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Electroweak Interactions in the SM and Beyond

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A short course on the EW Theory

We start from the basic principles and formalism (a fast recall).
Then we go to present status and challenges

Content

- Formalism of gauge theories
- The SU(2)xU(1) symmetric lagrangian
- The symmetry breaking sector
- Beyond tree level
- Precision tests
- Problems of the SM
- Beyond the SM

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Spontaneous Symmetry Breaking

Borrowed from the theory of phase transitions:

Ferromagnet (Landau-Ginzburg, classical)

At zero magnetic field B

$$F = F(M, T) = F_0(T) + \frac{1}{2}\mu^2(T)\vec{M}^2 + \frac{1}{4}\lambda(T)(\vec{M}^2)^2 + \dots$$

Free energy

Magnetisation

Temperature

M small
(analogue of renorm.ty)
λ(T)0: stability

F is rotation invariant.
Minimum condition:

$$\frac{\partial F}{\partial M} = 0 \longrightarrow [\mu^2(T) + \lambda(T)\vec{M}^2]\vec{M} = 0$$

Two cases:

A $\mu^2(T) > 0$
Solution: $M_0 = 0$

B $\mu^2(T) < 0$
Solution: $M_0^2 = -\mu^2/\lambda$

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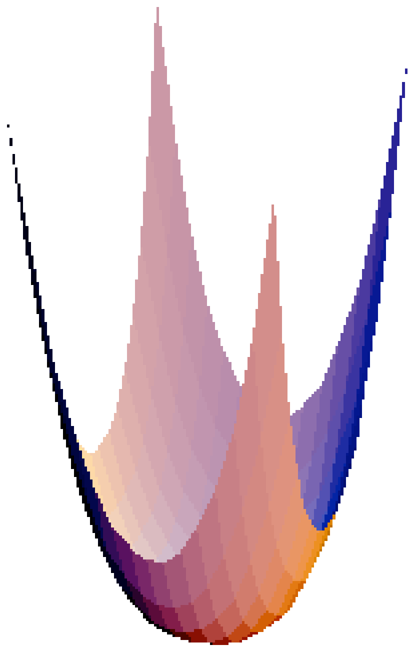
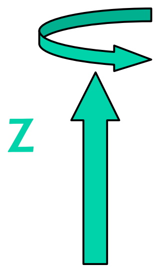
Critical temperature T_C: $\mu^2(T_C) = 0$

A $\mu^2(T) > 0$

Solution: $M_0 = 0$



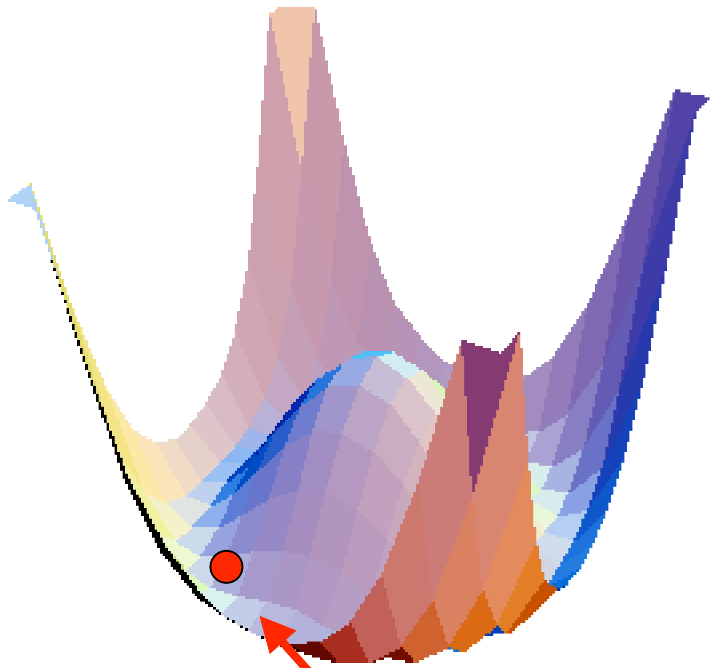
Unique minimum: no SSB



Here the actual symm. is rotation around z

B $\mu^2(T) < 0$

Solution: $M_0{}^2 = -\mu^2/\lambda$



A line of minima: SSB

The symmetry is broken when the system chooses one particular minimum point

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Goldstone Theorem: When SSB of a continous symmetry occurs there is a zero mass mode in the spectrum with the quantum numbers of the broken generator.

$\Phi_i(x) \longrightarrow \Phi'_i(x) = U_{ij}\, \Phi_j(x)$

$\delta\phi_a \sim i\sum \varepsilon^A t^A{}_{ij}\phi_j \sim i\varepsilon t_{ij}\phi_j$

$U = \exp[i\sum_A t^A\varepsilon^A] \sim 1 + i\sum_A t^A\varepsilon^A + o(\varepsilon^2)$

t^A: generators
ε^A: parameters

Hamiltonian density $\longrightarrow$ $H = |\partial_\mu \phi|^2 + V(\phi)$

ϕ^0: minimum of H (note constant: no gradients)

- minimum $\longrightarrow$ $\left.\dfrac{\partial V}{\partial \phi_i}\right|_{\phi=\phi^0} = 0$
- symmetry $\longrightarrow$ $\delta V = \dfrac{\partial V}{\partial \phi_i}\cdot \delta\phi_i = \dfrac{\partial V}{\partial \phi_i} t_{ij}\phi_j = 0$
- another derivative at the minimum $\longrightarrow$ $\left.\dfrac{\partial^2 V}{\partial \phi_k \delta \phi_i}\right|_{\phi=\phi^0} t_{ij}\phi^0_j + \cancel{\left.\dfrac{\partial V}{\partial \phi_i}\right|_{\phi=\phi^0}} t_{ik} = 0$

$$\left.\frac{\partial^2 V}{\partial\phi_k\delta\phi_i}\right|_{\phi=\phi^0} t_{ij}\phi_j^0 = M^2_{ki}t_{ij}\phi_j^0 = M^2\overrightarrow{(t\phi_0)} = 0$$

This is an eigenvalue equation for the (mass)^2 matrix M^2:

Either $\overrightarrow{(t\phi_0)} = 0$ for all t^A $\Rightarrow$ All generators leave ϕ^0 ("the vacuum") inv. symmetry

Or for some t^A $\overrightarrow{(t\phi_0)} \neq 0$ $\Rightarrow$ Non vanishing eigenvector of M^2 with zero eigenvalue Goldstone boson

For each broken generator t^A, there is a GB with the quantum numbers of t^A

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SSB: quantum versus classical

- For finite ‡ d.o.f. quantum effects remove degeneracy

e.g. Schroedinger eqn.: $V(x) = -\mu^2 x^2 + \lambda x^4$

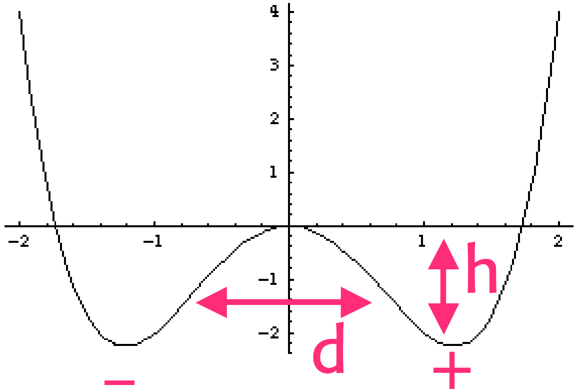
$<+|V|+> = <-|V|-> = a$

$<+|V|-> = <-|V|+> = b$

$$V = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \rightarrow$$

Eigenvectors:
$\sim |+> \pm |->$
Eigenvalues:
$= a \pm b$

$b \sim \exp[-dh]$ (tunnel)



Vacuum is unique!

While, for   d.o.f. and   volume

$<v|H|v'> = \delta_{vv'}$

and vacuum is degenerate

- Also, classical potential corrected by quantum effects

$$V_{eff} \sim -\mu^2\Phi^2 + \lambda\Phi^4 + \gamma\Phi^4(\log\Phi^2/\mu^2 + c) + \ldots$$

Classical
tree level

Quantum corr's
loop expansion

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SSB in gauge theories: Higgs mechanism

In general SSB → Goldstone bosons with quantum numbers of broken generators t^A

$$M^2_{ki} = \left. \frac{\partial^2 V}{\partial \phi_k \delta \phi_i} \right|_{\phi=\phi^0}$$

$\boxed{M^2 t^A \Phi^0 = 0}$

$t^A \Phi^0$ not 0

In gauge theory with Higgs mechanism

Symmetry broken by vacuum expectation values (vev) of Higgs field (scalar fields otherwise Lorentz also broken)

→ No physical Goldstone bosons. Become 3rd helicity state of gauge bosons with t^A quantum numb's that take mass

The Higgs potential has an orbit of minima, and the Higgs fields, like magnetisation, take a particular direction

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Symmetry restauration possible at high T (early Universe)

Simplest abelian U(1) model (Higgs)

"wrong" sign

$$L = -\frac{1}{4}F^2_{\mu\nu} + |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}\mu^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$

Invariant under $(U = \exp[iQe\varepsilon(x)])$: $\begin{cases} A_\mu \Rightarrow A_\mu' = A_\mu + \partial_\mu\varepsilon(x) \\ \phi \Rightarrow \phi' = e^{ie\varepsilon(x)}\phi \end{cases}$

$(Q\phi=\phi)$

If $\phi^0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{\lambda}}$ (real 0) $(\phi^0 = \text{constant} = \langle 0|\phi|0\rangle)$

one must shift (small oscill.s about field=0):

$$\phi(x) \Rightarrow \frac{\rho(x) + v}{\sqrt{2}}\exp[ie\chi(x)/\sqrt{2}] \qquad A_\mu \Rightarrow A_\mu + \frac{1}{v}\partial_\mu\chi(x)$$

$(\langle 0|\rho|0\rangle = \langle 0|\chi|0\rangle = 0)$

$$L = -\frac{1}{4}F^2_{\mu\nu} + \frac{1}{2}e^2v^2A^2_\mu + \frac{1}{2}e^2\rho^2A^2_\mu + e^2\rho vA^2_\mu + L_v(\rho)$$

mass term

No $\chi(x)$, A_μ massive
(same number of d.o.f.!)

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$$L = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}\mu^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$

$$\phi^0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\phi(x) \Rightarrow \frac{\rho(x)+v}{\sqrt{2}}\exp[ie\chi(x)/\sqrt{2}]$$

$$L = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\rho^2A_\mu^2 + e^2\rho vA_\mu^2 + L_v(\rho)$$

$$L_v(\rho) = \frac{1}{2}\mu^2 \cdot \frac{(\rho(x)+v)^2}{2} - \frac{1}{4}\lambda \cdot \frac{(\rho(x)+v)^4}{4}$$

Expanding:

$$L_v(\rho) = \frac{1}{2}\rho^2\left(\frac{1}{2}\mu^2 - \frac{3}{4}\lambda v^2\right) + \ldots = \frac{1}{2}\rho^2\left(\frac{1}{2}\mu^2 - \frac{3}{2}\mu^2\right) + \ldots = -\frac{1}{2}\rho^2\mu^2 + \ldots$$

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The ρ mass has the right sign!

The Higgs mechanism was discovered in condensed matter physics. e.g.: Superconductor in Landau-Ginzburg approx'n

Free energy

Wrong sign

$$F = F_0 + \frac{1}{2}\vec{B}^2 + \frac{1}{4m}\left|(\vec{\nabla} - 2ie\vec{A})\phi\right|^2 - \alpha\,|\phi|^2 + \beta\,|\phi|^4$$

$|\phi|^2$: Cooper pair density (e-e-: charge -2e and mass 2m)

"Wrong" sign of α leads to ϕ not 0 at minimum

- No propagation of massless phonons ($\omega = k\,v$)
- Mass term for A -> exponential decrease of B Inside the superconductor (Meissner effect)

$L = L_{symm} + L_{Higgs}$

In general $\phi = \phi^i$ (several multiplets)

$$L_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) - [\bar{\psi}_L \Gamma \psi_R \phi + \mathrm{h.c.}]$$

$$V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

No more than quartic for renormalisation

Only weak-isospin doublet Higgs ϕ contribute to fermion masses (ψ_L doublets, ψ_R singlets)

All non trivial repres.s break SU(2)xU(1) and give masses to $W^\pm$ and Z

Minimal model: only one Higgs ϕ doublet

Fermion masses: $[\bar{\psi}_L \Gamma \psi_R \phi + \mathrm{h.c.}]$

singlet

doublet doublet

With one Higgs doublet: $g_f \bar{\Psi}_{fL} \Psi_{fR} \phi \longrightarrow m_f = g_f v$

Ugly: each mass one new coupling

Large mass ratios (m_t/m_e, m_t/m_u…) imply large coupling ratios

Fermion masses demand a more fundamental theory (at M_{Pl}?)

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Gauge Boson Masses

$$L_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) + \ldots.$$

$$D^\mu\phi = \left[\partial_\mu + ig\sum_A t^A W^A_\mu + ig'\frac{Y}{2}B^\mu\right]\phi$$

Recall:
$W_3 = c_W Z + s_W A$
$B = -s_W Z + c_W A$
$tg\theta_W = s_w/c_w = g'/g$

Zero photon mass -> Q unbroken
$Qv=(t^3+Y/2)v=0$: only neutral components of ϕ have vev0 → e.g for a doublet:

$$\phi = \begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix} = \begin{bmatrix}0\\ v\end{bmatrix} \equiv v$$

$|t^+v|^2=v^2$
$|t^3v|^2=v^2/4$

- $m_W^2 W_\mu{}^\dagger W^\mu = g^2\left|\frac{t^+}{\sqrt{2}}v\right|^2 W_\mu{}^\dagger W^\mu$
- $\frac{1}{2}m_Z^2 Z_\mu Z^\mu = \left|\left(g c_W t^3 - g' s_W \frac{Y}{2}\right)v\right|^2 Z_\mu Z^\mu =$

Qv=0 → $= (gc_W + g's_W)^2\left|t^3 v\right|^2 Z_\mu Z^\mu = \left(\frac{g}{c_W}\right)^2\left|t^3 v\right|^2 Z_\mu Z^\mu$

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Thus, for one doublet ϕ: $\boxed{m_W^2 = \frac{1}{2}g^2 v^2 = m_Z^2\cos^2\theta_W}$

For doublet ϕ: $$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$ (Tree level)

In general: $$\rho_0 = \frac{\sum_\phi \frac{1}{2} \langle t^+ t^- + t^- t^+ \rangle v_\phi^2}{\sum_\phi 2 \langle t^3 t^3 \rangle v_\phi^2} = \frac{\sum_\phi \langle t(t+1) - t^3 t^3 \rangle v_\phi^2}{\sum_\phi 2 \langle t^3 t^3 \rangle v_\phi^2}$$

In general, at tree level, $\rho_0 = 1 + \Delta\rho_0$. In the SM with radiative corrections: $\rho_{SM} = (1 + \Delta\rho_{SM})\, \rho_0$

$$\Delta\rho_{SM} = \frac{3 G_F m_t^2}{8\pi^2 \sqrt{2}} + \ldots \sim 1\%$$

Exp. puts a strong bound on $\Delta\rho_0$:

$(\rho_0)_{Exp} = 1.0004 \pm 0.0006$
($m_H \sim 115$ GeV) PDG'03

Note: $v = 2^{-3/4} G_F^{-1/2} \sim 174$ GeV

$$m_W^2 = \frac{1}{2} g^2 v^2 \quad \text{and} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

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Higgs couplings

H: physical Higgs field

Note: normalisation

Charged $\quad \partial_\mu \phi^\dagger \partial^\mu \phi$

Neutral $\quad \frac{1}{2}\partial_\mu H \partial^\mu H$

$$\phi(x) = \begin{bmatrix} \phi^+(x) \\ \phi^0(x) \end{bmatrix} = \begin{bmatrix} 0 \\ v + \frac{H(x)}{\sqrt{2}} \end{bmatrix}$$

$$D^\mu \phi = \left[\partial_\mu + ig\sum_A t^A W_\mu^A + ig'\frac{Y}{2}B^\mu\right]\phi$$

Recall:

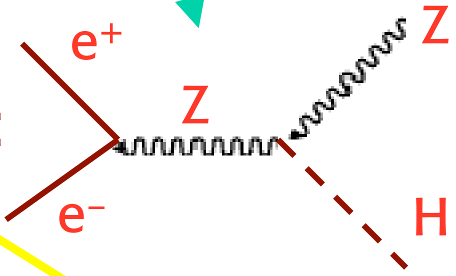
$$m_W^2 = \frac{1}{2}g^2 v^2 = m_Z^2 \cos^2\theta_W$$

$$L_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) + \ldots\ldots = \frac{1}{2}\partial_\mu H \partial^\mu H + L(H, W, Z)$$

$$L(H, W, Z) = g^2\frac{v}{\sqrt{2}}W_\mu{}^\dagger W^\mu H + \frac{g^2}{4}WWHH + g^2\frac{v}{2\sqrt{2}c_W^2}ZZH + \frac{g^2}{8c_W^2}ZZHH$$

$$g^2\frac{v}{\sqrt{2}}W_\mu{}^\dagger W^\mu H = g m_W W_\mu{}^\dagger W^\mu H$$

LEP:



Fermions (after diag.)

$$\frac{m_f}{v}\bar{\psi}_{fL}\psi_{fR}\frac{H}{\sqrt{2}} \sim 2^{1/4}G_F^{1/2} m_f \bar{\psi}_{fL}\psi_{fR}H$$

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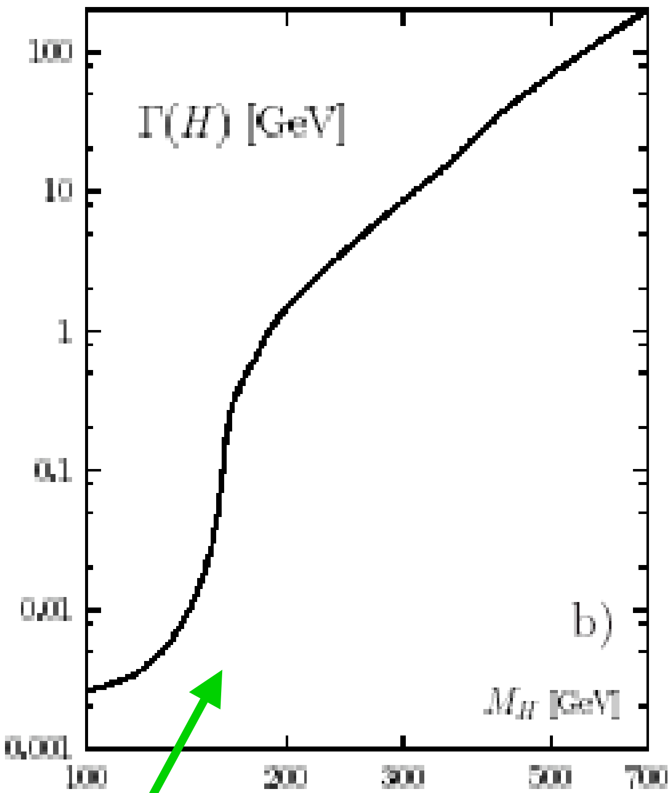
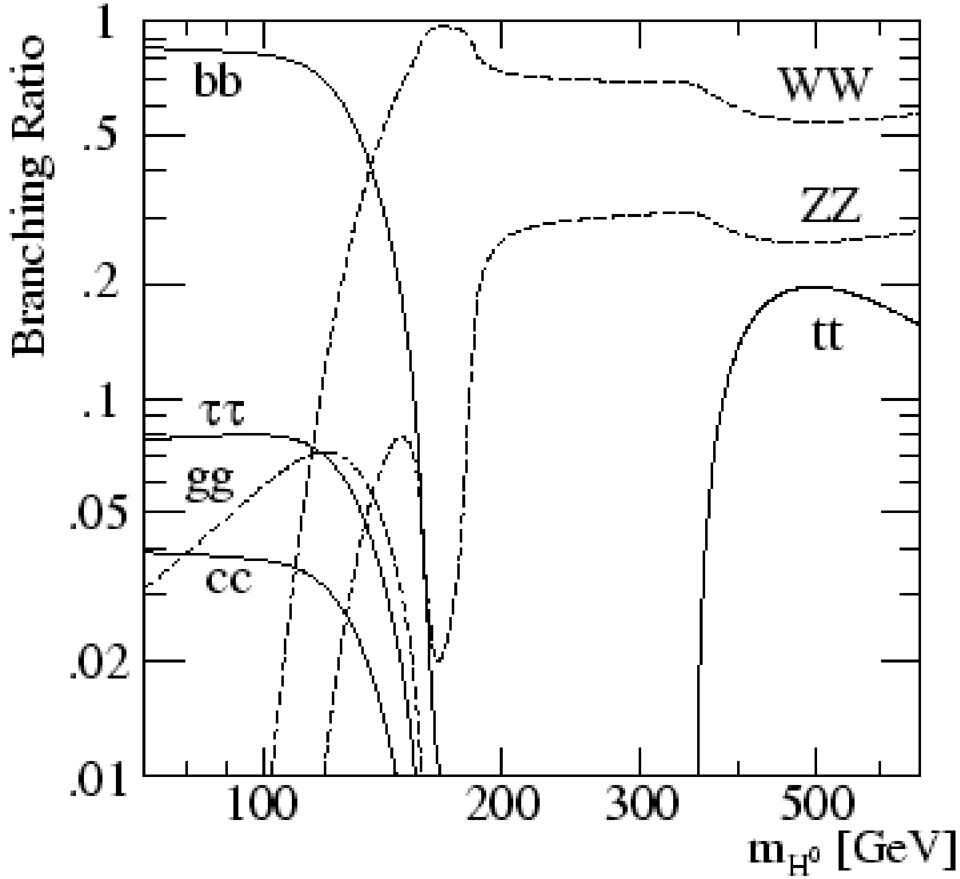
Higgs width and branching ratios





Γ_H: ~few MeV near the LEP limit,
~few GeV for intermediate mass, $\sim 1/2(m_H)^3$
(Γ_H, m_H in TeV) for heavy mass.

Note

- In spite of $m_D \sim m_\tau$ and colour, $B(H\to\tau\tau) \sim 3B(H\to cc)$

  Due to QCD running masses $m_c \to m_c(m_H) \sim 0.6$ GeV

- In spite of $m_t > m_W$, $B(H\to WW) \sim 3\text{-}4\ B(H\to tt)$ for heavy H

  Due to behaviour of W polarization sums

$(k+k')^2 = m_H^2$

$$\sum_{A,B} e_\mu^{A*} e_\nu^{A} e^{B\mu *} e^{B\nu} = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right)\left(-g^{\mu\nu} + \frac{k'^\mu k'^\nu}{m_W^2}\right) = \frac{1}{4}\left(\frac{m_H}{m_W}\right)^4 - \left(\frac{m_H}{m_W}\right)^2 + 3$$

and $\Gamma(H\to tt) \sim \beta_t^3$ (P-wave), $\Gamma(H\to WW) \sim \beta_W$

$\beta_i^2 = 1 - 4m_i^2/m_H^2$

$$\Gamma_t = N_C \frac{g^2}{32\pi}\left(\frac{m_t}{m_H}\right)^2 \beta_t^3 m_H$$

$$\Gamma_W = \frac{g^2}{64\pi}\left(\frac{m_H}{m_W}\right)^2 \beta_W m_H \left[1 - \frac{4m_W^2}{m_H^2} + 12\left(\frac{m_W}{m_H}\right)^4\right]$$

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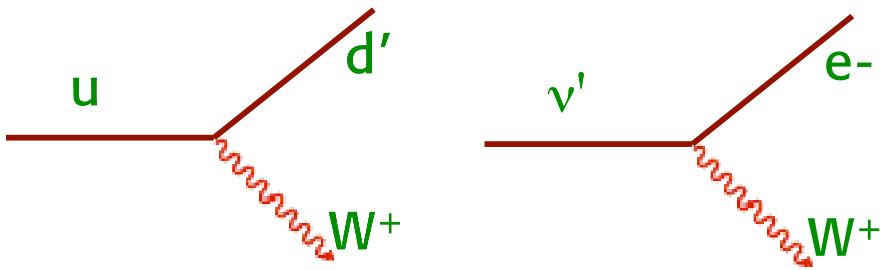
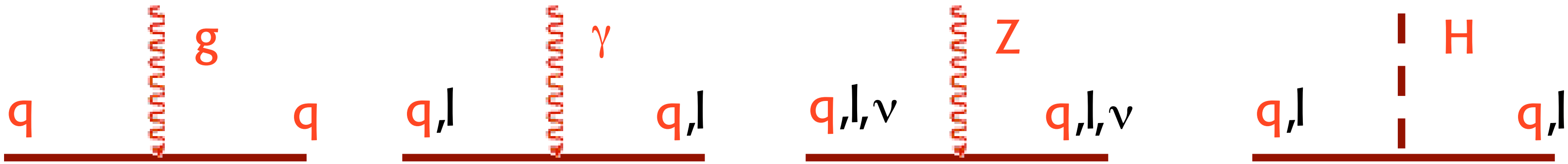
Quarks and leptons exist in different flavours

within one family and across families

$$\begin{bmatrix} u & u & u & \nu_e \\ d & d & d & e \end{bmatrix} \qquad \begin{bmatrix} c & c & c & \nu_\mu \\ s & s & s & \mu \end{bmatrix} \qquad \begin{bmatrix} t & t & t & \nu_\tau \\ b & b & b & \tau \end{bmatrix}$$

At tree level only charged-current weak int's change flavour





QCD

QED

Neutral curr.s
GIM needed

Higgs
only 1 Higgs
per charge sector

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Fermion masses

$$L_{Higgs} = \ldots - [\bar{\psi}_L \Gamma \psi_R \phi + \text{h.c.}]$$

Yukawa matrix

Only Higgs doublets ϕ can contribute

Masses arise when ϕ is replaced by its vev v

If more doublets

$$M_\psi = \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L \qquad M = \Gamma v \; (= \Sigma_i \Gamma^i v^i)$$

By separate rotations of the L and R fields one can make M_ψ real and diagonal: $U^+_{L,R} U_{L,R} = U_{L,R} U^+_{L,R} = 1$

$$\psi_L^{diag} = U_L \psi_L$$
$$\psi_R^{diag} = U_R \psi_R$$

$$\Rightarrow \quad \boxed{M_{diag} = U^+_L M U_R = U^+_R M^+ U_L}$$

M commutes with Q $\Rightarrow$ Separate rotations for up, down, ch. leptons, ν's

e.g $U^u_{L,R}$, $U^d_{L,R}$ etc

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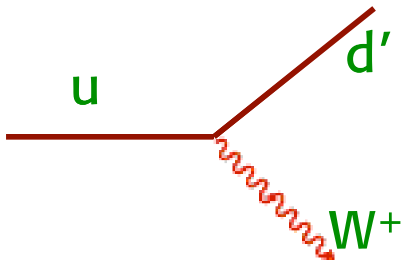
CKM Matrix



Left-doublets

$$\begin{bmatrix} u \\ d' \end{bmatrix} \begin{bmatrix} c \\ s' \end{bmatrix} \begin{bmatrix} t \\ b' \end{bmatrix}$$

W-eigenstates

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = V_{CKM} \begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

mass eigenstates

V_{CKM} unitary (change of basis): $V^+V = VV^+ = 1$

Neutral current diagonal in both bases:

$$(\bar{d}', \bar{s}', \bar{b}') \begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = (\bar{d}, \bar{s}, \bar{b}) \underbrace{V^+V}_{1} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

or

$$\bar{d}'d' + \bar{s}'s' + \bar{b}'b' = \bar{d}d + \bar{s}s + \bar{b}b$$

An equal number of up and down needed

Glashow-Iliopoulos-Maiani '70

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The neutral current couplings are:

$$\frac{g}{\cos\theta_W}\bar{\psi}\gamma_\mu[t_L^3\cdot\frac{1-\gamma_5}{2}+t_R^3\cdot\frac{1+\gamma_5}{2}-Q\sin^2\theta_W]\psi Z^\mu$$

zero for q&l

For GIM to work all states with equal Q must have the same t^3_L and t^3_R

was not true in old Cabibbo theory: $(u,d_C)_L$ doublet , s_{CL} singlet

$d_C = \cos\theta_C d + \sin\theta_C s$
$s_C = -\sin\theta_C d + \cos\theta_C s$

In the t^3 part there is $\bar{d}_C d_C$ but not $\bar{s}_C s_C$ and the FC terms $\cos\theta_C \sin\theta_C(\bar{d}s+\bar{s}d)$ are present

The charged current couplings are:

$$\frac{g}{2\sqrt{2}}\bar{u}\gamma^\mu(1-\gamma_5)d\cdot W_\mu \Longrightarrow V_{CKM}=U_L^{u\dagger}U_L^d$$

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Note: kinetic terms diagonal in both bases $\overline{u_L}i\gamma^\mu\partial_\mu u_L+\ldots$

More Higgs doublets?

Beware of FCNC, e. g.

To avoid FCNC (and CP viol) in the Higgs sector you need to have at most 1 Higgs for u-type quarks,
1 Higgs for d-type quarks, 1 Higgs for e-type leptons,
(1 Higgs for ν-type leptons)

In fact diagonalisation of masses $M=\Gamma^1 v^1+\Gamma^2 v^2+\ldots$ guarantees diagonalisation of couplings $\Gamma^1\phi^1+\Gamma^2\phi^2+\ldots$ only for a single term (then masses and couplings are proportional)

For example, in SUSY models there are H^u and H^d that give mass to $t^3=+1/2$ and $t^3=-1/2$ states, respectively.

Counting Parameters in V_{CKM}

Assume there are N down quarks: $D' = V\,D$, $V \sim N\mathrm{x}N$ unitary matrix

$V \sim N\mathrm{x}N$ unitary matrix → N^2 complex numbers $- N^2$ unitary conditions → N^2 real parameters

Freedom of phase def.:
2N quarks -> 2N - 1 relative phases
(currents $\bar{\Psi}\Psi$ insensitive to overall phase)

TOTAL:
$N^2-(2N-1)=(N-1)^2$
physical parameters

cfr: a NxN orthogonal matrix has N(N-1)/2 parameters

$OO^T=O^TO=1 \rightarrow N^2-N(N+1)/2=N(N-1)/2$

N	$(N-1)^2$	$N(N-1)/2$	angles	phases
2	1	1	1 (θ_C)	0
3	4	3	3	1
4	9	6	6	3

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A phase in V_{CKM} $\Longrightarrow$ CP Violation

$\overline{U}_L \gamma_\mu V_{CKM} D_L W^\mu + \overline{D}_L \gamma_\mu V^+{}_{CKM} U_L W^{+\mu}$ ← h.c.

Parity: $P\psi_L P^{-1} = P\psi_R$

Charge conj.: $C\psi_L C^{-1} = C\overline{\psi_R}^T$

Time Rev.: $T\psi_L T^{-1} = TK\psi_L$

$\overline{\psi}$: creates f , ann. $\overline{f}$

ψ : ann. f, creates $\overline{f}$

Complex conj. of c-numbers: T antiunitary

$Tc\psi T^{-1} = c^* T\psi T^{-1}$ $[x,p]=i\hbar$

$$(CP)\overline{U}_L \gamma_\mu V_{CKM} D_L W^\mu (CP)^{-1} = \overline{D}_L \gamma_\mu V^T{}_{CKM} U_L W^{+\mu}$$

If V is real then $V^T=V^+$ and CP invariance holds, otherwise is violated. Note CPT always holds:

$$(CPT)\overline{U}_L \gamma_\mu V_{CKM} D_L W^\mu (CPT)^{-1} = \overline{D}_L \gamma_\mu V^+{}_{CKM} U_L W^{+\mu}$$

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Any Lorentz inv, hermitian, local L is CPT inv.

A simple example

Three charged scalar fields A, B, C for the decay A -> B+C

$$L = \lambda A B^+ C^+ + \text{h.c.} = \lambda A B^+ C^+ + \lambda^* A^+ B C$$

All products are normal-ordered

$$(CP)L\ (CP)^{-1} = \lambda A^+ B C + \lambda^* A B^+ C^+$$

(Under CP A<-> A^+ etc)

$$(TCP)L\ (TCP)^{-1} = \lambda^* A^+ B C + \lambda A B^+ C^+$$

TCP is always true while CP invariance holds for λ real

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

Maiani

$s_{12} = \sin\theta_C$

$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}\,s_{23} \\ \dots & \dots & c_{13}\,c_{23} \end{bmatrix}$$

PDG'02

$s_{12} \sim 0.2196 \pm 0.0026$

$s_{23} \sim (41.2 \pm 2.0)\,10^{-3}$

$s_{13} \sim (3.6 \pm 0.7)\,10^{-3}$

Wolfenstein parametrisation:

$s_{12} = \lambda$

$s_{23} = A\lambda^2$

$s_{13}\, e^{-i\delta} = A\lambda^3(\rho - i\eta)$

$$V \sim \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + o(\lambda^4)$$

$A = 0.85 \pm 0.05$

$(\rho^2 + \eta^2)^{1/2} = 0.40 \pm 0.08$

G. Altarelli

More precisely

$s_{12} = \lambda$
$s_{23} = A\lambda^2$
$s_{13}\, e^{-i\delta} = A\lambda^3(\rho - i\eta)$

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4, \qquad V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1+4A^2),$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4, \qquad V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\varrho + i\eta)],$$

$$V_{us} = \lambda + \mathcal{O}(\lambda^7), \qquad V_{ub} = A\lambda^3(\varrho - i\eta), \qquad V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^8),$$

$$V_{ts} = -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\varrho + i\eta)], \qquad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta})$$

$\bar{\rho} = \rho(1 - \lambda^2/2)$
$\bar{\eta} = \eta(1 - \lambda^2/2)$

Unitarity Triangles

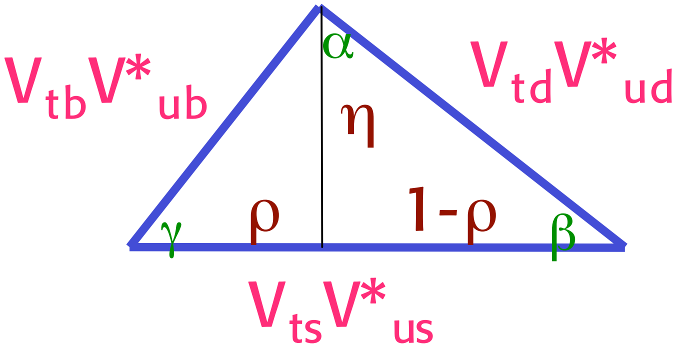
$VV^{+} = 1 \rightarrow V_{hk}V^{*}_{hl} = \delta_{kl}$

For example: $V_{ta}V^{*}_{ua} = 0$

a → d s b

$$A\lambda^3(1-\rho-i\eta) - A\lambda^3 + A\lambda^3(\rho+i\eta) = 0$$

Can be drawn as a triangle



(other 5 triangles are either equivalent $[V_{ab}V^{*}_{ad}]$ or too flat)
All have same area ~J

In SM all CP violation is proportional to J

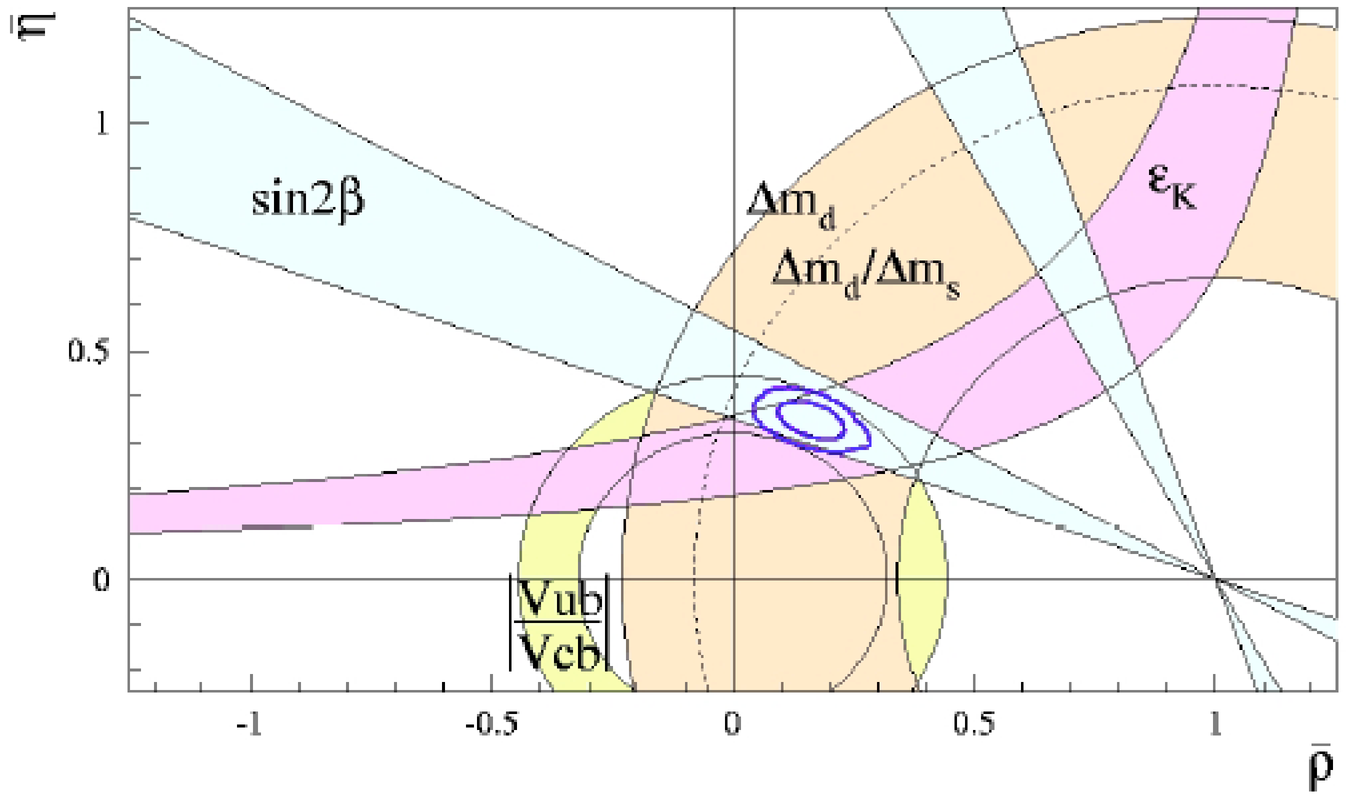
$$2\bullet\text{Area} = J = \eta A^2\lambda^6 \sim \eta(0.85)^2(0.224)^6 \sim \eta\, 9\cdot 10^{-5}$$

$J \sim s_{12}\, s_{13}\, s_{23} \sin\delta$ Jarlskog

G. Altarelli

Note: $V_{td}=|V_{td}|e^{-i\beta}$, $V_{ub}=|V_{ub}|e^{-i\gamma}$

Lubicz, Durham ‘03, hep-ph/0307195



$$\bar{\rho} = \rho(1 - \lambda^2/2) = 0.178 \pm 0.046$$
$$\bar{\eta} = \eta(1 - \lambda^2/2) = 0.341 \pm 0.028$$