)$$

For necessary calculations we represent the perturbed potential as

$$\varphi(x,y,t) = e^{\mp\frac{x}{2L}}\ \bar{\varphi}(x,y,t) \quad (7)$$

.

Then Hasegawa-Mima equation can be rewritten as

$$\beta\frac{\omega_{ci}}{T_e}\frac{\partial\varphi}{\partial t} - \frac{1}{M\omega_{ci}}\frac{\partial}{\partial t}\left(\nabla_\perp^2 - \frac{1}{4L^2}\right)\varphi \mp \frac{1}{ML}\frac{\partial\varphi}{\partial y} - \frac{Ze}{M^2\omega_{ci}^2}J(\varphi, \nabla_\perp^2\varphi) = 0 \quad (8)$$

The factor $\exp(\mp x/2L)$, as well as bar over φ has been omitted

3. Non-linear Interactions of Drift Waves and Zonal-Flows in EPI Plasmas

We consider a standard three wave-coupling process in EPI plasma. The coupling between the pump drift waves and side band modes drives the low frequency large-scale one-dimensional modes propagating along the x-axis i.e. the zonal flows.

We decompose the perturbed electric potential into three parts

$$\varphi = \tilde{\varphi} + \hat{\varphi} + \bar{\varphi}. \qquad (9)$$

The function $\bar{\varphi}$ is taken in the form

$$\bar{\varphi} = \bar{\varphi}_0 \exp\left(-i\Omega t + i q_x x\right) + c.c \quad (10)$$

and amplitude $\bar{\varphi}_0$ of zonal flow mode is assumed to be constant

and

$$\hat{\varphi} = \sum_{\mathbf{k}} \left[\hat{\varphi}_+(\mathbf{k}) \exp(i\mathbf{k}_+ \cdot \mathbf{r} - i\omega_+ t) + \hat{\varphi}_-(\mathbf{k}) \exp(i\mathbf{k}_- \cdot \mathbf{r} - i\omega_- t)\right. \quad (11)$$

Finally,

$$\tilde{\varphi} = \sum_{\mathbf{k}} \left[\tilde{\varphi}_+(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}} t) + \tilde{\varphi}_-(\mathbf{k}) \exp(-i\mathbf{k} \cdot \mathbf{r} + i\omega_{\mathbf{k}} t)\right] \quad (12)$$

where $\hat{\varphi}_\pm(\mathbf{k})$ is the sideband amplitude. The energy and momentum conservation is fulfilled $\omega_\pm = \Omega \pm \omega_{\mathbf{k}}$ and $k_\pm = q_x \mathbf{e}_x \pm \mathbf{k}$.

For the problem of zonal flow generation, the following conditions are satisfied

$$|\Omega/\omega_{\mathbf{k}}| \sim |q_x/\mathbf{k}_\perp| \ll 1$$

The averaging over the fast small-scale fluctuations gives the eqn. for zonal-flow evolution

$$\frac{\partial}{\partial t}\left[\beta\frac{\omega_{ci}}{T_e} - \frac{1}{M\omega_{ci}}\left(\nabla_\perp^2 - \frac{1}{4L^2}\right)\right]\bar{\varphi} = \frac{Ze}{M^2\omega_{ci}^2}\langle J(\varphi, \nabla_\perp^2\varphi)\rangle \quad \textbf{(13)}$$

In terms of Fourier components

$$i\Omega\left[\beta\frac{\omega_{ci}}{T_e} + \frac{1}{M\omega_{ci}}\left(q_x^2 + \frac{1}{4L^2}\right)\right]\bar{\varphi}_0 = -\frac{Zeq_x^2}{M^2\omega_{ci}^2}\sum_{\mathbf{k}} k_y[2k_x(\tilde{\varphi}_+\hat{\varphi}_- + \tilde{\varphi}_-\hat{\varphi}_+) + q_x(\tilde{\varphi}_-\hat{\varphi}_+ - \tilde{\varphi}_+\hat{\varphi}_-)]. \quad \textbf{(14)}$$

The right hand side of this eqn. corresponds to the driving force of the zonal-flow which is known as Reynolds stresses.

For the turbulent part contributions we have

$$\frac{\partial}{\partial t}\left[\beta\frac{\omega_{ci}}{T_e}-\frac{1}{M\omega_{ci}}(\nabla^2{}_\perp-\frac{1}{4L^2})\right]\hat{\varphi}_\pm \mp\frac{1}{ML}\frac{\partial}{\partial y}\hat{\varphi}_\pm=\frac{Ze}{M^2\omega_{ci}{}^2}\left[J\left(\tilde{\varphi}_\pm,\nabla_\perp{}^2\bar{\varphi}_0\right)+J\left(\bar{\varphi}_0,\nabla_\perp{}^2\tilde{\varphi}_\pm\right)\right] \quad \textbf{(15)}$$

Using the expansion over small parameters Ω and q_x the solution of this eqn. can be represented as

$$\hat{\varphi}_\pm \approx i\frac{Zek_y q_x k_\perp^2}{M^2\omega_{ci}^2 CD}\left[\pm 1+\frac{q_x^2 V_g^{'}}{2D}-\frac{2k_x q_x}{M\omega_{ci}C}\right]\bar{\varphi}_0\tilde{\varphi}_\pm . \quad \textbf{(16)}$$

Here

$$D = \Omega - q_x V_g(\mathbf{k}),\ C = \beta\omega_{ci}/T_e + M^{-1}\omega_{ci}^{-1}\left(k_\perp^2 + (2L)^{-1}\right)$$

$$V_g(\mathbf{k}) = \frac{\partial \omega_{\mathbf{k}}}{\partial k_x} = -2\frac{k_x \omega_{\mathbf{k}}}{M\omega_{ci} C}$$

and

$$V_g'(\mathbf{k}) = \frac{\partial V_g}{\partial k_x} = -2\frac{\omega_{\mathbf{k}}}{M\omega_{ci} C}\left(1 - \frac{4k_x^2}{M\omega_{ci} C}\right).$$

Substituting into the basic eqn. we arrive at the following zonal-flow dispersion relation

$$1 - \sum_{\mathbf{k}} \frac{F(\mathbf{k})}{\left[\Omega - q_x V_g(\mathbf{k})\right]^2} = 0, \qquad (17)$$

where

$$F(\mathbf{k}) = \frac{Z^2 e^2 q_x^4 k_y^2 k_\perp^2 V_g'(\mathbf{k})}{M^3 \omega_{\mathrm{ci}}^3 \omega A} |\tilde{\varphi}_+|^2$$

and

$$A = \beta \frac{\omega_{\mathrm{ci}}}{T_e} + \frac{1}{M\omega_{\mathrm{ci}}} \left(q_x^2 + \frac{1}{4L^2} \right)$$

4. Zonal-flow Instabilities in case of monochromatic wave packet

In the case of the monochromatic wave packet, $F(\mathbf{k}) \sim \delta(\mathbf{k} - \mathbf{k}_0)$ **and we get hydrodynamic-type coherent instability described by the eqn.**

$$(\Omega - q_x V_g)^2 = F(\mathbf{k}_0) = -\Gamma^2 ,$$

where

$$\Gamma^2 = -\frac{Z^2 e^2 q_x^4 k_{y0}^2 k_{\perp 0}^2 V_g'(\mathbf{k}_0)}{2M^3 \omega_{ci}^3 A \omega_{\mathbf{k}_0}} I_{\mathbf{k}_0}, \qquad \textbf{(18)}$$

and

$$I_{\mathbf{k}0} = 2\tilde{\varphi}_+ \tilde{\varphi}_- = 2\,|\tilde{\varphi}_+|^2$$

The necessary condition for instability is $\frac{V'_{g0}}{\omega_{\mathbf{k}0}} < 0$, **then instability condition becomes**

$$\beta \frac{M\omega_{ci}^2}{T_e} + k_y^2 - 3k_x^2 > 0. \qquad \textbf{(19)}$$

When $k_\perp r_s \gg 1$, **in this approximation**

$$\frac{V_g'}{\omega_{\mathbf{k}0}} = -\frac{2}{k_\perp^4}\left(k_y^2 - 3k_x^2\right) > 0 \qquad \textbf{(20)}$$

Thus, the instability condition applies to drift waves with the wave vectors in the cone $-k_y/\sqrt{3} < k_y/\sqrt{3}.$

The maximum growth rate is obtained at the axis of the cone when $k_x = 0.$ **In this case, the mode is purely growing with the growth rate**

$$\Gamma = \frac{q_x^{\,2} \omega_{ci} |k_{y0}| r_s^{\,3}}{\left[\beta + r_s^{\,2}\left(q_x^{\,2} + 1/4L^2\right)\right]^{1/2}} I_{k0}^{\,1/2} \qquad (21)$$

Here $\tilde{\varphi}_+$ is normalized to the value T_e/Ze.

For $q_x r_s \sim 1$ we can estimate the growth rate as

$$\Gamma \approx \omega_{ci} |k_{y0}| r_s I_{k0}^{1/2} \qquad (22)$$

This estimation shows that Γ increases as k in the short wavelength limit $(k_\perp r_s \gg 1)$.

5. Zonal-flow Instabilities in case of non-monochromatic wave packet

Consider a single non-monochromatic packet of drift waves, taking the spectrum of $I_{\mathbf{k}}$ in the Gaussian form

$$I_{\mathbf{k}} = \frac{1}{\pi^{1/2}|\Delta k_x|} \exp\left(-\frac{(k_x - k_{x0})^2}{(\Delta k_x)^2}\right) I_{\mathbf{k}0} \qquad \textbf{(23)}$$

$k_y = k_{y0}$ **The summation over $\mathbf{k}$ in dispersion relation is now understood as the integral over k_x. Then $\omega = \omega(k_x)$ and $V_g = V_g(k_x)$ Let us consider the case when**

$$\Delta k_x / k_{x0} \ll 1$$

Then expand V_g in a series in the vicinity of k_{x0}, then dispersion relation becomes as

$$\hat{\Omega}^2 = (\Omega - q_x V_{g0})^2 = -\Gamma^2 \left(1 + \frac{3}{2}\frac{(q_x V'_{g0})^2}{\hat{\Omega}^2}(\Delta k_x)^2\right) \qquad (24)$$

One can see that weak spectrum broadening leads to decrease of the growth rate of hydrodynamic instability. Spectrum broadening can be neglected only if

$$|\Delta k_x / k_{x0}| < |\Gamma / q_x V_{g0}|$$

When broadening of the wave packet is arbitrary, we get the following zonal flow dispersion relation

$$\hat{\Omega} = i\frac{|q_x V'_{g0} \Delta k_x|}{\sqrt{\pi}} \left(1 - \frac{(q_x V'_{g0} \Delta k_x)^2}{2\Gamma^2}\right). \qquad (25)$$

Then we can find the instability condition

$$\Gamma^2 > (q_x V'_{g0} \Delta k_x)^2 / 2.$$

The growth rate obtained from eqn. 5.3 attains the maximum when

$$|\Delta k_x| = \left(\frac{2}{3}\right)^{\frac{1}{2}} \left|\frac{\Gamma}{q_x V'_{g0}}\right|. \qquad (26)$$

Conclusion

- **Generation of the zonal-flow by small scale electrostatic drift waves in EPI plasma is investigated.**
- **Our investigation provides an important nonlinear mechanism for the transfer of spectral energy from small-scale drift waves to the large scale enhanced zonal flows in EPI plasmas.**
- **Explicit expressions for the maximum growth are obtained.**
- **In contrast to usual electron-ion plasma existence of positrons in the plasma causes modification of both the zonal-flow growth rate and instability conditions.**

Thanks