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Lecture AB-II

RARE MESON DECAYS MEDIATED BY MAJORANA NEUTRINOS

Based on:

[1] A. Ali, A. V. Borisov, and N. B. Zamorin, Eur. Phys. J. C, v. 21, no. 1, p. 123 - 132 (2001) [hep-ph/0104123]

[2] A. Ali, A. V. Borisov, and M. V. Sidorova, Phys. At. Nucl. (2006) (accepted for publ.)

OUTLINE

- **Introduction**

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- **Rare meson decays** $M^+ \to M'^- \ell^+ \ell'^+$

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  *The model-dependent Bethe-Salpeter amplitudes for the pseudoscalar mesons

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- **Conclusion**

***Lepton-number violating processes mediated by Majorana neutrinos**

The Majorana mass term violates lepton number by two units, $\Delta L = \pm 2$.

Neutrinoless double beta decay $(0\nu\beta\beta)$ [a possible signal in Heidelberg – Moscow experiment (2001)?]

$$(A, Z) \to (A, Z+2) + e^- + e^-$$

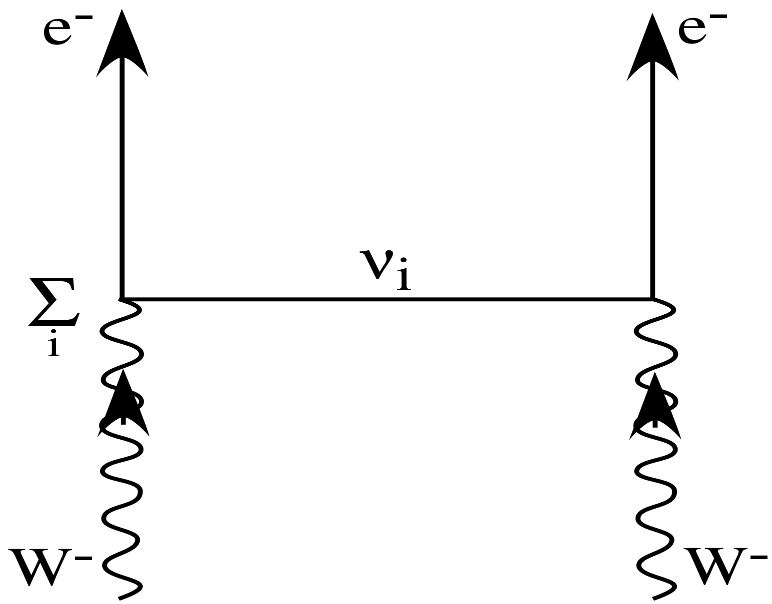


Fig. 1. The fundamental subprocess for $0\nu\beta\beta$. The Standard model weak interaction acts at each vertex:

$$L_{CC} = -\frac{g}{\sqrt{2}} \bar{e}_L \gamma^\mu \sum_k U_{ek} \nu_{kL} W_\mu^+ + \mathrm{H.c.}$$

Rare meson decays, e.g.,

$$K^+ \rightarrow \pi^- + \ell^+ + \ell'^+ \ (\ell\ell' = ee, e\mu, \mu\mu).$$

Same-sign dilepton production in high-energy hadron-hadron and lepton-hadron collisions

$$p + p \rightarrow \ell^\pm + \ell'^\pm + X,$$

$$e^+ + p \rightarrow \bar{\nu}_e + \ell^+ + \ell'^+ + X.$$

W-pair production in lepton-lepton collisions

$$e^- + e^- \rightarrow W^- + W^-.$$

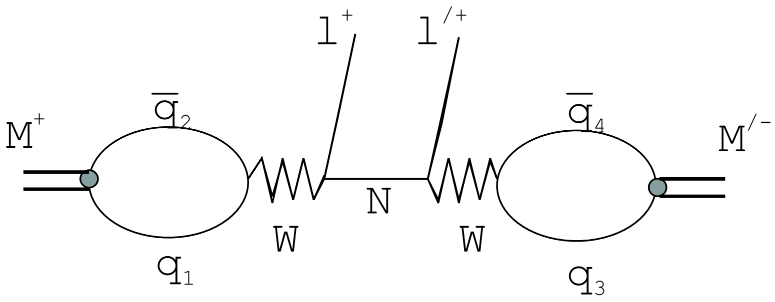
Nuclear $\mu^- \rightarrow e^+$ conversion (the subprocess $W^+ + \mu^- \rightarrow e^+ + W^-$)

$$(A, Z) + \mu_b^- \rightarrow e^+ + (A, Z-2)^*.$$

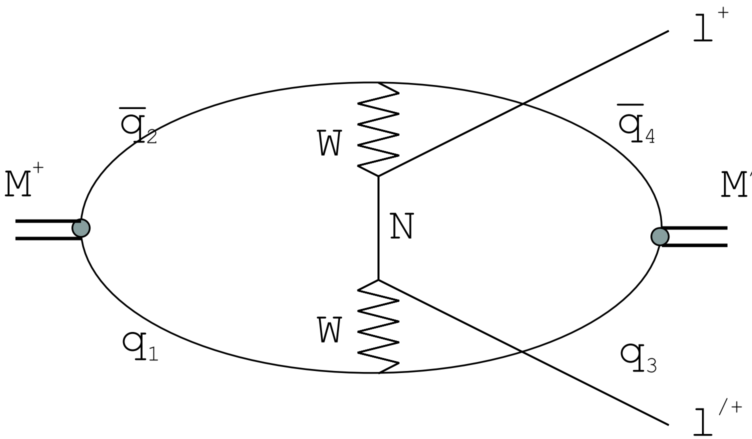
Rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$

*The amplitude of RMD

The lowest order amplitude of the process is given by the sum of the tree and the box diagrams (see Fig. 2).



(tree)



(box)

Fig. 2. Feynman diagrams for the RMD. Here N is a Majorana neutrino, bold vertices correspond to Bethe—Salpeter amplitudes for mesons as bound states of a quark and an antiquark. There are also two crossed diagrams with interchanged lepton lines.

The width of the decay $M^+(P) \to M'^-(P') + \ell^+(p) + \ell'^+(p')$

$$\Gamma_{\ell\ell'} = \left(1 - \frac{1}{2}\delta_{\ell\ell'}\right) \int (2\pi)^4 \delta^{(4)}(P' + p + p' - P) \frac{\left|A_t + A_b\right|^2}{2m_M} \frac{d^3P' d^3p d^3p'}{2^3 (2\pi)^9 P'^0 p^0 p'^0}.$$

The tree and box amplitudes are expressed in the Bethe—Salpeter formalism of Ref. [G. Esteve, A. Morales, and R. N\'u\~nes-Lagos, J. Phys. G 9 (1983) 357]

$$A_i = \frac{1}{(2\pi)^8} \int d^4q d^4q' H^{(i)}_{\mu\nu} L^{\mu\nu}_i, i = t, b;$$

The lepton tensors

$$L^{\mu\nu}_i = \frac{g^4}{4} \frac{g^{\mu\alpha}}{p_i^2 - m_W^2} \frac{g^{\nu\beta}}{p_i'^2 - m_W^2} \sum_N U_{\ell N} U_{\ell' N} \eta_N m_N$$
$$\times \left(\overline{v}^c(p) \left[\frac{\gamma_\alpha \gamma_\beta}{(p_i - p)^2 - m_N^2} + \frac{\gamma_\beta \gamma_\alpha}{(p_i - p')^2 - m_N^2} \right] \frac{1 - \gamma^5}{2} v(p') \right),$$

where η_N is the charge conjugation phase factor of the of the mass eigenstate $\nu_N = \eta_N \nu_N^c, |\eta_N| = 1;$

$$p_t = P, p_t' = P'; p_b = \frac{1}{2}(P - P') + q' - q, p_b' = \frac{1}{2}(P - P') - q' + q.$$

The hadron tensors

$$H_{\mu\nu}^{(t)} = \mathrm{Tr}\left\{\chi_P(q)V_{12}\gamma_\mu \frac{1-\gamma^5}{2}\right\}\mathrm{Tr}\left\{\bar{\chi}_{P'}(q')V_{43}\gamma_\nu \frac{1-\gamma^5}{2}\right\},$$

$$H_{\mu\nu}^{(t)} = \mathrm{Tr}\left\{\chi_P(q)V_{13}\gamma_\mu \frac{1-\gamma^5}{2}\bar{\chi}_{P'}(q')V_{42}\gamma_\nu \frac{1-\gamma^5}{2}\right\}.$$

Here V_{jk} are the elements of the CKM matrix, the model-dependent BS amplitudes

$$\chi_P(q) = \int d^4x e^{iq\cdot x}\chi_P(x) = \gamma^5(1-\delta_M \hat{P})\varphi_P(q)\phi_G,$$

where $\delta_M = (m_1 + m_2)/m_M^2$, m_M is the mass of meson having a quark q_1 and an antiquark $\bar{q}_2$, $m_{1,2}$ are the quark (current) masses, $q = (p_1 - p_2)/2$ Is the relative 4-momentum, $P = p_1 + p_2$ is the total 4-momentum of the meson, $\hat{P} = \gamma^\mu P_\mu$; the function $\varphi_P(q)$ is model dependent, and ϕ_G is the $SU(N_f)\times SU(N_c)$-group factor.

The BS amplitude of a meson in configuration space is defined as

$$\chi_P(x_1,x_2) = -\frac{i}{\sqrt{N_c}}\left\langle 0\left|T\left\{q_1^a(x_1)\bar{q}_{2a}(x_2)\right\}\right|M(P)\right\rangle = e^{-iP\cdot X}\chi_P(x),$$

$$\chi_P(x) = \int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot x}\chi_P(q),$$

where $X = \frac{1}{2}(x_1+x_2),\ x = x_1 - x_2; a = 1,2,3$ is the color index (the number of colors $N_c = 3$).

The decay constant f_M of the meson M is expressed through the BS amplitude as follows

$$if_M P^\mu = \left\langle 0\left|\bar{q}_{2a}(0)\gamma^\mu\gamma^5 q_1^a(0)\right|M(P)\right\rangle = -i\sqrt{N_c}\,\mathrm{Tr}\left[\gamma^\mu\gamma^5\chi_P(x=0)\right],$$

$$f_M = 4\sqrt{N_c}\,\delta_M\int\frac{d^4q}{(2\pi)^4}\varphi_P(q).$$

The normalization condition for the BS amplitude (see Z.-G. Wang, W.-M. Yang, and S.-L. Wan,hep-ph/0411142)

$$\int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}[\bar{\chi}(q) \frac{\partial}{\partial P_\mu}\left(\hat{q}+\frac{1}{2}\hat{P}-m_1\right)\chi(q)\left(\hat{q}-\frac{1}{2}\hat{P}-m_2\right)$$

$$+\bar{\chi}(q)\left(\hat{q}+\frac{1}{2}\hat{P}-m_1\right)\chi(q)\frac{\partial}{\partial P_\mu}\left(\hat{q}-\frac{1}{2}\hat{P}-m_2\right)] = 2P^\mu,$$

where $\bar{\chi} = \gamma^0\chi^+\gamma^0$, leads to the condition for the model dependent function $\varphi_P(q)$:

$$\left[1-\left(\frac{m_1+m_2}{m_M}\right)^2\right]\int \frac{d^4q}{(2\pi)^4}\left|\varphi_P(q)\right|^2 = 1.$$

We will use the ***relativistic Gaussian model***

$$\varphi_P(q) = \frac{4\pi}{\alpha^2}\left(1-\mu^2\right)^{-1/2} \exp\left\{-\frac{1}{2\alpha^2}\left[2\left(\frac{P\cdot q}{m_M}\right)^2 - q^2\right]\right\},$$

$$\alpha^2 = \frac{\pi}{4\sqrt{N_c}}\left(1-\mu^2\right)^{1/2}\frac{f_M}{\delta_M}, \quad \mu = m_M\delta_M = \frac{m_1+m_2}{m_M}.$$

The tree amplitude is expressed in a model independent way in terms of the meson decay constants:

$$A_t = -\frac{1}{4} f_M f_{M'} V_{12} V_{43} P_\mu P'_\nu L_t^{\mu\nu} .$$

The box amplitude depends in general on the details of hadron dynamics:

$$A_b = 2V_{13} V_{42} \delta_M \delta_{M'} (P_\mu P'_\nu + P_\nu P'_\mu - g_{\mu\nu} P \cdot P' + i \varepsilon_{\mu\nu\alpha\beta} P^\alpha P'^\beta)$$
$$\times \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q'}{(2\pi)^4} \varphi_P(q) \varphi_{P'}(q') L_b^{\mu\nu}(q - q', p, p'; P - P').$$

For all mesons in question, $m_M \ll m_W$, and we can use the leading current-current approximation in lepton tensors $L_i^{\mu\nu}$,

$$\frac{g^4}{4}\frac{1}{p_i^2 - m_W^2}\frac{1}{p_i'^2 - m_W^2} \to 8G_F^2 .$$

Further we will consider two limiting cases of heavy and light Majorana neutrinos.

***Heavy neutrinos:** $m_N \gg m_M$

For this case, the tree and box lepton tensors are equal to each other in the leading order of the expansion $1/m_N^2$:

$$L_t^{\mu\nu} = L_b^{\mu\nu} = -16 g^{\mu\nu} L(p,p'),$$

$$L(p,p') = G_F^2 \left\langle m_{\ell\ell'}^{-1} \right\rangle \left(\overline{v}^c(p) \frac{1-\gamma^5}{2} v(p) \right).$$

Here the effective inverse Majorana mass

$$\left\langle m_{\ell\ell'}^{-1} \right\rangle = \left| \sum_N U_{\ell N} U_{\ell' N} \eta_N \frac{1}{m_N} \right| .$$

The total amplitude of the decay is model independent in this limit:

$$A = A_t + A_b = 4K_V f_M f_{M'} (P \cdot P') L(p, p'),$$

$$K_V = V_{12} V_{43} + \frac{1}{N_c} V_{13} V_{42} .$$

The color suppression factor $1/N_c = 1/3$ is present in the box amplitude (independently of the value of the neutrino mass).

Indeed, in Fig. 4, in the tree diagram the color summation takes place independently in the two quark loops. Not so in the box diagram, where the color of the quark q_1 (antiquark $\bar{q}_2$) must be the same as the color of the quark q_3 (antiquark $\bar{q}_4$) .

The decay width

$$\Gamma_{\ell\ell'} = \frac{G_F^4 m_M^7}{128\pi^3} f_M^2 f_{M'}^2 \left|K_V\right|^2 \left\langle m_{\ell\ell'}^{-1} \right\rangle^2 \Phi_{\ell\ell'}.$$

Here $\Phi_{\ell\ell'}$ is the reduced phase space integral. For identical leptons ($\ell' = \ell$)

$$\Phi_{\ell\ell} = \int_{4z_0}^{z_1} dz(z-2z_0)\left[\left(1-\frac{4z_0}{z}\right)(z_1-z)(z_2-z)\right]^{1/2}(1+z_3-z)^2.$$

For distinct leptons ($\ell' \neq \ell$), assuming $m_{\ell'}/m_\ell \ll 1$, in the leading approximation

$$\Phi_{\ell\ell'} = 2\int_{z_0}^{z_1} \frac{dz}{z}(z-z_0)^2[(z_1-z)(z_2-z)]^{1/2}(1+z_3-z)^2.$$

Here the mass parameters

$$z_0 = \left(\frac{m_\ell}{m_M}\right)^2, z_{1,2} = (1 \mp \sqrt{z_3})^2, z_3 = \left(\frac{m_{M'}}{m_M}\right)^2,$$

The variable of integration

$$z = (P - P')^2 / m_M^2$$

is the normalized invariant mass of the lepton pair.
The branching ratios

$$B_{\ell\ell'}(MM') = \frac{\Gamma(M^+ \to M'^- \ell^+ \ell'^+)}{\Gamma(M^+ \to all)} = C_{\ell\ell'}(MM') \left\langle m_{\ell\ell'}^{-1} \right\rangle^2 .$$

The results of numerical calculations for the decays

$$K^+ \to \pi^- \ell^+ \ell'^+, \;\; D^+ \to K^- \ell^+ \ell'^+$$

are shown in Table 1.

Table 1. Experimental and indirect upper bounds on the branching ratios $B_{\ell\ell'}$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by heavy Majorana neutrinos (with $m_N \gg m_M$)

Rare decay	Exp. upper bound on $B_{\ell\ell'}$	$C_{\ell\ell'}$ [MeV^2]	Ind. bound on $B_{\ell\ell'}$
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	8.5×10^{-10}	5.9×10^{-32}
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	2.4×10^{-10}	1.1×10^{-24}
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	1.0×10^{-9}	5.1×10^{-24}
$D^+ \to K^- e^+ e^+$	1.2×10^{-4}	2.2×10^{-9}	1.5×10^{-31}
$D^+ \to K^- \mu^+ \mu^+$	1.3×10^{-5}	2.0×10^{-9}	8.9×10^{-24}
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	4.2×10^{-9}	2.1×10^{-23}

For $K^+ \to \pi^- \ell^+ \ell'^+$

$$f_\pi = 130.7\ \text{MeV},\quad f_K = 159.8\ \text{MeV};$$

$$V_{12} = V_{42} = V_{us} = 0.2200, V_{43} = V_{13} = V_{ud} = 0.9738;$$

$$A_b = \frac{1}{N_c} A_t .$$

For $D^+ \to K^- \ell^+ \ell'^+$

$$f_D = 228\ \text{MeV}\ \ \text{(lattice QCD)};$$

$$V_{12} = V_{cd} = -0.224, V_{43} = V_{us} = 0.2200,$$

$$V_{13} = V_{cs} = 0.996, V_{42} = V_{ud} = 0.9738$$

with strong (double) Cabibbo suppression of the tree amplitude:

$$\frac{A_t}{A_b} = N_c \frac{V_{cd} V_{us}}{V_{cs} V_{ud}} \simeq -0.15.$$

The direct bounds on the inverse Majorana masses are unphysical:

$$\left\langle m_{\ell\ell'}^{-1} \right\rangle_{\exp} = (B_{\ell\ell'}^{\exp} / C_{\ell\ell'})^{1/2} \gg (m_M - m_{M'})^{-1}$$

in contrast with

$$\left\langle m_{\ell\ell'}^{-1} \right\rangle < \sum_N \frac{1}{m_N}.$$

The ***indirect*** bounds on $B_{\ell\ell'}$ have been obtained with use of experimental data from other processes:

$$\left\langle m_{ee}^{-1} \right\rangle < \left(1.2\times 10^{8}\ \text{GeV}\right)^{-1} \text{ [from } 0\nu\beta\beta\text{];}$$

$$\sum_N \left|U_{eN}\right|^2 < 6.6\times 10^{-3}, \sum_N \left|U_{\mu N}\right|^2 < 6.0\times 10^{-3} \text{ [from the precision electroweak data],}$$

$$m_N > 90.7\ \text{GeV}$$

[from the direct searches for heavy neutral leptons with Majorana coupling to μ]

The above limits yield the upper bounds

$$\langle m_{\mu\mu}^{-1} \rangle < (1.5\times10^{4}\ \mathrm{GeV})^{-1}, \langle m_{e\mu}^{-1} \rangle < (1.4\times10^{4}\ \mathrm{GeV})^{-1},$$

that have been used for calculating the indirect bounds on $\mathrm{B}_{\ell\ell'}$.

***Light neutrinos:** $m_N \ll m_\ell,\, m_{\ell'}$

In this case, the lepton tensors, neglecting m_N^2 in the dominators, can be approximated as

$$L_i^{\mu\nu} = 8G_F^2 \langle m_{\ell\ell'} \rangle \left(\bar{v}^c(p) \left[\frac{\gamma^\mu \gamma^\nu}{(p_i - p)^2} + \frac{\gamma^\nu \gamma^\mu}{(p_i - p')^2} \right] \frac{1-\gamma^5}{2} v(p') \right),$$

where the effective Majorana mass is introduced as

$$\langle m_{\ell\ell'} \rangle = \left| \sum_N U_{\ell N} U_{\ell' N} \eta_N m_N \right|.$$

The width of RMD is a sum of the contributions of the tree and box amplitudes, as well as their interference:

$$\Gamma = \Gamma_t + \Gamma_b + \Gamma_{tb}.$$

The tree contribution is model independent,

$$\Gamma_t = \frac{G_F^4 m_M^3}{16\pi^3} f_M^2 f_{M'}^2 \left|V_{12}V_{43}\right|^2 \langle m_{\ell\ell'} \rangle^2 \phi_{\ell\ell'}.$$

Here the reduced phase space integral has a rather complicated expression but the realistic limit of massless leptons, $m_\ell / m_M \to 0$ and $m_{\ell'} / m_M \to 0,$
It can be approximated as

$$\phi_{\ell\ell'} \simeq \left(1 - \frac{1}{2}\delta_{\ell\ell'}\right)\varphi(z_3),$$

$$\varphi(z_3) = (1 - z_3)\left[2z_3 + \frac{1}{6}(1 - z_3)^2\right] + z_3(1 + z_3)\ln z_3,$$

$$z_3 = (m_M / m_{M'})^2.$$

The box and interferential tree-box contributions depend on the form of the BS amplitudes for mesons. For the Gaussian model described beforehand, after integration over relative quark-antiquark 4-momenta q and q' in the initial and final mesons with use of the Fock—Schwinger proper-time representation for the neutrino propagator,

$$(\Delta + i0)^{-1} = -i\int_0^\infty ds \exp[is(\Delta + i0)],$$

and trivial integration over one of the phase-space angular variables we have obtained the six- and five-dimensional integral representations for Γ_b and Γ_{tb}, respectively.

The results of numerical calculation of the ratios

$$c_{\ell\ell'} = \frac{B_{\ell\ell'}}{\langle m_{\ell\ell'}\rangle^2} = c_t + c_b + c_{tb}$$

are shown in Table 2.

Table 2. The contributions of the tree and box amplitudes and their interference to $c_{\ell\ell'}$ and indirect upper bounds on the branching ratios $\mathbf{B}_{\ell\ell'}$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by light Majorana neutrinos (with $m_N \ll m_\ell, m_{\ell'}$)

Rare decay	c_t [MeV^{-2}]	c_b [MeV^{-2}]	c_{tb} [MeV^{-2}]	$c_{\ell\ell'}$ [MeV^{-2}]	Ind. bound on $\mathrm{B}_{\ell\ell'}$
$K^+ \to \pi^- e^+ e^+$	5.3×10^{-20}	3.6×10^{-22}	-8.8×10^{-21}	4.4×10^{-20}	2.3×10^{-33}
$K^+ \to \pi^- \mu^+ \mu^+$	1.4×10^{-20}	1.2×10^{-22}	-2.1×10^{-21}	1.2×10^{-20}	6.2×10^{-34}
$K^+ \to \pi^- e^+ \mu^+$	6.2×10^{-20}	7.2×10^{-22}	-1.8×10^{-20}	4.0×10^{-20}	9.1×10^{-34}
$D^+ \to K^- e^+ e^+$	4.1×10^{-23}	3.6×10^{-21}	8.5×10^{-22}	4.5×10^{-21}	2.4×10^{-34}
$D^+ \to K^- \mu^+ \mu^+$	4.0×10^{-23}	3.3×10^{-21}	7.6×10^{-22}	4.1×10^{-21}	2.2×10^{-34}
$D^+ \to K^- e^+ \mu^+$	8.2×10^{-23}	7.3×10^{-21}	1.7×10^{-21}	9.1×10^{-21}	2.0×10^{-34}

We have used the current quark masses

$$m_u = 4\ \text{MeV},\ m_d = 7\ \text{MeV},\ m_s = 150\ \text{MeV},\ m_c = 1.26\ \text{GeV}$$

and the upper bounds on the effective Majorana masses

$$\langle m_{\ell\ell} \rangle < 0.23\ \text{eV}\ (\ell = e, \mu),$$
$$\langle m_{e\mu} \rangle < 0.15\ \text{eV}.$$

These bounds have been derived assuming three light neutrinos ($m_1 < m_2 < m_3$) with use of the bound

$$m_3 < 0.23\ \text{eV [from recent cosmological data and neutrino oscillations]}$$

and the best fit for the neutrino mixing matrix from the neutrino oscillation measurements

$$U_{\text{bf}} = \begin{pmatrix} 0.84 & 0.55 & 0.00 \\ -0.39 & 0.59 & 0.71 \\ 0.39 & -0.59 & 0.71 \end{pmatrix}.$$

As well as for the case of heavy neutrinos, the direct bounds on the effective Majorana masses of light neutrinos are unphysical:

$$\langle m_{\ell\ell'} \rangle_{\exp} = \left(B_{\ell\ell'}^{\exp} / c_{\ell\ell'} \right)^{1/2} \gg m_{\ell}, m_{\ell'} .$$

For example, from $K^+ \to \pi^- e^+ \mu^+$ it follows

$$\langle m_{e\mu} \rangle_{\exp} = 73\,\mathrm{GeV} \gg m_e, m_\mu .$$

The results obtained for other rare meson decays are shown in Tables 3 and 4.

Table 3. Bounds on $\langle m_{\ell\ell'}^{-1}\rangle^{-1}$ and indirect bounds on the branching ratios $\mathbf{B}_{\ell\ell'}(M)$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by heavy Majorana neutrinos ($m_N \gg m_M$) and present experimental bounds

Rare decay	Exp. upper bounds on $\mathrm{B}_{\ell\ell'}(M)$	Theor. estimate for $\mathrm{B}_{\ell\ell'}(M)/\langle m_{\ell\ell'}^{-1}\rangle^2$ [MeV^2]	Bounds on $\langle m_{\ell\ell'}^{-1}\rangle^{-1}$ [keV]	Ind. bounds on $\mathrm{B}_{\ell\ell'}(M)$
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	8.6×10^{-10}	1200	2.2×10^{-30}
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	2.5×10^{-10}	300	3.5×10^{-20}
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	8.4×10^{-10}	1300	1.2×10^{-19}
$D^+ \to \pi^- e^+ e^+$	9.6×10^{-5}	2.2×10^{-9}	4.8	5.5×10^{-30}
$D^+ \to \pi^- \mu^+ \mu^+$	1.7×10^{-5}	2.0×10^{-9}	11	2.8×10^{-19}
$D^+ \to \pi^- e^+ \mu^+$	5.0×10^{-5}	4.2×10^{-9}	9.2	5.9×10^{-19}
$D^+ \to K^- e^+ e^+$	1.2×10^{-4}	2.2×10^{-9}	4.3	5.5×10^{-30}
$D^+ \to K^- \mu^+ \mu^+$	1.2×10^{-4}	2.1×10^{-9}	4.1	3.0×10^{-19}
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	4.3×10^{-9}	5.7	6.1×10^{-19}
$D_s^+ \to \pi^- e^+ e^+$	6.9×10^{-4}	1.9×10^{-8}	5.2	4.8×10^{-29}
$D_s^+ \to \pi^- \mu^+ \mu^+$	8.2×10^{-5}	1.8×10^{-8}	15	2.5×10^{-18}
$D_s^+ \to \pi^- e^+ \mu^+$	7.3×10^{-4}	3.6×10^{-8}	7.1	5.1×10^{-18}
$D_s^+ \to K^- e^+ e^+$	6.3×10^{-4}	2.2×10^{-9}	1.9	5.5×10^{-30}
$D_s^+ \to K^- \mu^+ \mu^+$	1.8×10^{-4}	2.0×10^{-9}	3.4	2.8×10^{-19}
$D_s^+ \to K^- e^+ \mu^+$	6.8×10^{-4}	4.2×10^{-9}	2.5	5.9×10^{-19}
$B^+ \to \pi^- e^+ e^+$	3.9×10^{-3}	$(0.3 \div 1.9) \times 10^{-9}$	$0.3 \div 0.7$	4.8×10^{-30}
$B^+ \to \pi^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.3 \div 1.9) \times 10^{-9}$	$0.2 \div 0.5$	2.7×10^{-19}
$B^+ \to \pi^- e^+ \mu^+$	6.4×10^{-3}	$(0.6 \div 3.8) \times 10^{-9}$	$0.3 \div 0.8$	5.4×10^{-19}
$B^+ \to \pi^- \tau^+ \tau^+$		$(0.2 \div 1.2) \times 10^{-9}$		1.7×10^{-19}
$B^+ \to \pi^- e^+ \tau^+$		$(1.0 \div 6.2) \times 10^{-10}$		4.0×10^{-19}
$B^+ \to \pi^- \mu^+ \tau^+$		$(1.0 \div 6.2) \times 10^{-10}$		8.8×10^{-20}
$B^+ \to K^- e^+ e^+$	3.9×10^{-3}	$(0.2 \div 1.5) \times 10^{-10}$	$0.07 \div 0.20$	3.8×10^{-31}
$B^+ \to K^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.2 \div 1.5) \times 10^{-10}$	$0.05 \div 0.13$	2.1×10^{-20}
$B^+ \to K^- e^+ \mu^+$	6.4×10^{-3}	$(0.5 \div 2.9) \times 10^{-10}$	$0.09 \div 0.21$	4.1×10^{-20}
$B^+ \to K^- \tau^+ \tau^+$		$(1.3 \div 8.4) \times 10^{-12}$		1.2×10^{-21}
$B^+ \to K^- e^+ \tau^+$		$(0.7 \div 4.4) \times 10^{-11}$		6.2×10^{-21}
$B^+ \to K^- \mu^+ \tau^+$		$(0.7 \div 4.4) \times 10^{-11}$		6.2×10^{-21}

Table 4. Bounds on $\langle m_{\ell\ell'} \rangle$ and indirect bounds on the branching ratios $\mathbf{B}_{\ell\ell'}(M)$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by light Majorana neutrinos $(m_N \ll m_\ell, m_{\ell'})$ and present experimental bounds

Rare decay	Exp. upper bounds on $B_{\ell\ell'}(M)$	Theor. estimate for $B_{\ell\ell'}(M)/\langle m_{\ell\ell'} \rangle^2$ [MeV^{-2}]	Bounds on $\langle m_{\ell\ell'} \rangle$ [TeV]	Ind. bounds on $B_{\ell\ell'}(M)$
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	5.1×10^{-20}	0.11	5.1×10^{-32}
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	1.4×10^{-20}	0.47	1.1×10^{-30}
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	6.2×10^{-20}	0.09	5.0×10^{-30}
$D^+ \to \pi^- e^+ e^+$	9.6×10^{-5}	1.2×10^{-21}	280	1.2×10^{-33}
$D^+ \to \pi^- \mu^+ \mu^+$	1.7×10^{-5}	1.2×10^{-21}	120	9.7×10^{-32}
$D^+ \to \pi^- e^+ \mu^+$	5.0×10^{-5}	2.3×10^{-21}	150	1.9×10^{-31}
$D^+ \to K^- e^+ e^+$	1.2×10^{-4}	2.3×10^{-21}	230	2.3×10^{-33}
$D^+ \to K^- \mu^+ \mu^+$	1.2×10^{-4}	2.3×10^{-21}	230	1.8×10^{-31}
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	4.5×10^{-21}	170	3.7×10^{-31}
$D_s^+ \to \pi^- e^+ e^+$	6.9×10^{-4}	1.5×10^{-20}	210	1.5×10^{-32}
$D_s^+ \to \pi^- \mu^+ \mu^+$	8.2×10^{-5}	1.5×10^{-20}	74	1.2×10^{-30}
$D_s^+ \to \pi^- e^+ \mu^+$	7.3×10^{-4}	3.1×10^{-20}	150	2.5×10^{-30}
$D_s^+ \to K^- e^+ e^+$	6.3×10^{-4}	5.6×10^{-22}	1100	5.6×10^{-34}
$D_s^+ \to K^- \mu^+ \mu^+$	1.8×10^{-4}	5.6×10^{-22}	570	4.5×10^{-32}
$D_s^+ \to K^- e^+ \mu^+$	6.8×10^{-4}	1.1×10^{-21}	780	8.9×10^{-32}
$B^+ \to \pi^- e^+ e^+$	3.9×10^{-3}	$(0.3 \div 1.8) \times 10^{-23}$	$(1.5 \div 3.6) \times 10^4$	1.8×10^{-35}
$B^+ \to \pi^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.3 \div 1.8) \times 10^{-23}$	$(2.2 \div 5.5) \times 10^4$	1.5×10^{-33}
$B^+ \to \pi^- e^+ \mu^+$	6.4×10^{-3}	$(0.6 \div 3.6) \times 10^{-23}$	$(1.3 \div 3.3) \times 10^4$	2.9×10^{-33}
$B^+ \to \pi^- \tau^+ \tau^+$		$(1.5 \div 9.6) \times 10^{-25}$		7.8×10^{-35}
$B^+ \to \pi^- e^+ \tau^+$		$(0.4 \div 2.4) \times 10^{-23}$		1.9×10^{-33}
$B^+ \to \pi^- \mu^+ \tau^+$		$(0.4 \div 2.4) \times 10^{-23}$		1.9×10^{-33}
$B^+ \to K^- e^+ e^+$	3.9×10^{-3}	$(0.2 \div 1.2) \times 10^{-24}$	$(0.6 \div 1.4) \times 10^5$	1.2×10^{-36}
$B^+ \to K^- \mu^+ \mu^+$	9.1×10^{-3}	$(0.2 \div 1.2) \times 10^{-24}$	$(0.9 \div 2.2) \times 10^5$	9.7×10^{-35}
$B^+ \to K^- e^+ \mu^+$	6.4×10^{-3}	$(0.4 \div 2.4) \times 10^{-24}$	$(0.5 \div 1.3) \times 10^5$	1.9×10^{-34}
$B^+ \to K^- \tau^+ \tau^+$		$(1.0 \div 6.1) \times 10^{-25}$		4.9×10^{-35}
$B^+ \to K^- e^+ \tau^+$		$(0.2 \div 1.2) \times 10^{-24}$		9.7×10^{-35}
$B^+ \to K^- \mu^+ \tau^+$		$(0.2 \div 1.2) \times 10^{-24}$		9.7×10^{-35}

Compare our results for the decay

$$K^+ \to \pi^- \mu^+ \mu^+$$

with the results of Ref. [M. A. Ivanov and S. G. Kovalenko, Phys. Rev. D **71,** 053004 (2005); hep-ph/0412198] where a more complicated model for the BS amplitudes for mesons has been used.

For heavy neutrinos ($m_N \gg m_K \simeq 493.7\ \mathrm{MeV}$):

$$C_{\mu\mu} = \frac{\mathrm{B}_{\mu\mu}}{\langle m_{\mu\mu}^{-1} \rangle^2} = 2.4 \times 10^{-10}\ \mathrm{MeV}^2;$$

$$C_{\mu\mu}^{\mathrm{IK}} = \frac{m_N^2 \left(\Gamma_{\mu\mu} / |U_{\mu N}|^4 \right)^{\mathrm{IK}}}{\Gamma\left(K^+ \to \mathrm{all}\right)} = \left(2.4, 2.6, 2.6, 2.6\right) \times 10^{-10}\ \mathrm{MeV}^2$$

for $m_N = (1, 250, 500, 750)\ \mathrm{GeV}.$

For light neutrinos ($m_N \ll m_\mu \simeq 105.6\ \mathrm{MeV}$):

$$c_{\mu\mu} = \frac{\mathrm{B}_{\mu\mu}}{\langle m_{\mu\mu} \rangle^2} = 1.2\times10^{-20}\ \mathrm{MeV}^{-2}, \quad \frac{\Gamma_{\mu\mu}}{\Gamma^{(t)}_{\mu\mu}} = \frac{c_{\mu\mu}}{c_t} = 0.86;$$

$$c^{\mathrm{IK}}_{\mu\mu} = \frac{\left(\Gamma_{\mu\mu} / |U_{\mu N}|^4\right)^{\mathrm{IK}}}{m_N^2 \Gamma\left(K^+ \to \mathrm{all}\right)} = \left(1.2, 1.2, 1.2, 1.2, 0.75\right)\times10^{-20}\ \mathrm{MeV}^{-2},$$

$$\left(\frac{\Gamma_{\mu\mu}}{\Gamma^{(t)}_{\mu\mu}}\right)^{IK} = (0.85, 0.85, 0.85, 0.85, 1.00)$$

for $m_N = (10^{-6}, 5\times10^{-4}, 0.25, 1, 500)$ MeV.

Rough estimates of the strongly Cabibbo suppressed decays $D^+ \to K^- \ell^+ \ell'^+$:

$$c_{\ell\ell'} \sim D c_t , \quad D = \left[1 + \frac{1}{N_c} \left| \frac{V_{cs} V_{ud}}{V_{cd} V_{us}} \right| \right]^2 = \left[1 + \frac{1}{3} \left| \frac{0.996 \times 0.9738}{(-0.224) \times 0.2200} \right| \right]^2 \simeq 57;$$

$$\left(c_{ee}, c_{\mu\mu}, c_{e\mu} \right) \sim (2,2,5) \times 10^{-21}\ \mathrm{MeV}^{-2}.$$

These estimates are in agreement (up to a factor about 2) with calculated ones (see Table 2):

$$\left(c_{ee}, c_{\mu\mu}, c_{e\mu} \right) = (4.5, 4.1, 9.1) \times 10^{-21}\ \mathrm{MeV}^{-2}.$$

Conclusion

We have examined lepton-number violating rare meson decays

$$M^+ \to M'^- \ell^+ \ell'^+ \quad (\ell, \ell' = e, \mu)$$

mediated by heavy or light Majorana neutrinos. The effects of meson structure are taken into account in the framework of a Gaussian model for Bethe—Salpeter amplitudes. We have shown that the present direct experimental bounds on the branching ratios of the rare decays are too weak to set reasonable limits on effective Majorana masses, $\langle m_{\ell\ell'} \rangle$ (for light neutrinos) and $\langle m_{\ell\ell'}^{-1} \rangle^{-1}$ (for heavy ones).

Therefore, a very substantial improvement of the experimental reach on the rare meson decays is needed.

Conversely, if a same-sign dilepton signal is seen in any of the rare meson decays in foreseeable future, it will be due to new physics other than one induced by Majorana neutrinos, e.g., such as *R*-parity violating supersymmetry.