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Physics at the LHC, II

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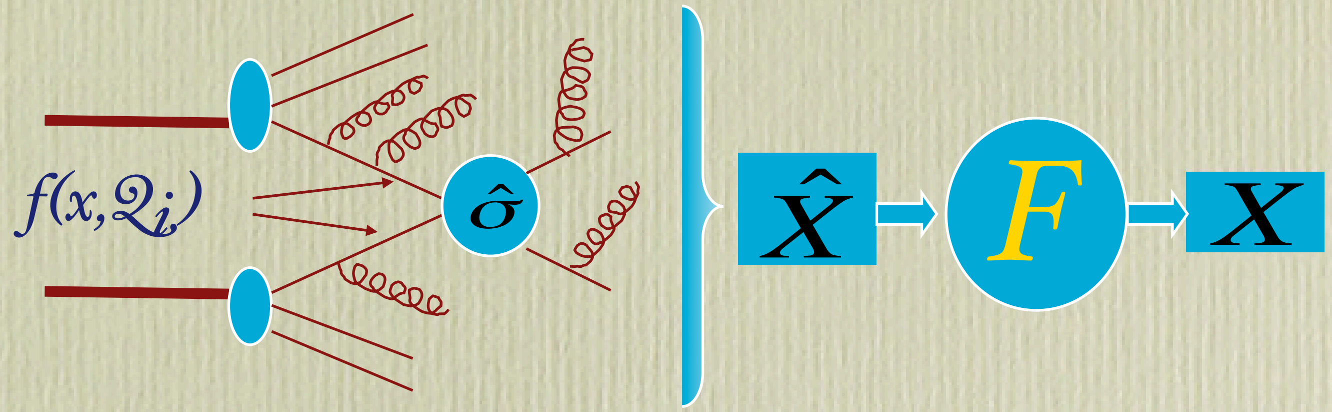
Contents

- The structure of the proton:
  - parton densities
  - their evolution
- Some benchmark SM processes and their applications:
  - Drell-Yan
  - Jets
  - Top quark production

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i)\, f_k(x_2, Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i, Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i, Q_f)$$



$$f_j(x, Q)$$

- sum over all initial state histories leading, at the scale Q, to:

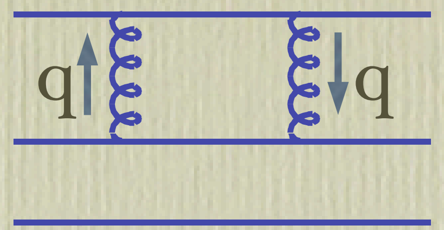
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$$F(\hat{X} \to X; Q_i, Q_f)$$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

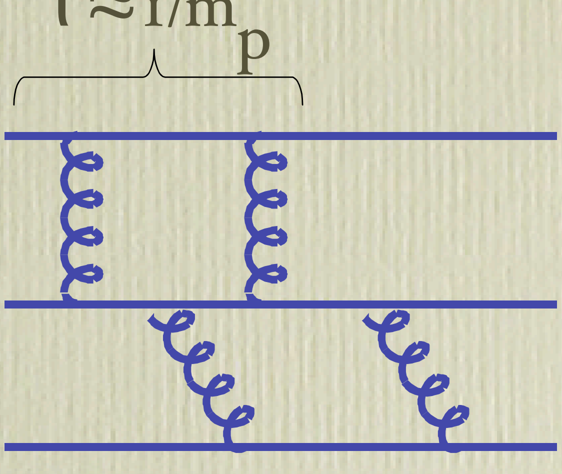
Universality of parton densities and factorization, a naive proof

Exchange of hard gluons among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\stackrel{q>Q}{\sim} \int_q^Q \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)

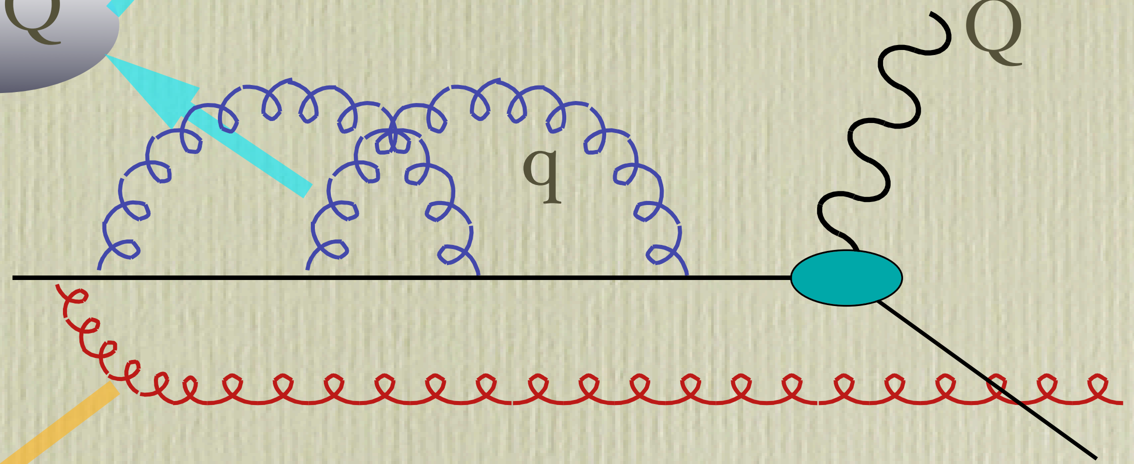


If a hard probe ($Q \gg m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$

q>Q



This gluon cannot be reabsorbed because the quark is gone

q<Q

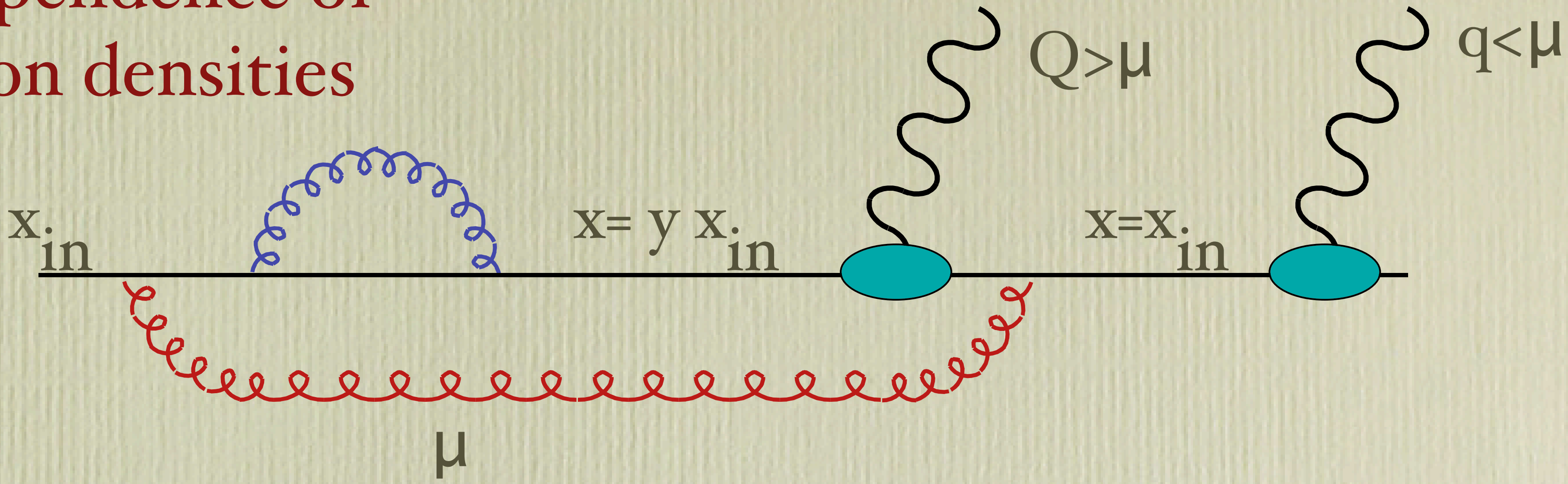
1) $x_{before} \neq x_{after}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere ⇒

Universality of f(x)

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy P(y,q^2)\, \delta(x - y x_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu) P(x/y,\mu^2)$$

One can prove that:

$$P(x,Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} P(x)$$

calculable in pQCD

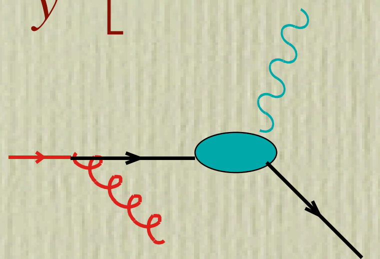
and therefore (Altarelli-Parisi equation):

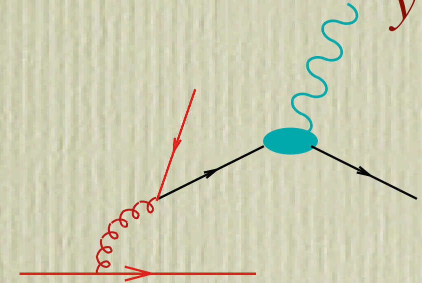
$$\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f(y,\mu) P(x/y)$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t = \log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) + g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$

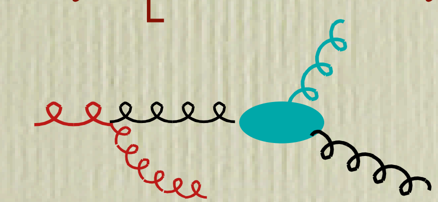


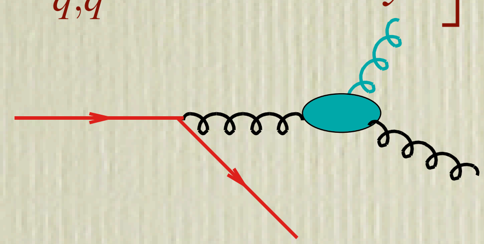


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) + \sum_{q,\bar{q}} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$





$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Example: charm in the proton

$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q) P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density:

$$g(x,Q) \sim A/x$$

we get:

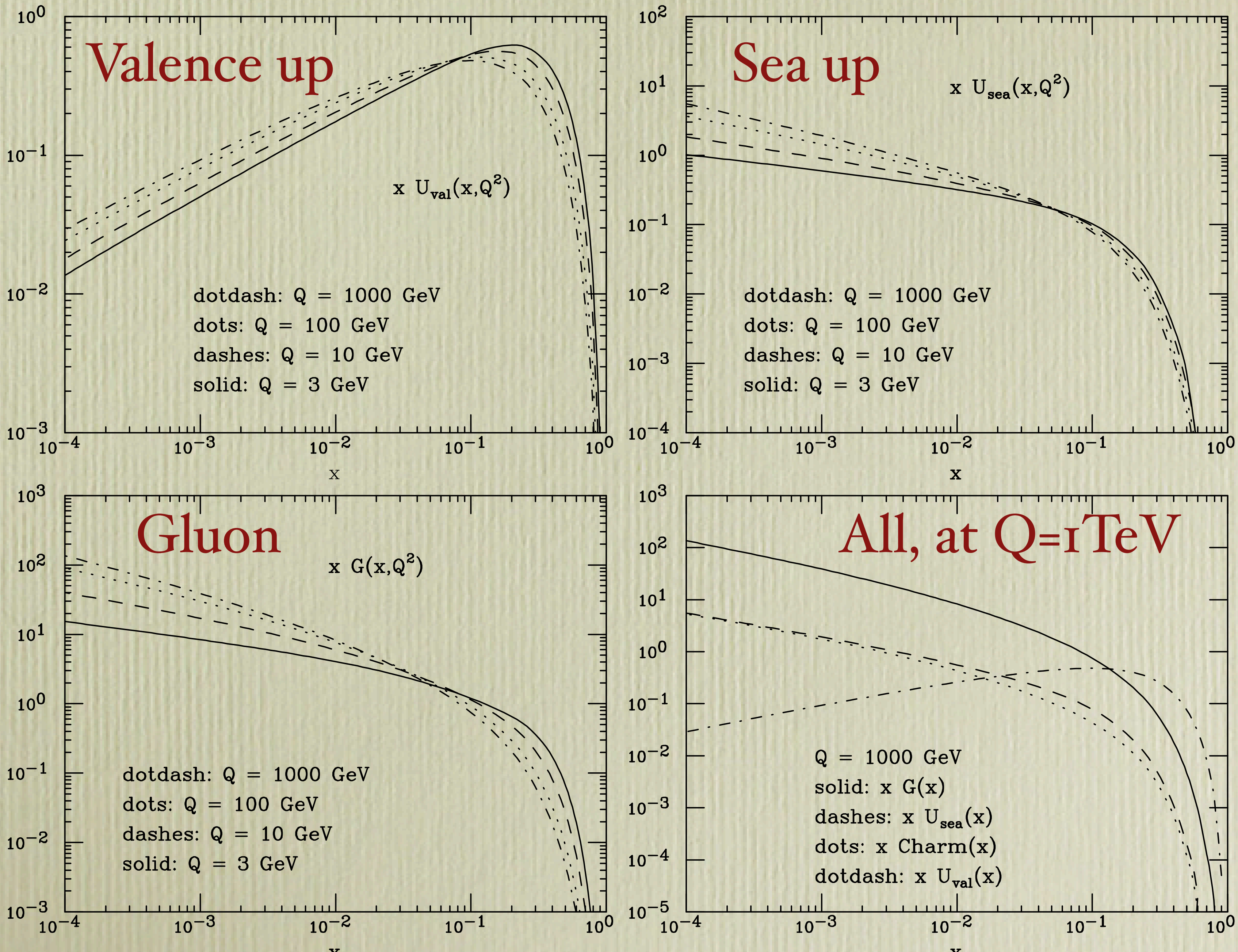
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q) P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2+(1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

and therefore:

$$c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

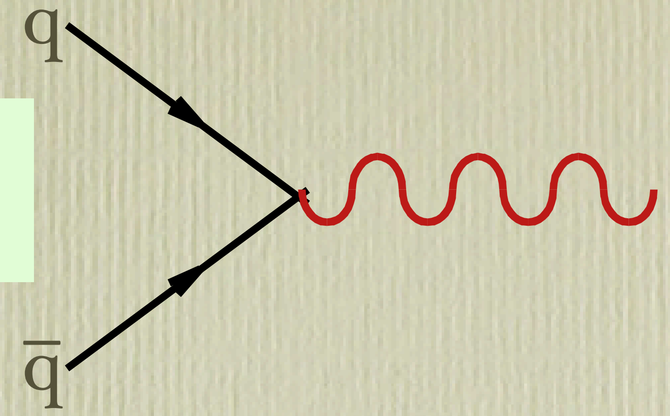
Examples of PDFs and their evolution



Note:
sea ≈10% glue

Note:
charm≈up at high Q

Drell-Yan processes:



$W \to \ell\nu$

$Z \to \ell^+\ell^-$

Goals:

- Tests of QCD: σ(W,Z) known up to NNLO (2-loops)
- Measure m(W) (➙ constrain m(H))
- constrain PDFs (e.g. $f_{up}(x)/f_{down}(x)$)
- search for new gauge bosons: $q\bar{q} \to W', Z'$
- Probe contact interactions: $q\bar{q}\ell^+\ell^-$

LO Cross-section calculation

$$\sigma(pp \to W) = \sum_{q,q'} \int dx_1 dx_2 \, f_q(x_1, Q) \, f_{\bar{q}'}(x_2, Q) \, \frac{1}{2\hat{s}} \int d[PS] \overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2$$

where:

$$\overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2 = \frac{1}{3}\frac{1}{4} 8 g_W^2 \, |V_{qq'}|^2 \hat{s} = \frac{2}{3} \frac{G_F \, m_W^2}{\sqrt{2}} \, |V_{qq'}|^2 \hat{s}$$

$$\begin{aligned} d[PS] &= \frac{d^3 p_W}{(2\pi)^3 p_W^0} (2\pi)^4 \, \delta^4(P_{in} - p_W) \\ &= 2\pi \, d^4 p_W \, \delta(p_W^2 - m_W^2) \, \delta^4(P_{in} - p_W) = 2\pi \, \delta(\hat{s} - m_W^2) \end{aligned}$$

leading to (exercise!):

$$\sigma(pp \to W) = \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \, \tau \int_\tau^1 \frac{dx}{x} f_i(x, Q) \, f_j(\frac{\tau}{x}, Q) \equiv \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \, \tau L_{ij}(\tau)$$

where:

$$\frac{\pi A_{u\bar{d}}}{m_W^2} = 6.5\text{nb} \quad \text{and} \quad \tau = \frac{m_W^2}{S}$$

Some useful relations and definitions

Rapidity: $y = \frac{1}{2}\log\frac{E_W + p_W^z}{E_W - p_W^z}$

Pseudorapidity: $\eta = -\log(\tan\frac{\theta}{2})$

where:

$\tan\theta = \frac{p_T}{p^z}$ and $p_T = \sqrt{p_x^2 + p_y^2}$

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using $\tau = \frac{\hat{s}}{S} = x_1 x_2$ and

$$\begin{cases} E_W = (x_1 + x_2)\, E_{\text{beam}} \\ p_W^z = (x_1 - x_2)\, E_{\text{beam}} \end{cases} \Rightarrow y = \frac{1}{2}\log\frac{x_1}{x_2}$$

prove the following relations:

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y} \qquad dx_1\, dx_2 = dy\, d\tau$$

$$dy = \frac{dx_1}{x_1} \qquad d\tau\,\delta(\hat{s} - m_W^2) = \frac{1}{S}$$

Study the function τL(τ)

Assume, for example, that $f(x) \sim \frac{1}{x^{1+\delta}}\,, \quad 0 < \delta < 1$

Then: $L(\tau) = \int_\tau^1 \frac{dx}{x} \frac{1}{x^{1+\delta}} (\frac{x}{\tau})^{1+\delta} = \frac{1}{\tau^{1+\delta}} \log(\frac{1}{\tau})$

and: $$\sigma_W = \sigma_W^0 \left(\frac{S}{m_W^2}\right)^\delta \log\left(\frac{S}{m_W}\right)$$

Therefore the W cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of e+e- collisions, where cross-sections tend to decrease with CM energy. Note also the following relation, which allows the measurement of the total width of the W boson from the determination of the leptonic rates of W and Z bosons,

$$\Gamma_W = \frac{N(e^+e^-)}{N(e^\pm\nu)} \left(\frac{\sigma_{W^\pm}}{\sigma_Z}\right) \left(\frac{\Gamma^W_{e\nu}}{\Gamma^Z_{e^+e^-}}\right) \Gamma_Z$$

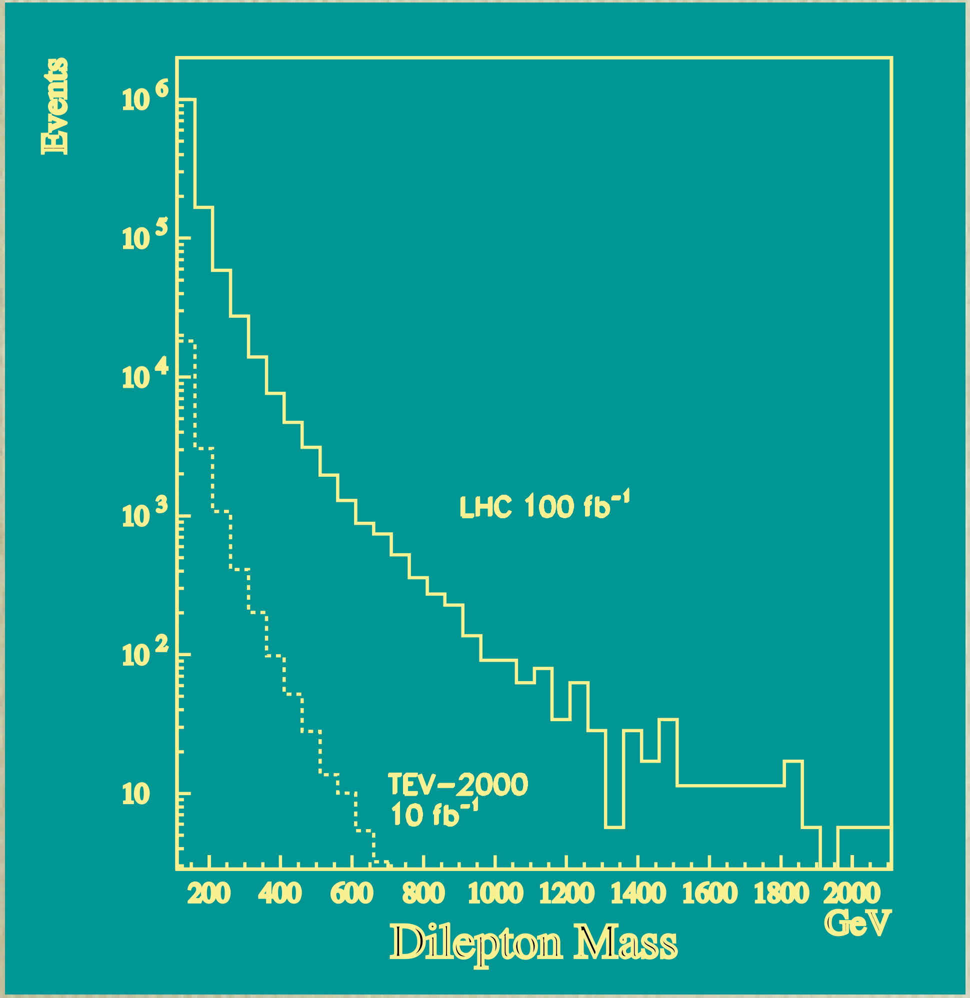
LHC data

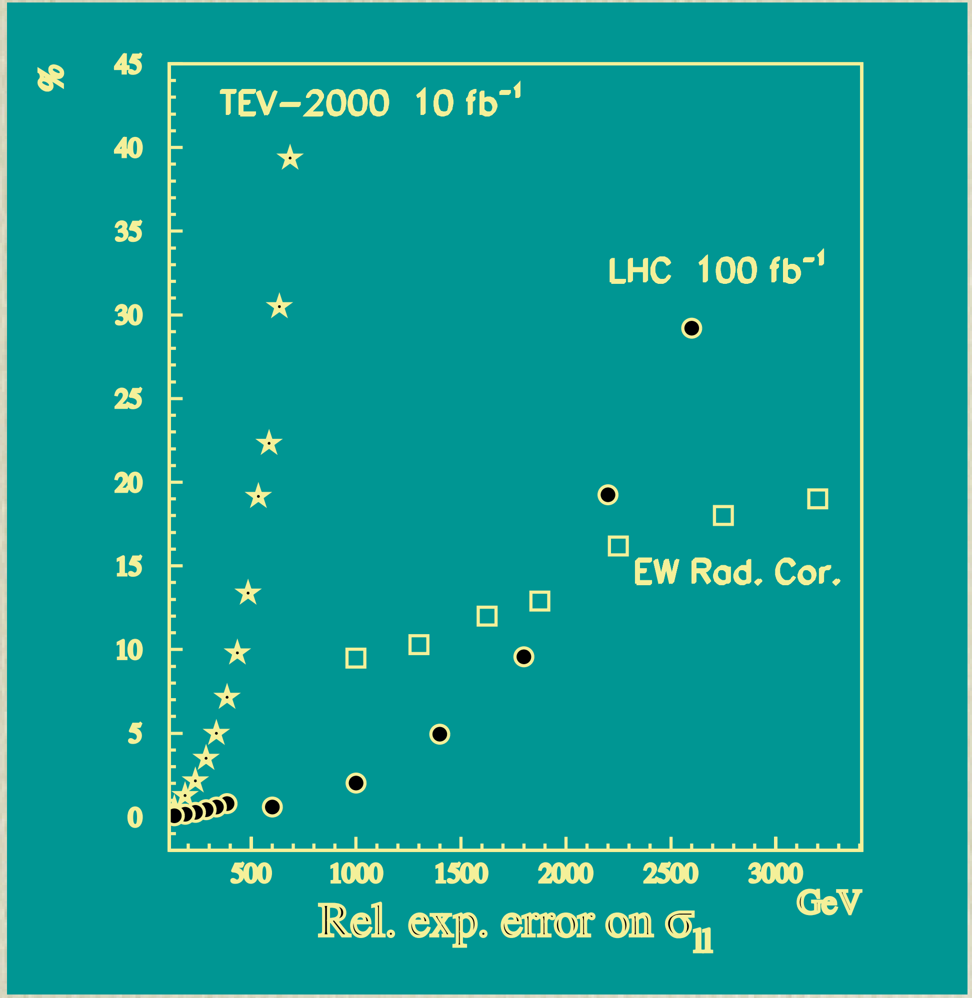
theory

LEP/SLC

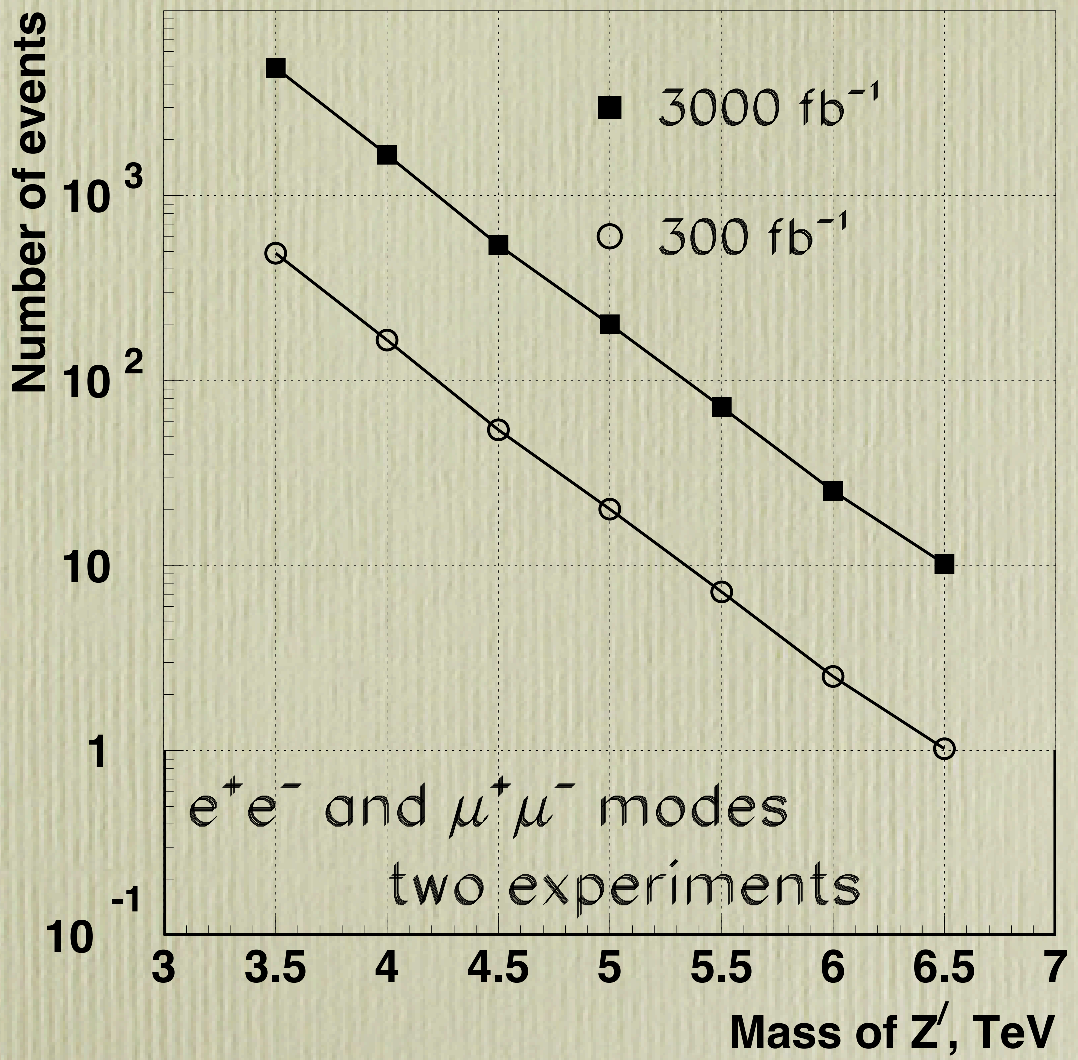
DY final states

m(ll)	Z peak	>110GeV	>400GeV
#(e,μ)/100fb-1	130M	2.6M	33K





Rates and discovery reach for SM-like new Z bosons



As seen from the plot in the previous page, the SM DY rate falls below 1 event/100 fb^{-1} once above m_{DY}>2TeV. In the high mass region the bg contamination (which includes also dilepton pairs from ttbar events) is totally negligible. A discovery based on observation of 10 events, leads to a reach of

5.3 TeV

for the standard high luminosity option, and of

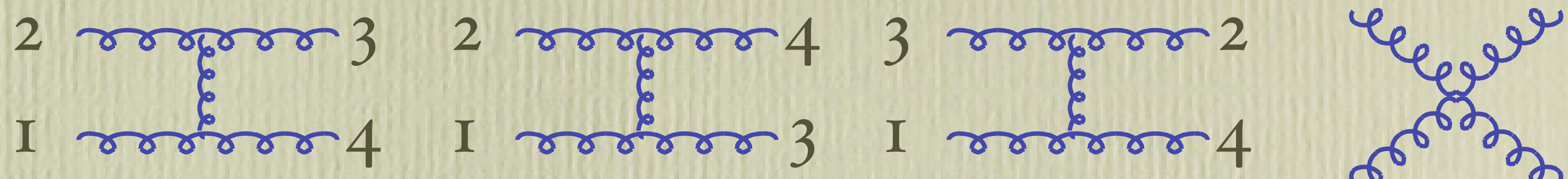
6.5 TeV

for the super-LHC upgrade

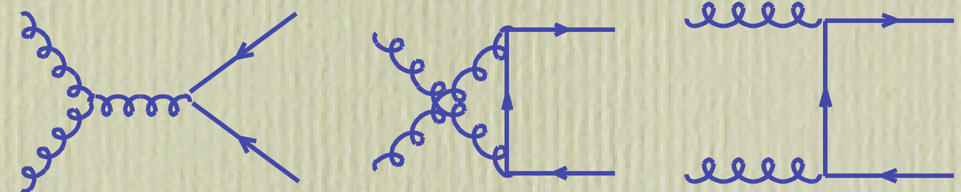
$m_{Z'}$(TeV)	2	3	4	5	6
$\Gamma_{Z'}$(GeV)	62	94	126	158	190

Jet production

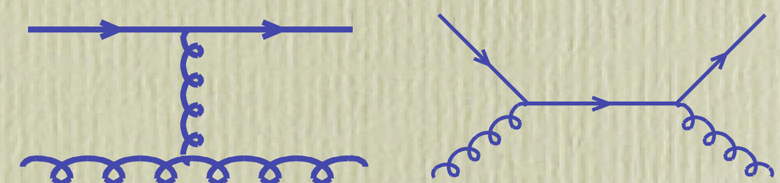
$gg \rightarrow gg$



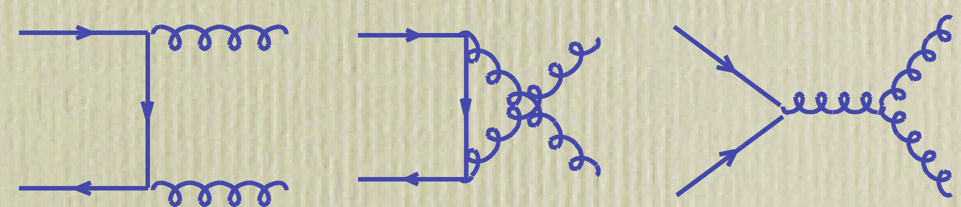
$gg \rightarrow q\bar{q}$



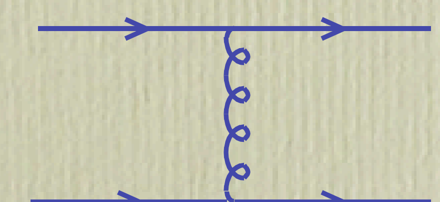
$qg \rightarrow qg$



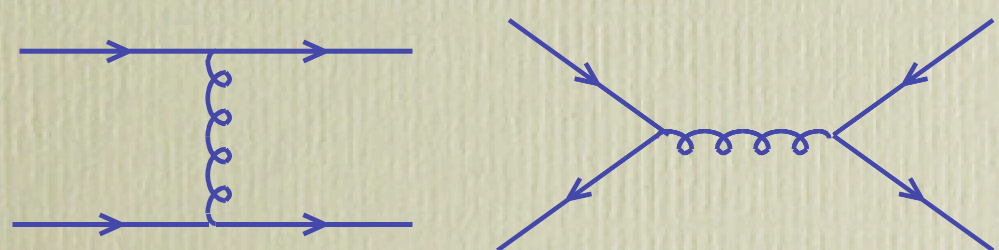
$q\bar{q} \rightarrow gg$



$qq' \rightarrow qq'$



$q\bar{q} \rightarrow q\bar{q}$



- Inclusive production of jets is the largest component of high-Q phenomena in hadronic collisions
- QCD predictions are known up to NLO accuracy
- Intrinsic theoretical uncertainty (at NLO) is approximately 10%
- Uncertainty due to knowledge of parton densities varies from 5-10% (at low transverse momentum, p_T to 100% (at very high p_T, corresponding to high-x gluons)
- Jet are used as probes of the quark structure (possible substructure implies departures from point-like behaviour of cross-section), or as probes of new particles (peaks in the invariant mass distribution of jet pairs)

Phase space and cross-section for LO jet production

$$d[PS] = \frac{d^3p_1}{(2\pi)^2 2p_1^0}\frac{d^3p_2}{(2\pi)^2 2p_2^0}(2\pi)^4\delta^4(P_{in}-P_{out})\,dx_1\,dx_2$$

(a) $\delta(E_{in}-E_{out})\,\delta(P^z_{in}-P^z_{out})\,dx_1\,dx_2 = \frac{1}{2E^2_{beam}}$

(b) $\frac{dp^z}{p^0} = dy \equiv d\eta$

$$d[PS] = \frac{1}{4\pi S}\,p_T\,dp_T\,d\eta_1\,d\eta_2$$

$$\frac{d^3\sigma}{dp_T d\eta_1 d\eta_2} = \frac{p_T}{4\pi S}\sum_{i,j} f_i(x_1)\,f_j(x_2)\,\frac{1}{2\hat{s}}\overline{\sum_{kl}}|M(ij\to kl)|^2$$

The measurement of pT and rapidities for a dijet final state uniquely determines the parton momenta x_1 and x_2. Knowledge of the partonic cross-section allows therefore the determination of partonic densities f(x)

Some more kinematics

Prove as an exercise that

$$x_{1,2} = \frac{p_T}{E_{beam}} \cosh y^* \, e^{\pm y_b}$$

where

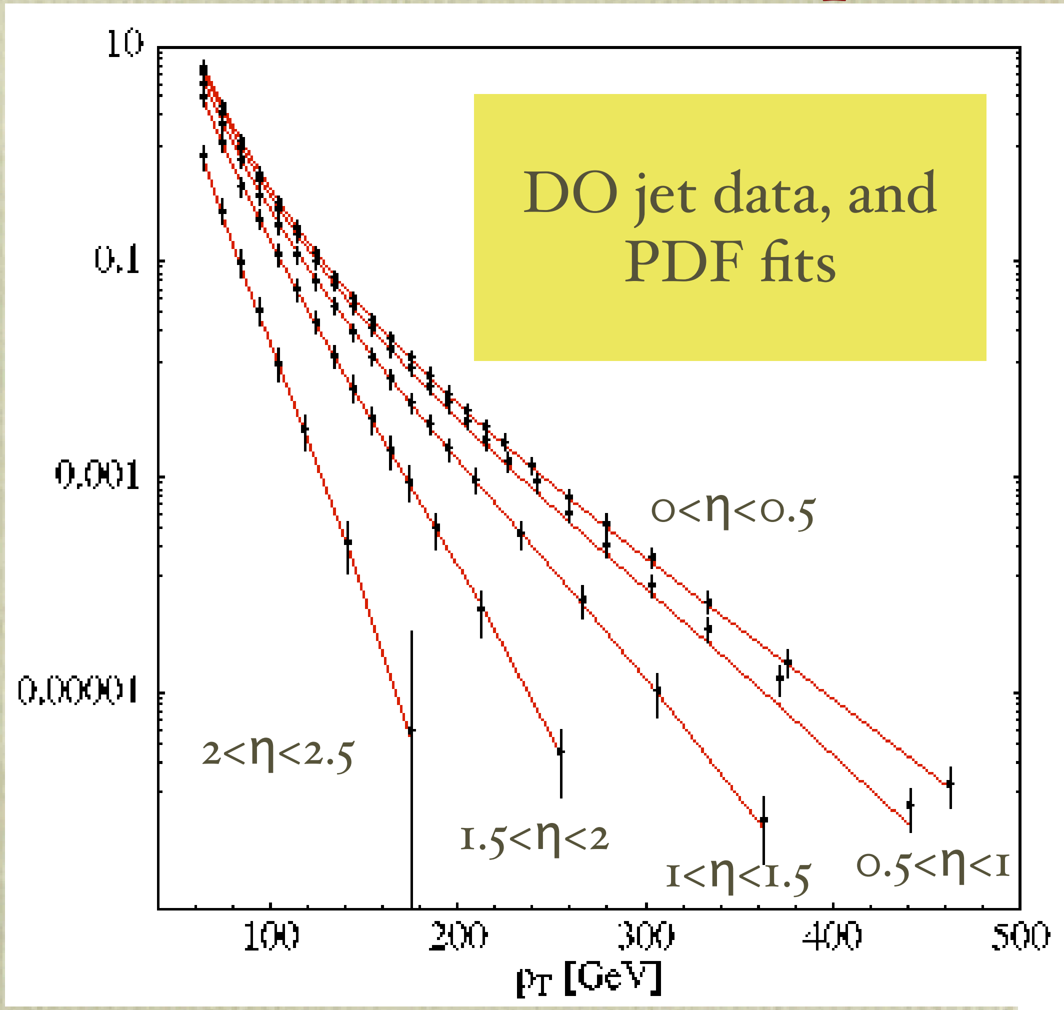
$$y^* = \frac{\eta_1 - \eta_2}{2} \,, \quad y_b = \frac{\eta_1 + \eta_2}{2}$$

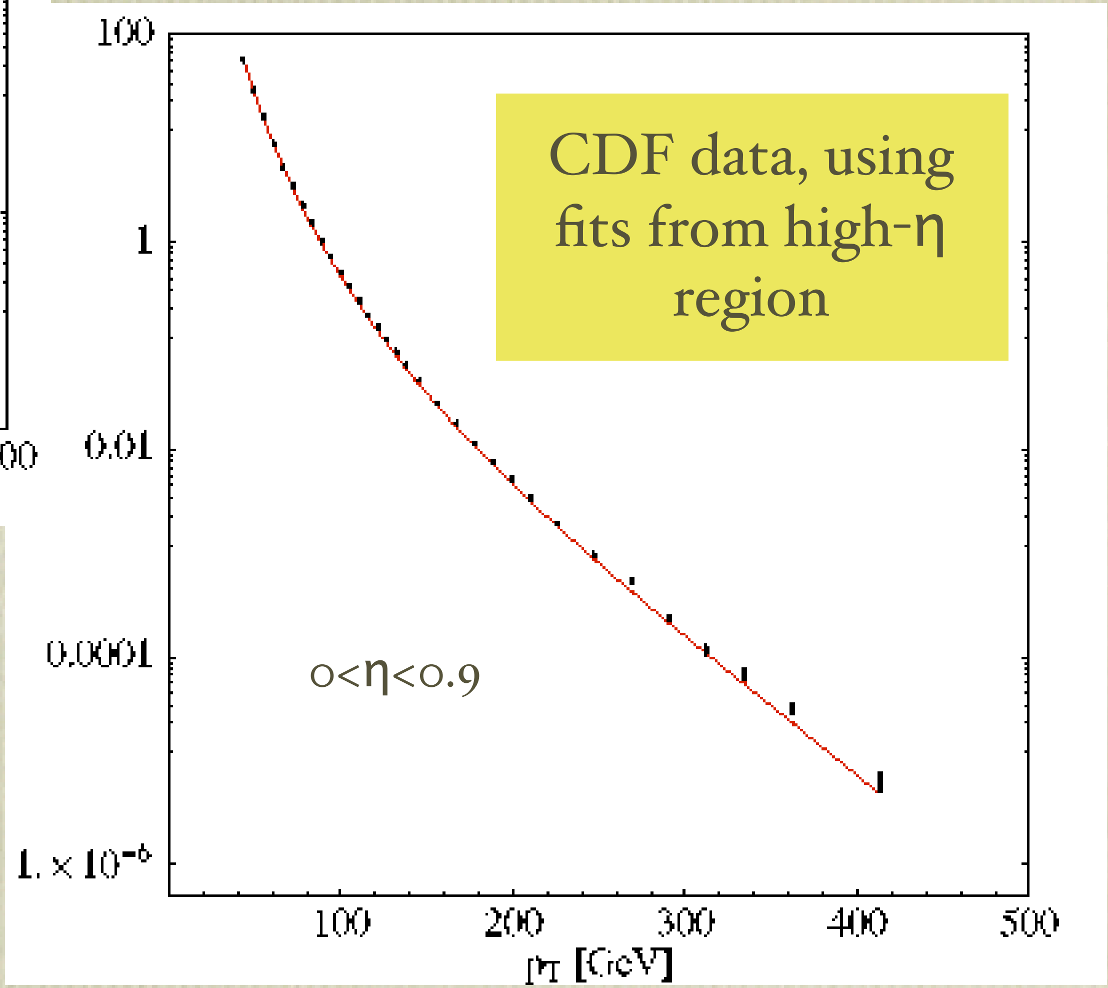
We can therefore reach large values of x either by selecting large invariant mass events:

$$\frac{p_T}{E_{beam}} \cosh y^* \equiv \sqrt{\tau} \to 1$$

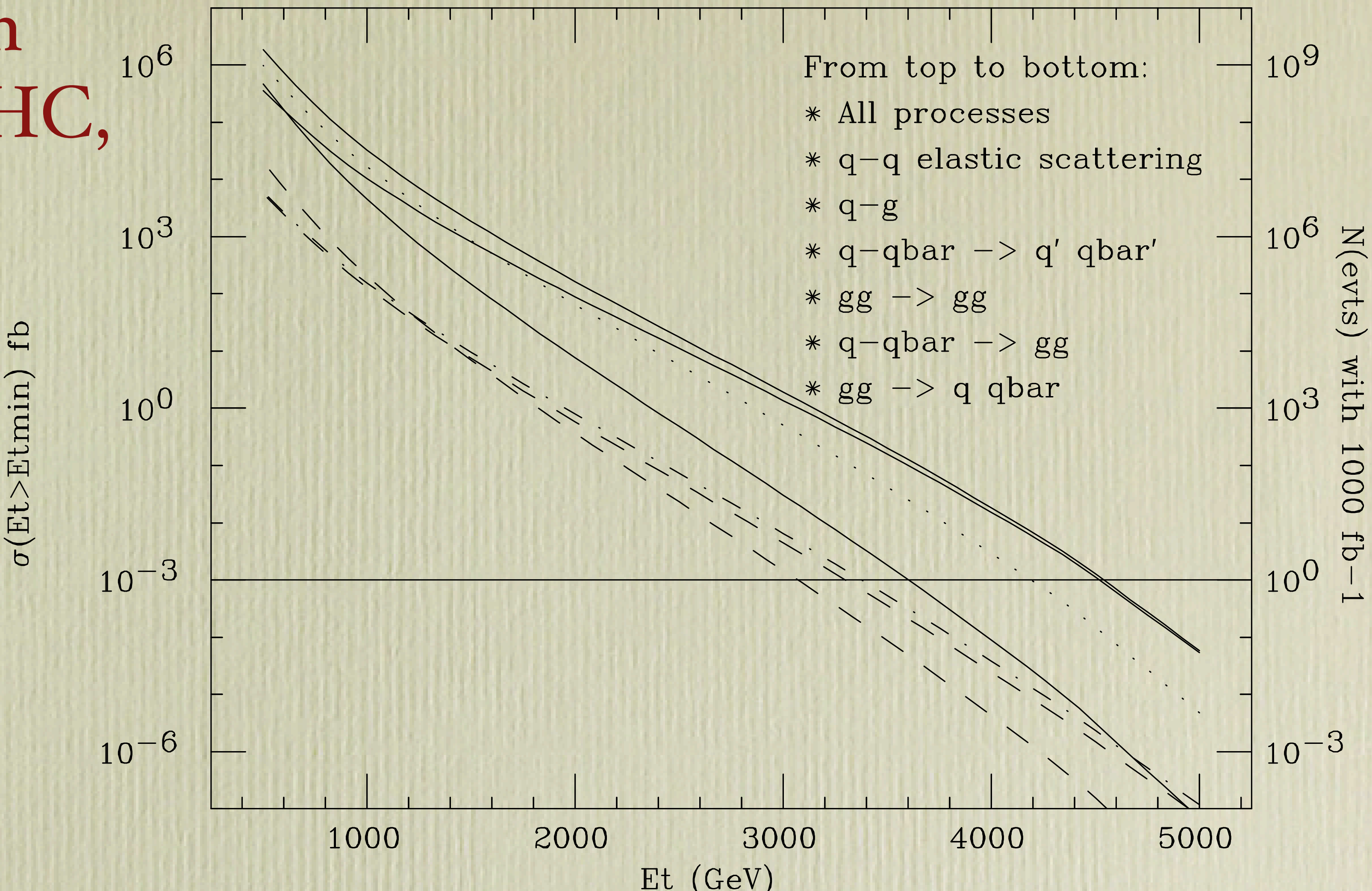
or by selecting low-mass events, but with large boosts (y_b large) in either positive of negative directions. In this case, we probe large-x with events where possible new physics is absent, thus setting consistent constraints on the behaviour of the cross-section in the high-mass region, which could hide new phenomena.

Example, at the Tevatron





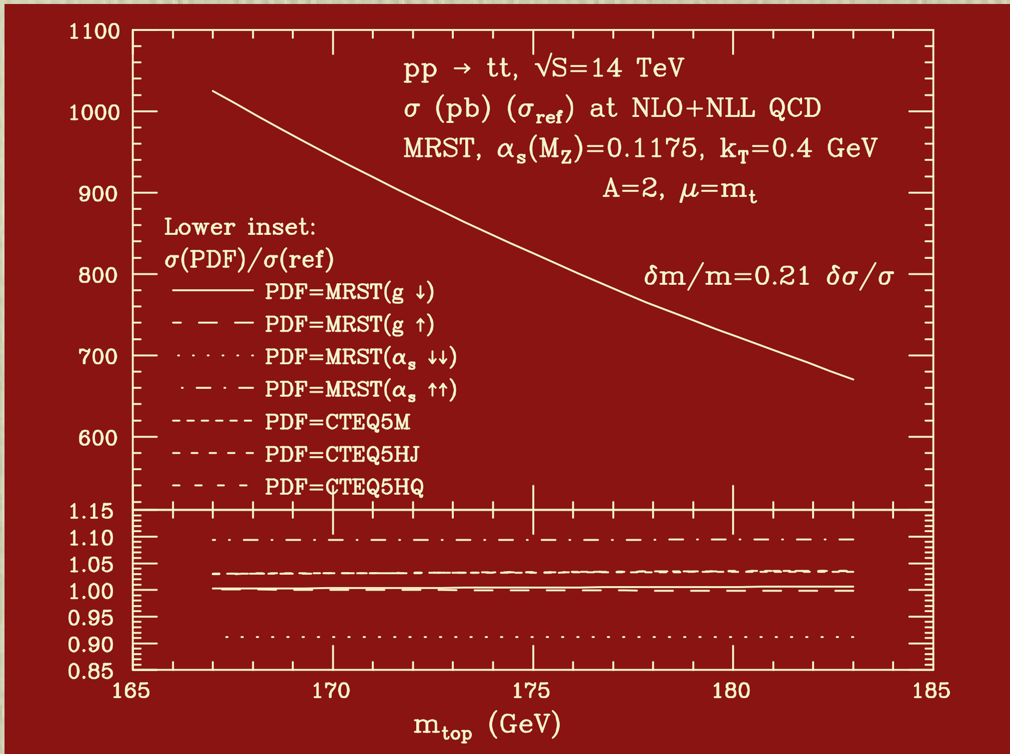
Jet production rates at the LHC, subprocess composition

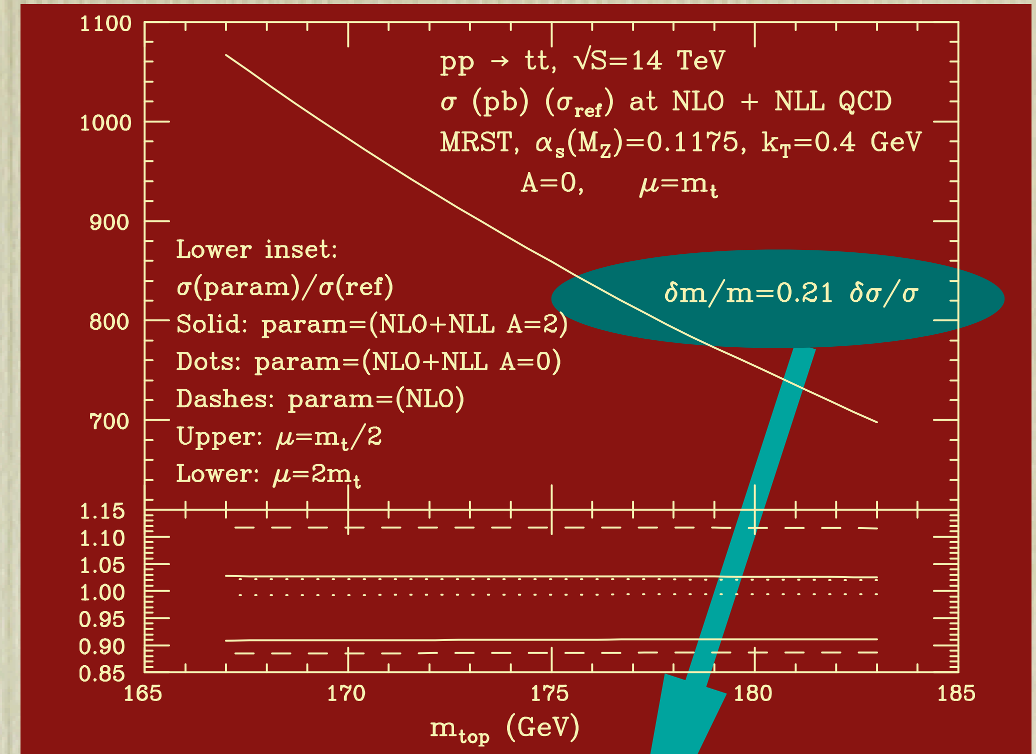


The presence of a quark substructure would manifest itself via contact interactions (as in Fermi's theory of weak interactions). On one side these new interactions would lead to an increase in cross-section, on the other they would affect the jets' angular distributions. In the dijet CMF, QCD implies Rutherford law, and extra point-like interactions can then be isolated using a fit. With the anticipated statistics of 300 fb-1, limits on the scale of the new interactions in excess of 40 TeV should be reached (to increase to 60 TeV with 3000 fb-1)

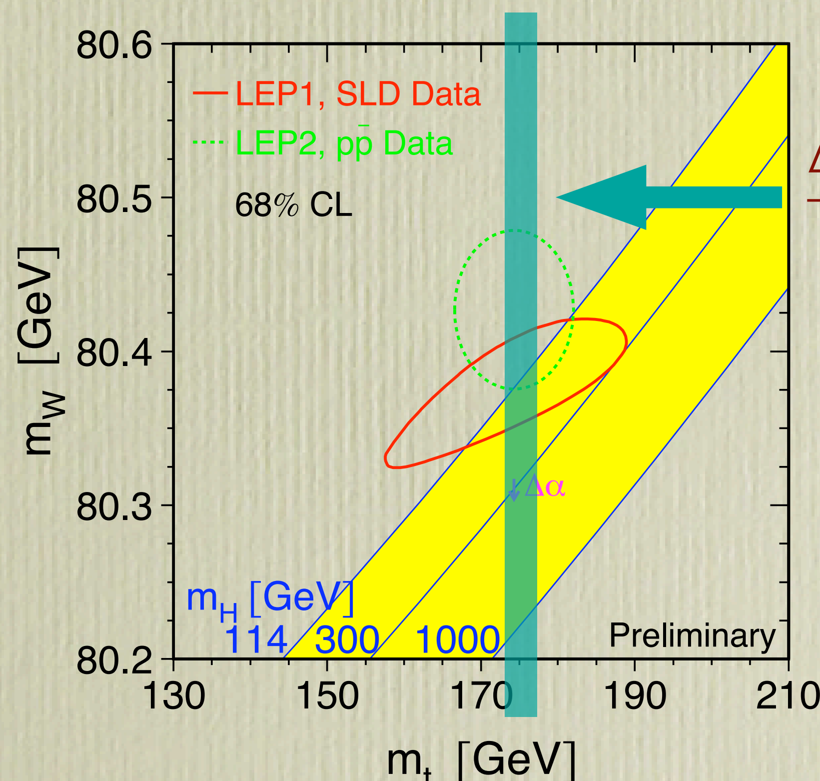
Top quark production

- Heaviest elementary particle known today
- m_{top} 175 GeV ⇒ top Yukawa coupling=1! The most natural value for a fermion mass: a special role in Nature for the top quark?
- LHC will be a "top Factory": σ~800 pb ⇒10^7 events/yr, 1Hz!
- Large statistics ⇒ statistically accurate determinations of the top properties:
  - mass (crucial to better constrain/predict Higgs mass)
  - production cross-section (accurate QCD tests)
- New physics BSM
  - rare decays (indirect searches for new physics, e.g. FCNC)
  - signal, parent, partner and background for new particle production:
    - gluino → top stop, stop → top neutralino, H^+→t bbar
    - top→H^+b



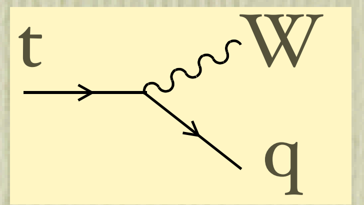


Theoretical systematics dominated today by PDF uncertainties! With the most recent analyses this is now at the level of 5% (see luminosity plots in previous lecture)



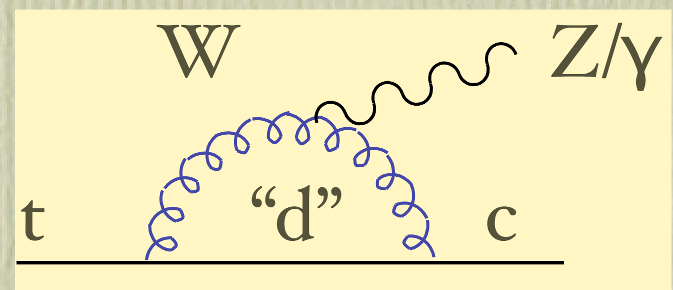
$$\frac{\Delta\sigma}{\sigma} \sim 5\% \Leftrightarrow \Delta m \sim 2 \text{ GeV}$$

Some rare top decays

BR(
) $\propto |V_{tq}|^2 = (10^{-4},\ 1.6\,10^{-3},\ 1) \sim (1,\ \lambda^4,\ \lambda^6)$ for $q = d, s, b$

Probability of not identifying b quark large, BR(t→W+d or s) very hard to measure

GIM suppression/CKM unitarity

BR(
) $\propto \left[\left(\frac{m_b}{m_t} \right)^2 V_{cb}\, \alpha_W \right]^2 \sim 10^{-13}$

Beyond any possible reach, unless new sources of FCNC. E.g., the SUSY partner of the above graph, with charginos and CKM-not-aligned down-type squarks.

t→WZb: m(b)+m(W)+m(Z)=176 GeV implies that the decay is just barely allowed by phase-space, once finite-width effects for the W and Z bosons are included. Very sensitive to m(top), could be an excellent probe of m(top). Unfortunately BR in the range of 10^{-6}, below experimental sensitivity (need to include BR(Z→ee) and BR (W→eν) as well)

Mode	SM BR	Allowed BSM	Wshop est reach
sW	1.6 E-3	0.25 (4th family)	missing
dW	~1 E-4	0.01 (4th family)	missing
bWZ	2 E-6	same	1 E-4
cWW	~1 E-13	1 E-6 (FCNC)	missing
cg	~5 E-11	1 E-3 (MSSM)	2 E-5 (cg->t)
cγ	~5 E-13	1 E-5 (MSSM)	3 E-5
cZ	~1 E-13	1 E-4	1 E-4
cH	< E-13	1 E-4	missing