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Electroweak Interactions in the SM and Beyond

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A short course on the EW Theory

We start from the basic principles and formalism (a fast recall).
Then we go to present status and challenges

Content

- Formalism of gauge theories
- The SU(2)xU(1) symmetric lagrangian
- The symmetry breaking sector
- Beyond tree level
- Precision tests
- Problems of the SM
- Beyond the SM

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General formalism of non abelian gauge theories

$\Phi_a(x)$: multiplet of fields (a=1,2,…,n)

Internal symmetry: $\Phi_a(x) \rightarrow \Phi'_a(x) = U_{ab}\,\Phi_b(x)$

internal: x unchanged

$\Phi' = U\Phi$

A=1,2,…,N

$U = \exp[i\sum_A t^A \varepsilon^A] \sim 1 + i\sum_A t^A \varepsilon^A + o(\varepsilon^2)$

Infinitesimal transformation

t^A: generators
ε^A: parameters

Generators may: commute — abelian
not commute — non abelian

$[t^A, t^B] = iC_{ABC}t^C$

C_{ABC}: structure constants
define the group
depend on normalisation

$\mathrm{Tr}\, t^A t^B = 1/2\, \delta_{AB}$
in fund. repres.

$\mathrm{Tr}[t^A, t^B]\, t^C = \frac{i}{2} C_{ABC}$ → compl. antisymmetric

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$U = \exp[i\Sigma_A t^A \varepsilon^A]$

Global symm.: ε^A constant
Local or gauge symm.: $\varepsilon^A = \varepsilon^A(x)$

Consider a lagrangian density invariant under a global symmetry:

$$L[\Phi, \partial_\mu \Phi] = L[\Phi', \partial_\mu \Phi'] = L[U\Phi, \partial_\mu U\Phi]$$

In general it is not invariant under gauge symmetry:

$$\partial_\mu(U\Phi) = U(\partial_\mu \Phi) + (\partial_\mu U)\Phi \neq U(\partial_\mu \Phi)$$

But $L[\Phi, D_\mu \Phi]$ is gauge invariant if $(D_\mu \Phi)' = U(D_\mu \Phi)$

D_μ is the covariant derivative, a linear operator that generalizes ∂_μ → $D_\mu = \partial_\mu + ig\sum_A t^A V^A_\mu(x) = \partial_\mu + igV_\mu(x)$

gauge fields

Def.: $V_\mu = \sum_{A=1}^{N} t^A V^A_\mu$

Solution: $V'_\mu = UV_\mu U^{-1} - \frac{1}{ig}(\partial_\mu U)U^{-1}$

This is how the gauge fields must transform

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$$D_\mu = \partial_\mu + ig\sum_A t^A V^A_\mu(x) = \partial_\mu + igV_\mu(x)$$

$$V'_\mu = UV_\mu U^{-1} - \frac{1}{ig}(\partial_\mu U)U^{-1}$$

Here is the proof that $(D_\mu\Phi)' = U(D_\mu\Phi)$

$$(D_\mu\Phi)' = (\partial_\mu + igV_\mu')\Phi' =$$

$$= [\partial_\mu + igUV_\mu U^{-1} - (\partial_\mu U)U^{-1}]U\Phi =$$

$$= U\partial_\mu\Phi + (\partial_\mu U)\Phi + igUV_\mu\Phi - (\partial_\mu U)\Phi =$$

$$= U(\partial_\mu + igV_\mu)\Phi = U(D_\mu\Phi)$$

Note: The abelian case (QED) $\quad U = \exp[iQ\varepsilon(x)]$ ← Electric charge

$$V_\mu = \sum_{A=1}^{N} t^A V^A_\mu \rightarrow QV_\mu \rightarrow QV'_\mu = QV_\mu - \frac{1}{ie}\cdot iQ\partial_\mu\varepsilon(x)e^{iQ\varepsilon}\cdot e^{-iQ\varepsilon}$$

$g = e$

finally:

Ordinary gauge invariance for the photon

$$V'_\mu = V_\mu - \frac{1}{e}\cdot\partial_\mu\varepsilon(x)$$

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Kinetic term for V^A_μ

$[D_\mu, D_\nu]\Phi \equiv igF_{\mu\nu}\Phi$

From $(D_\mu\Phi)' = U(D_\mu)\Phi$
one gets $(F_{\mu\nu}\Phi)' = U\, F_{\mu\nu}\Phi$
or $F_{\mu\nu}'\Phi' = U\, F_{\mu\nu}U^{-1}U\Phi$

$$F_{\mu\nu}' = U\, F_{\mu\nu}U^{-1}$$

Adjoint representation

Thus:

$$\mathrm{Tr}\, F_{\mu\nu}'\, F^{\mu\nu\prime} = \mathrm{Tr}\, U\, F_{\mu\nu}U^{-1}\, U\, F^{\mu\nu}U^{-1} = \mathrm{Tr}\, U^{-1}\, U\, F_{\mu\nu}F^{\mu\nu} = \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$$

Note: $F_{\mu\nu} = \Sigma_A F^A_{\mu\nu} t^A$ and

$$\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} = \Sigma_{A,B} F^A_{\mu\nu} F^{B\mu\nu} \underbrace{\mathrm{Tr}\, t^A t^B}_{1/2\ \delta^{AB}} = 1/2\,\Sigma_A F^A_{\mu\nu} F^{A\mu\nu}$$

Thus a gauge invariant lagrangian is given by:

$$L_{YM} = -1/2\ \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + L[\Phi, D_\mu\Phi]$$

Yang, Mills

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$$[D_\mu, D_\nu]\Phi = igF_{\mu\nu}\Phi$$

$$[\partial_\mu + igV_\mu, \partial_\nu + igV_\nu]\Phi = ig\{\partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]\}\Phi$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]$$

or, from $F_{\mu\nu} = \sum_A F^A{}_{\mu\nu} t^A$ and $[t^A, t^B] = iC_{ABC} t^C$

$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - gC_{ABC} V^B_\mu V^C_\nu$$

Note the abelian limit

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

The Electro-Weak Theory

At first sight unification of electromagnetism and of weak interactions looks difficult:

- QED is a vector theory, charged weak currents are V-A, neutral currents are a mixture of V and A

  → violation of C and P

- γ is massless, $W^{\pm}$, Z are very massive

In the SM the first problem is solved by making particles of different chiralities to transform differently:

the SM is a "chiral" theory

The second problem leads to the concept of spontaneously broken gauge symmetry and the Higgs mechanism.

Chirality

ψ : Dirac field

Def.:

$$\begin{cases} \psi_L = \dfrac{1-\gamma_5}{2}\psi \\ \psi_R = \dfrac{1+\gamma_5}{2}\psi \end{cases} \Longrightarrow \begin{cases} \overline{\psi_L} = \psi_L{}^{\dagger}\gamma_0 = \bar{\psi}\dfrac{1+\gamma_5}{2} \\ \overline{\psi_R} = \psi_R{}^{\dagger}\gamma_0 = \bar{\psi}\dfrac{1-\gamma_5}{2} \end{cases}$$

$$\gamma^5=\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \gamma_5{}^+=\gamma_5, \quad \gamma_5{}^2=1, \quad \{\gamma_\mu,\gamma_5\}=0$$

In the Bjorken-Drell basis: $\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}, \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(all entries are 2x2 matrices)

$P_\pm = 1/2(1\pm\gamma_5)$ are projectors:
$P_+P_+=P_+$; $P_-P_-=P_-$; $P_+P_-=P_-P_+=0$;
$P_++P_-=1$

$P_\pm$ project over definite "chirality". For a massless fermion chirality = helicity

$$\bar{\psi}\Gamma\psi = \bar{\psi}\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\Gamma\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\psi$$

Two classes of Dirac matrices:

$\Gamma_C = 1, \gamma_5, \sigma_{\mu\nu}$: commute with γ_5

$$\Longrightarrow \quad \bar{\psi}\Gamma_C\psi = \overline{\psi_L}\Gamma_C\psi_R + \overline{\psi_R}\Gamma_C\psi_L$$

e.g. a mass term

$$\bar{\psi}M\psi = \overline{\psi_L}M\psi_R + \overline{\psi_R}M\psi_L$$

chirality flip

$\Gamma_A = \gamma_\mu, \gamma_\mu\gamma_5$: anticommute with γ_5

$$\Longrightarrow \quad \bar{\psi}\Gamma_A\psi = \overline{\psi_L}\Gamma_A\psi_L + \overline{\psi_R}\Gamma_A\psi_R$$

e.g. cov. derivative term

chirality no-flip

$$\bar{\psi}i\widehat{D}\psi = \overline{\psi_L}i\widehat{D}\psi_L + \overline{\psi_R}i\widehat{D}\psi_R \qquad (\widehat{D} \equiv \gamma_\mu D^\mu)$$

Note:

$$\bar{\psi} M \psi = \overline{\psi_L} M \psi_R + \overline{\psi_R} M \psi_L$$

A mass term can be symmetric only if Ψ_L and Ψ_R have the same transformation properties.

$$\bar{\psi} i \widehat{D} \psi = \overline{\psi_L} i \widehat{D} \psi_L + \overline{\psi_R} i \widehat{D} \psi_R$$

A covariant derivative term can be symmetric also if Ψ_L and Ψ_R have different transformation properties.

In the SM the symmetry group is SU(2)XU(1), but all Ψ_L are SU(2) doublets and all Ψ_R are SU(2) singlets.

$$\begin{bmatrix} u \\ d \end{bmatrix}_L, \quad u_R, \quad d_R \qquad\qquad \begin{bmatrix} \nu \\ e \end{bmatrix}_L, \quad \nu_R\,(?), \quad e_R$$

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The Standard Electro-Weak Theory

$L = L_{symm} + L_{Higgs}$

Glashow, Weinberg, Salam

L_{symm} (introduced by Glashow in '61 for leptons) is a gauge theory for massless fermions based on $SU(2)XU_Y(1)$

$$L_{symm} = -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \overline{\psi_L}i\widehat{D}\psi_L + \overline{\psi_R}i\widehat{D}\psi_R$$

* There is a $\Psi_{L,R}$ term for each quark or lepton multiplet

* $D_\mu\Psi_{L,R} = [\partial_\mu + ig\sum_A t^A_{L,R}W^A_\mu + ig'\cdot\frac{1}{2}Y_{L,R}B_\mu]\Psi_{L,R}$

* $F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu - g\varepsilon_{ABC}W^B_\mu W^C_\nu$

* $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$[t^A,t^B]=i\varepsilon_{ABC}t^C$ ← Levi-Civita SU(2)

$Tr\, t^At^B = 1/2\, \delta^{AB}$ fixes norm of g,g'

Embedding of the electric charge in SU(2)XU(1)

$$Q = t^3_L + \frac{Y_L}{2} = t^3_R + \frac{Y_R}{2}$$

All Ψ_L are weak isospin doublets
All Ψ_R are weak isospin singlets

$Q=t^3+Y/2$

	t^3_L	t^3_R	Y_L	Y_R	Q
u_L	+1/2		1/3		2/3
d_L	−1/2		1/3		−1/3
u_R		0		4/3	2/3
d_R		0		−2/3	−1/3
ν_L	+1/2		−1		0
e_L	−1/2		−1		−1
e_R		0		−2	−1

Gauge couplings to fermions

$$D_\mu = [\partial_\mu + ig\sum_A t^A_{L,R} W^A_\mu + ig' \cdot \frac{1}{2} Y_{L,R} B_\mu]$$

Charged Currents

$$g(t^1 W^1 + t^2 W^2) = g[\frac{t^1 + it^2}{\sqrt{2}} \cdot \frac{W^1 - iW^2}{\sqrt{2}} + hc] = g\left(\frac{t^+ W^-}{\sqrt{2}} + \frac{t^- W^+}{\sqrt{2}}\right)$$

$$t^\pm = t^1 \pm it^2 \qquad W^\pm = \frac{W^1 \pm iW^2}{\sqrt{2}}$$

Putting together L and R:

$$g\bar{\psi}\gamma^\mu\left[\frac{t^+_L}{\sqrt{2}} \cdot \frac{1-\gamma_5}{2} + \frac{t^+_R}{\sqrt{2}} \cdot \frac{1+\gamma_5}{2}\right]\psi W^-_\mu + \text{h.c.}$$

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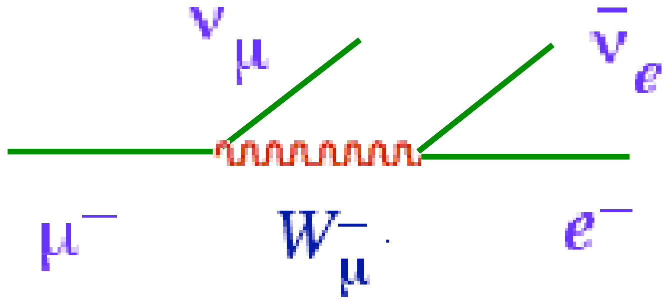
As $t^+_R = 0$ for quarks and leptons, CC are pure V-A

$$\mu^- \Rightarrow \nu_\mu + e^- + \bar{\nu}_e$$



$t^+_R=0$: $$g\bar{\psi}\gamma^\mu\left[\frac{t_L^+}{\sqrt{2}}\cdot\frac{1-\gamma_5}{2}\right]\psi W^-_\mu$$

$$(\nu_\mu,\mu^-)t^+\begin{bmatrix}\nu_\mu\\ \mu^-\end{bmatrix} = (\nu_\mu,\mu^-)\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\nu_\mu\\ \mu^-\end{bmatrix} = \nu_\mu\mu^-$$

$$g^2\overline{\nu_\mu}\gamma^\lambda\frac{(1-\gamma_5)}{2\sqrt{2}}\mu\cdot\frac{1}{q^2-m_W^2}.\bar{e}\gamma_\lambda\frac{(1-\gamma_5)}{2\sqrt{2}}\nu_e$$

negligible (q^2) — we anticipate the W mass (m_W^2)

$$\frac{g^2}{8m_W^2}\overline{\nu_\mu}\gamma^\lambda(1-\gamma_5)\mu.\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e = \frac{G_F}{\sqrt{2}}\overline{\nu_\mu}\gamma^\lambda(1-\gamma_5)\mu.\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e$$

Relation with old Fermi theory (tree level) →

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

$G_F=1.16639(1).10^{-5}$ GeV^{-2}

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Neutral Currents

$$g t^3 W^3 + g' \frac{Y}{2} B$$

Relation with γ and Z:

$$\begin{cases} W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{cases}$$

Def. of $\sin\theta_W$

Photon couplings: pure vector, ~Q

A_μ multiplies: $g \sin\theta_W \cdot t^3 + g' \cos\theta_W \cdot \frac{Y}{2}$

Since $(t^3+Y/2)_{L,R}=Q$ for $g\sin\theta_W = g'\cos\theta_W = e$ or $g'/g = \mathrm{tg}\theta_W$ we obtain:

$$e\bar{\psi}\gamma_\mu[(t^3_L + \frac{Y_L}{2}) \cdot \frac{1-\gamma_5}{2} + (t^3_R + \frac{Y_R}{2}) \cdot \frac{1+\gamma_5}{2}]\psi A^\mu = e\bar{\psi}\gamma_\mu Q\psi A^\mu$$

Neutral Currents

$$g t^3 W^3 + g' \frac{Y}{2} B$$

Relation with γ and Z:

$$\begin{cases} W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{cases}$$

Def. of $\sin\theta_W$

Z couplings are now fixed:

$$g\cos\theta_W \cdot t^3 - g'\sin\theta_W \cdot \frac{Y}{2} = (g\cos\theta_W + g'\sin\theta_W)t^3 - g'\sin\theta_W \cdot Q$$

$\frac{Y}{2}$: Q-t^3

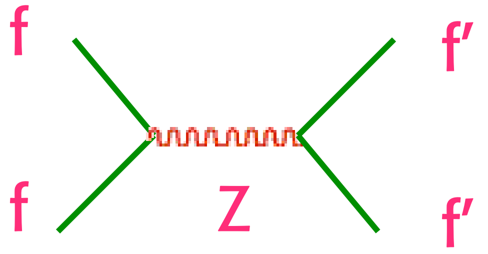
$$g' = g \cdot \frac{\sin\theta_W}{\cos\theta_W}$$

$$= \frac{g}{\cos\theta_W}(t^3 - Q\sin^2\theta_W)$$

Finally: $$\frac{g}{\cos\theta_W}\bar{\psi}\gamma_\mu[t^3_L \cdot \frac{1-\gamma_5}{2} + t^3_R \cdot \frac{1+\gamma_5}{2} - Q\sin^2\theta_W]\psi Z^\mu$$

$$\frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu[t^3_L(1-\gamma_5) + t^3_R(1+\gamma_5) - 2Q\sin^2\theta_W]\psi Z^\mu = \frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu[\ldots]\psi Z^\mu$$

As for CC we can derive the effective 4-fermion interaction at low energies



$$\frac{g^2}{4\cos^2\theta_W}\bar{\psi}\gamma_\mu[\ldots]\psi \cdot \frac{1}{q^2 - m_Z^2} \cdot \bar{\psi}'\gamma^\mu[\ldots]\psi'$$

At $q^2 << m_Z^2$, recalling that $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$

$$L_{eff} = \sqrt{2}G_F \cdot \frac{m_W^2}{m_Z^2\cos^2\theta_W} \cdot \bar{\psi}\gamma_\mu[\ldots]\psi \cdot \bar{\psi}'\gamma^\mu[\ldots]\psi'$$

ρ_0

We shall see that $\rho_0=1$ to a very good approximation.
Thus the intensities of NC and CC processes are comparable

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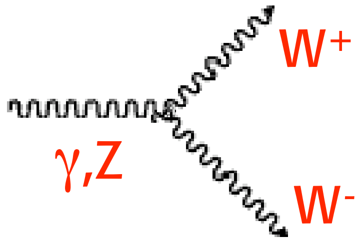
3- and 4-gauge couplings

$$L_{symm} = -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \ \ldots$$

$$F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu - g\varepsilon_{ABC} W^B_\mu W^C_\nu \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

3-gauge coupling: $-\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} \text{--->} 2\cdot 2\cdot \frac{1}{4} g\varepsilon_{ABC}\partial_\mu W^A_\nu W^{\mu B} W^{\nu C}$



must be ε_{123}

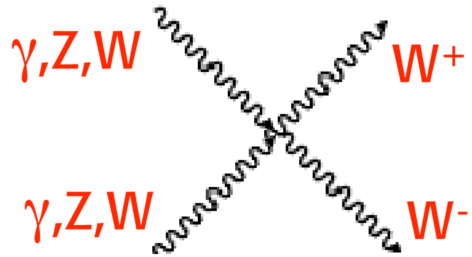
1,2 -> $W^{+,-}$

3 -> γ,Z

Only W_3 not B!

$W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$

$g_{\gamma WW} = g\sin\theta_W = e$
(obvious)
$g_{ZWW} = g\cos\theta_W = e\,\mathrm{cotg}\,\theta_W$
(larger by factor ~1.83)



4-gauge coupling: $\frac{1}{4} g^2 W^B_\mu W^C_\nu (W^{\mu B} W^{\nu C} - W^{\mu C} W^{\nu B})$

3-gauge coupling: The SM prediction is very special

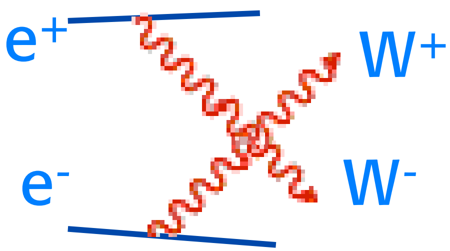
In general, assuming em gauge invariance and CP there are 6 parameters (5 for P and C conservation) for $(\gamma,Z)WW$

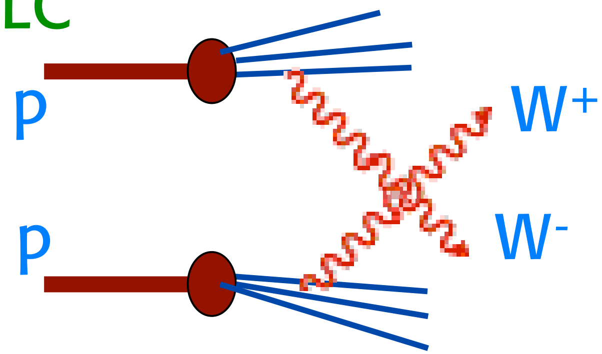
	SM
k_γ, k_Z	1
λ_γ, λ_Z	0
g_Z,(f_Z)	0

W magnetic moment: $e/2m_W(1+k_\gamma+\lambda_\gamma)$
W electric quad. mom: $-e/m_W^2\,(k_\gamma-\lambda_\gamma)$

Data are obtained from cross-section and distributions for $e^+e^- \to W^+W^-$ at LEP

The 4-gauge coupling is for LHC, NLC





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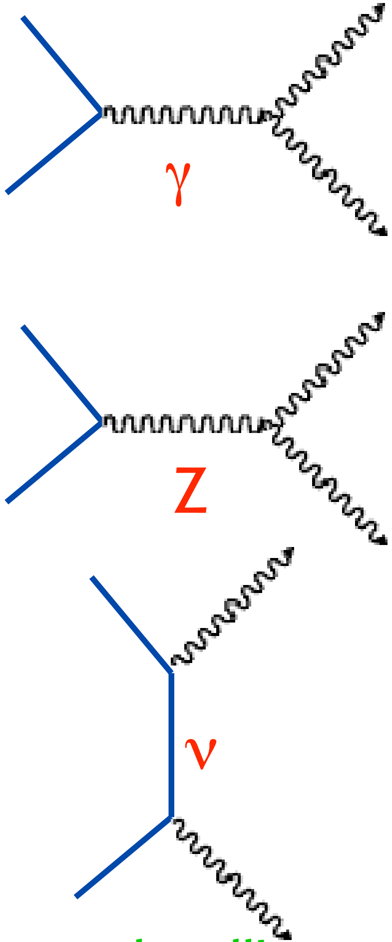
e^+e^- -> W^+W^-



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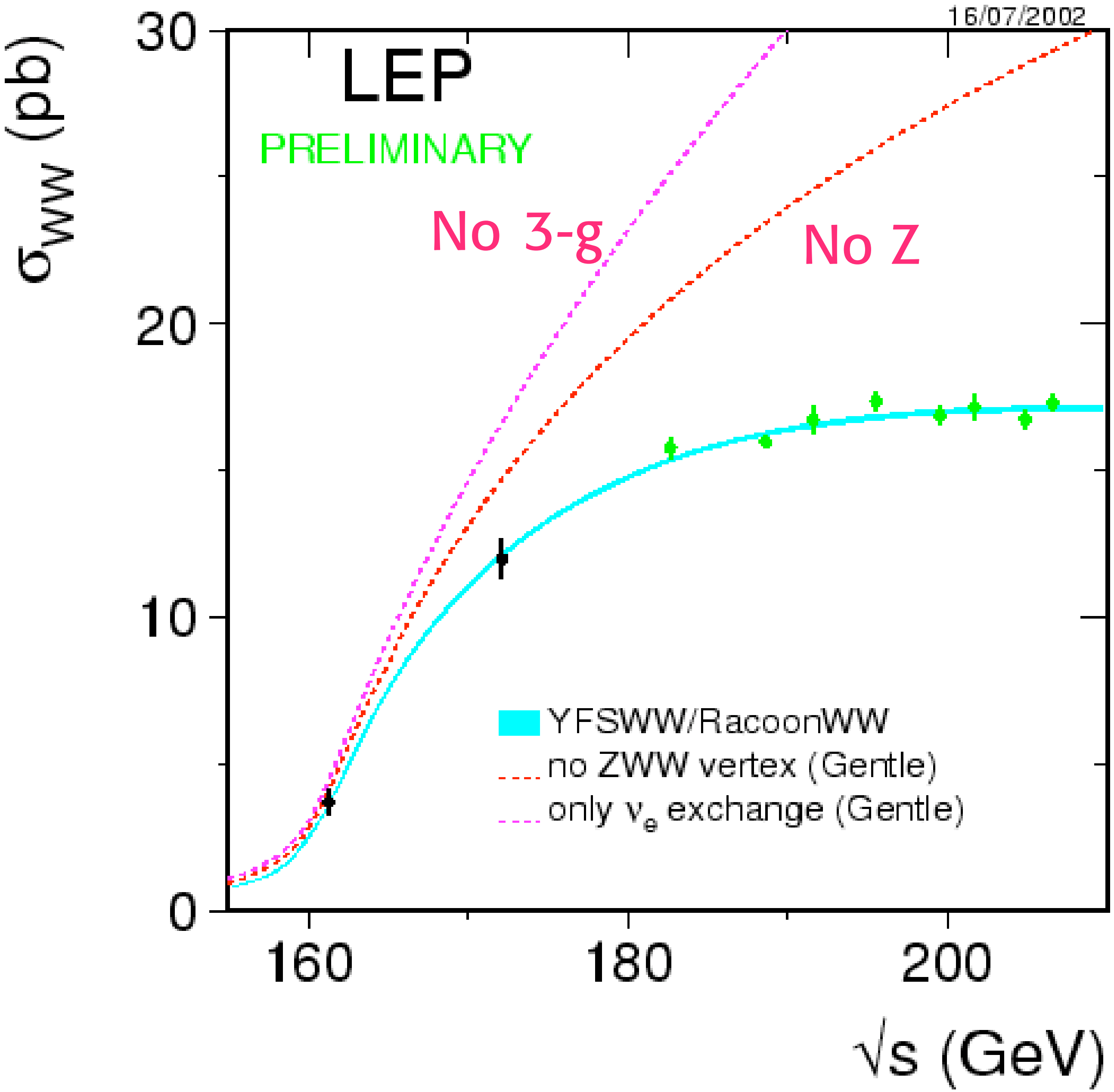

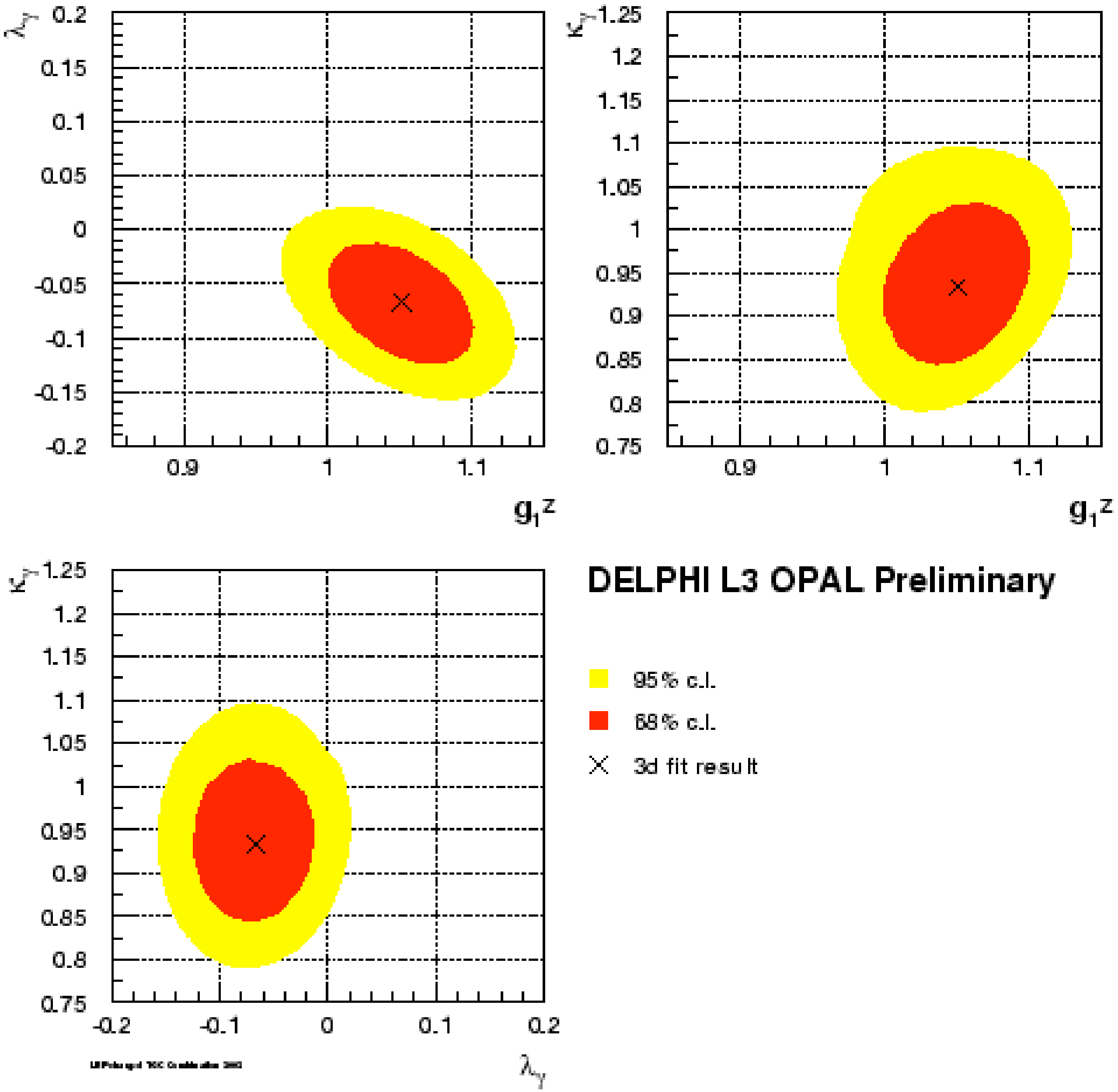
DELPHI L3 OPAL Preliminary
95% c.l.
68% c.l.
3d fit result
λ_γ
κ_γ
g_1^Z
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75

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