Generalized Parton Distributions
Recent Progress

(Mostly a summary of various talks at SIR2005@Jlab in May 2005

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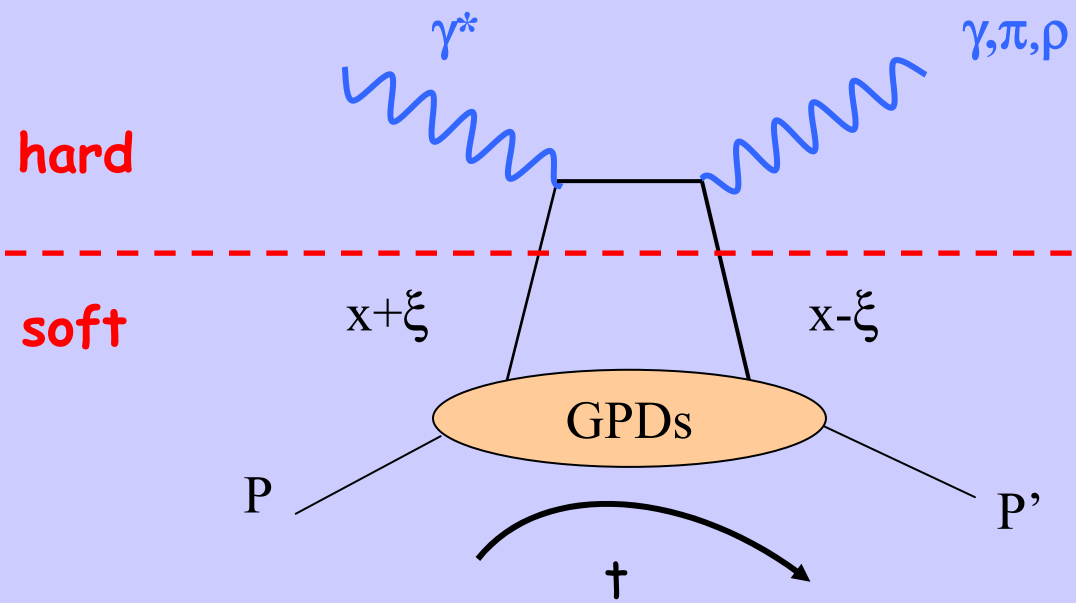


Factorisation:

Q^2 large, $-t<1$ GeV^2

What is a GPD?

- It is a proton matrix element which is a hybrid of elastic form factors and Feynman distributions
- GPDs depend upon:

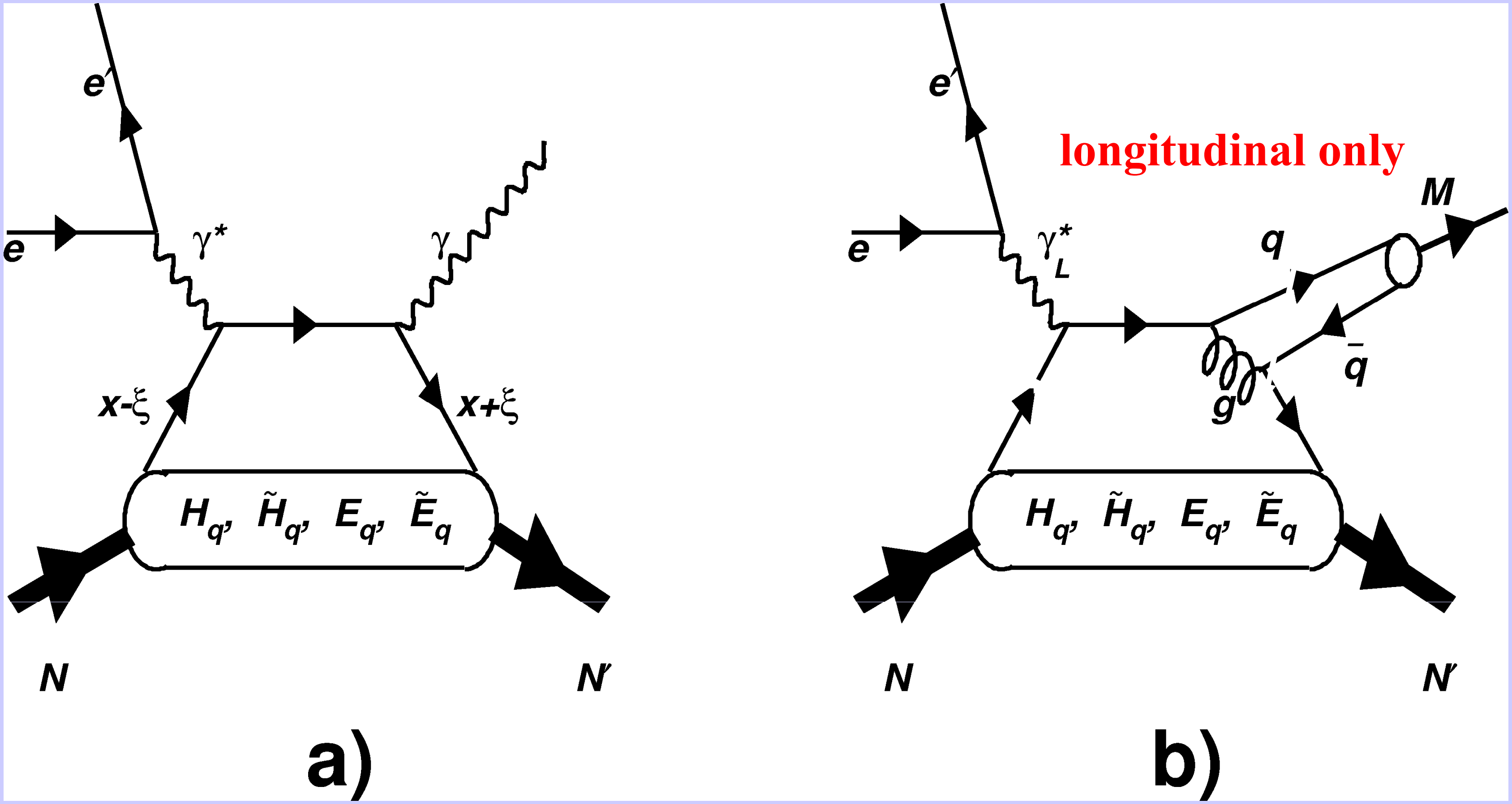
  x : *fraction of the longitudinal momentum carried by struck parton*

  t: *t-channel momentum transfer squared*

  ξ: *skewness parameter (a new variable coming from selection of a light-cone direction)*

  Q^2 : probing scale

DVCS DVMP



DVCS cannot separate u/d quark contributions.

M = ρ/ω select H, E, for u/d flavors
M = π, η, K select H, E

Formal definition of GPDs:

$$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \langle p' \left| \bar{q}(-\tfrac{1}{2}\lambda n)\gamma^{+} q(\tfrac{1}{2}\lambda n) \right| p \rangle = H(x,\xi,t)\bar{u}\gamma^{+}u + E(x,\xi,t)\bar{u}\frac{i\sigma^{+\nu}q_{\nu}}{2M}u$$

- x_i and x_f are the momentum fractions of the struck quark, and $x = \frac{1}{2}(x_i + x_f)$.
- $\xi = (x_f - x_i)/2$ is skewness. Depends on lightcone direction.
- $\int dx H(x,\xi,t) = F_1(t)$
- $\int dx E(x,\xi,t) = F_2(t)$

Relation of GPDs to Angular Momentum

Generalized form factor and quark angular momentum:

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\left[A^{q,g}_{20}(t)\,\gamma^{(\mu}P^{\nu)} + B^{q,g}_{20}(t)\,\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}\right]U(P)$$

Total quark angular momentum:

$$J^{u+d} = \frac{1}{2}\left[A^{u+d}_{20}(0) + B^{u+d}_{20}(0)\right] = \frac{1}{2}\left[\langle x\rangle^{u+d} + B^{u+d}_{20}(0)\right]$$

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

GPDs And Orbital Angular Momentum Distribution:

$$O^{\beta\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma^{(\beta} iD^{\mu_1} iD^{\mu_2} \cdots iD^{\mu_n)}\psi$$

Define generalized angular momentum tensor:

$$M^{\alpha\beta\mu_1\mu_2\cdots\mu_n} = \xi^\alpha O^{\beta\mu_1\mu_2\cdots\mu_n} - \xi^\beta O^{\alpha\mu_1\mu_2\cdots\mu_n} \text{ (minus traces)}$$

$$\int d^4\xi \langle p | M^{\alpha\beta\mu_1\mu_2\cdots\mu_n}(\xi) | p \rangle = J_n \times \text{tensor structures} \times (2\pi)^4 \delta^4(0)$$

reduced matrix element

$$\int d^3\xi \langle p | M^{12\cdots+++}(\xi) | p \rangle = S^{+\cdots+} + \tilde{L}^{+\cdots+} + \Delta\tilde{L}^{+\cdots+}$$

$$L(x) = \frac{1}{2}\left[xq(x) + xE(x) - \Delta q(x)\right]$$

Ji+Hoodbhoy

TMD Parton Distributions

- These appear in the processes in which hadron transverse-momentum is measured, often together with TMD fragmentation functions.
- The leading-twist ones are classified by Boer, Mulders, and Tangerman (1996,1998)
  - There are 8 of them

$q(x, k_\perp)$, $q_T(x, k_\perp)$,

$\Delta q_L(x, k_\perp)$, $\Delta q_T(x, k_\perp)$,

$\delta q(x, k_\perp)$, $\delta_L q(x, k_\perp)$,

$\delta_T q(x, k_\perp)$, $\delta_{T'} q(x, k_\perp)$

Wigner parton distributions (WPD)

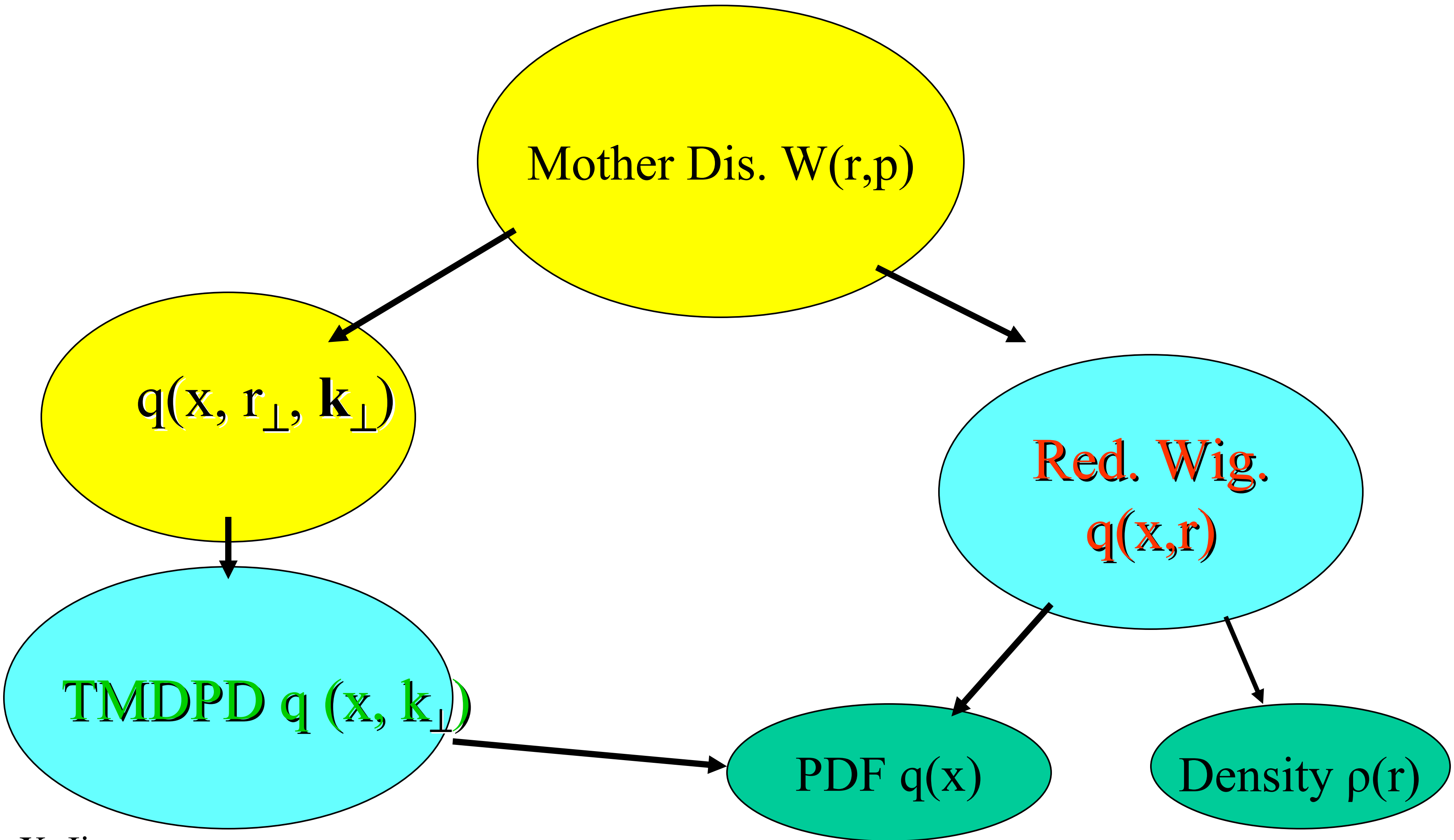
$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ ,$$

$$\langle O(x,p)\rangle = \int dxdp O(x,p)W(x,p)$$

- When integrated over p, one gets the coordinate space density $\rho(x)=|\psi(x)|^2$
- When integrated over x, one gets the coordinate space density $n(p)=|\psi(p)|^2$

X. Ji

Wigner parton distributions & offsprings (Ji)



X. Ji

Wigner distributions for quarks in proton

- Wigner operator (X. Ji,PRL91:062001,2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta \ ,$$

- Wigner distribution: "**density**" for quarks having *position **r** and 4-momentum* k^μ (off-shell)

$$W_\Gamma(\vec{r},k) = \frac{1}{2M}\int \frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2 \left| \hat{\mathcal{W}}(\vec{r},k) \right| -\vec{q}/2 \right\rangle$$
$$= \frac{1}{2M}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2 \left| \hat{\mathcal{W}}(0,k) \right| -\vec{q}/2 \right\rangle$$

Reduced Wigner Distributions and GPDs

- The 4D reduced Wigner distribution f(**r**,x) is related to Generalized parton distributions (GPD) H and E through simple FT,

$$f_\Gamma(\vec{r},x) = \frac{1}{2M}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} F_\Gamma(x,\xi,t) \ .$$

$$\frac{1}{2M}F_{\gamma^+}(x,\xi,t) = [H(x,\xi,t) - \tau E(x,\xi,t)] + i(\vec{s}\times\vec{q})^z \frac{1}{2M}[H(x,\xi,t)+E(x,\xi,t)]$$

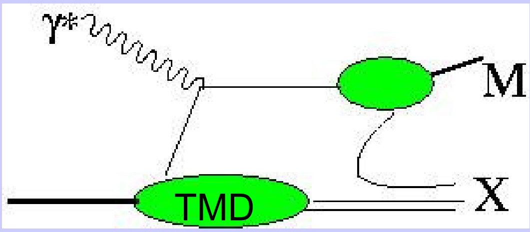
$t = -q^2$
$\xi \sim q_z$

H,E depend only on 3 variables. There is a rotational symmetry in the transverse plane..

X. Ji

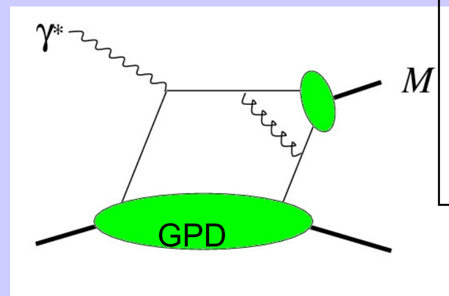
$W_p^u(x,\mathbf{k},\mathbf{r})$ "Parent" Wigner distributions

Probability to find a quark **u** in a nucleon **P** with a certain polarization in a position **r** and momentum **k**



d^3r

d^2k_T (FT)



TMD PDFs: $f_p^u(x,k_T), g_1, f^{\perp}_{1T}, h_1, h^{\perp}_{1L}$

GPDs: $H_p^u(x,\xi,t)$, $E_p^u(x,\xi,t)$, $\tilde{H}, \tilde{E}, \ldots$

Measure momentum transfer to quark
k_T distributions also important for exclusive studies

d^2k_T

$\xi=0, t=0$

dx

Measure momentum transfer to target
Exclusive meson data important in understanding of SIDIS measurements

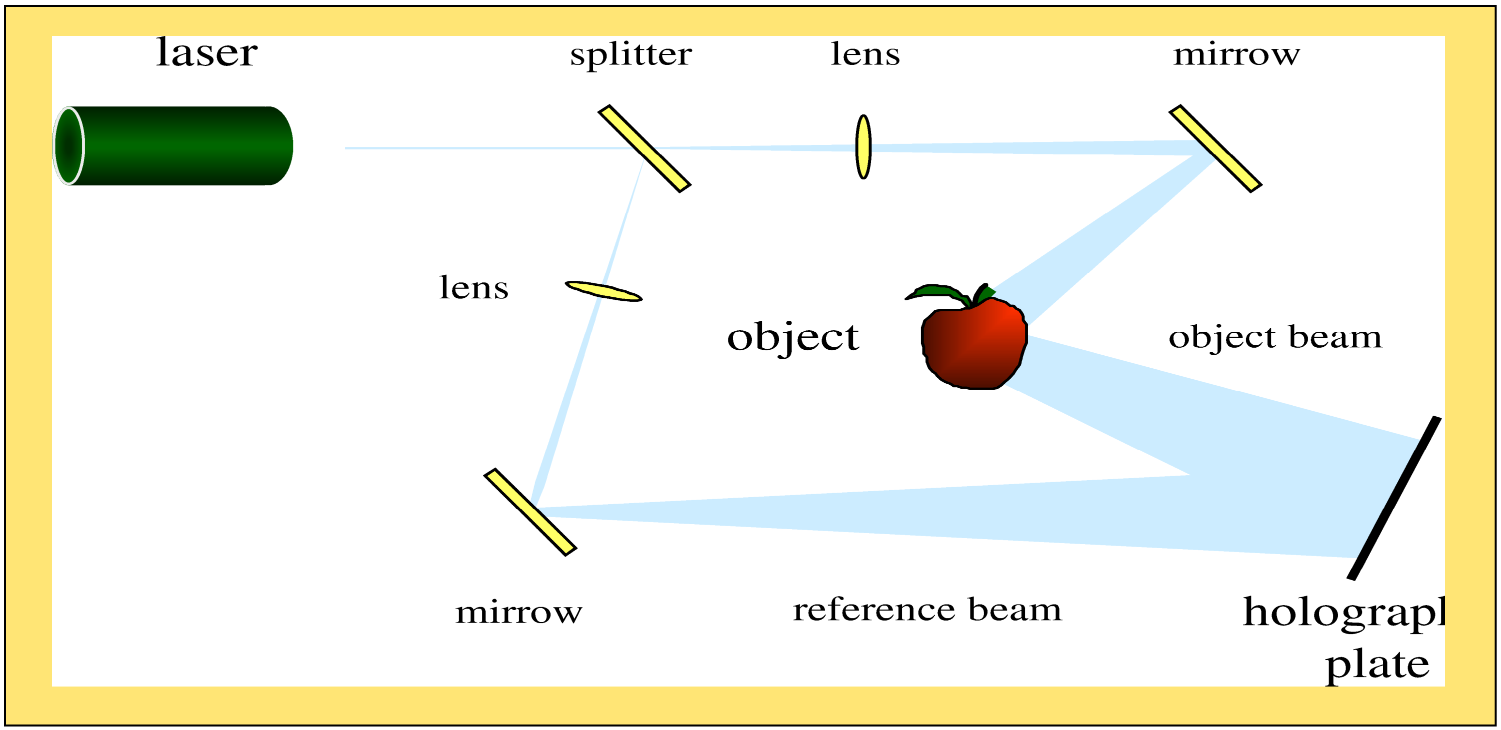
PDFs $f_p^u(x,k_T)$, g_1, h_1

FFs $F_{1p}^u(t), F_{2p}^u(t)$..

Some PDFs same in exclusive and semi-inclusive analysis

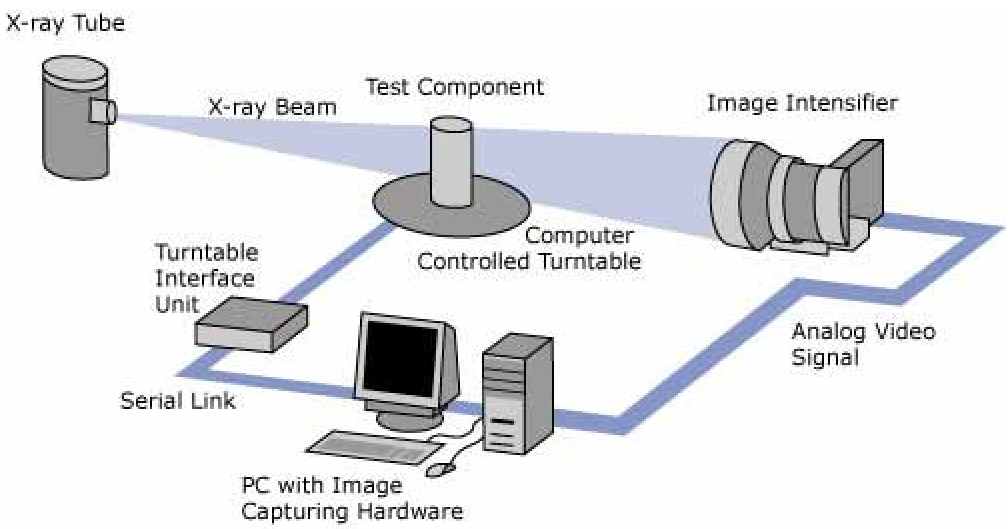
Analysis of SIDIS and DVMP are complementary

Holography is "lensless photography" in which an image is captured not as an image focused on film, but as an interference pattern at the film. Typically, coherent light from a laser is reflected from an object and combined at the film with light from a reference beam. This recorded interference pattern actually contains much more information that a focused image, and enables the viewer to view a true three-dimensional image which exhibits parallax.



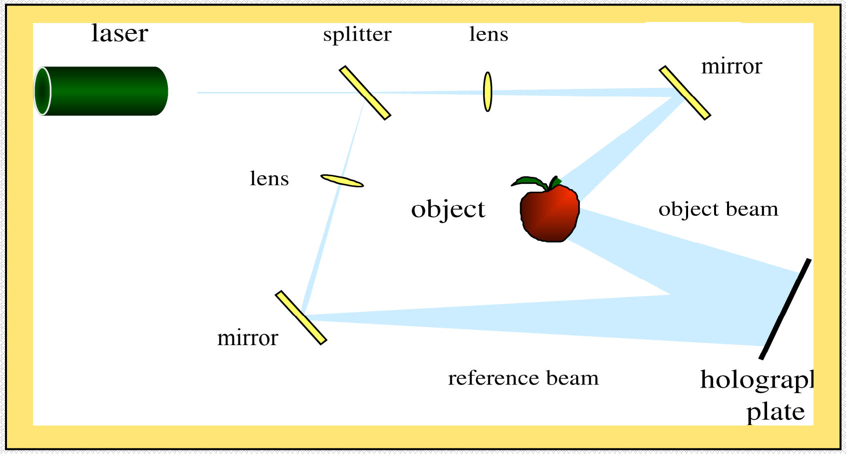
Computed Tomography

Computed Tomography (CT) is a powerful nondestructive evaluation (NDE) technique for producing 2-D and 3-D cross-sectional images of an object from flat X-ray images. Characteristics of the internal structure of an object such as dimensions, shape, internal defects, and density are readily available from CT images.



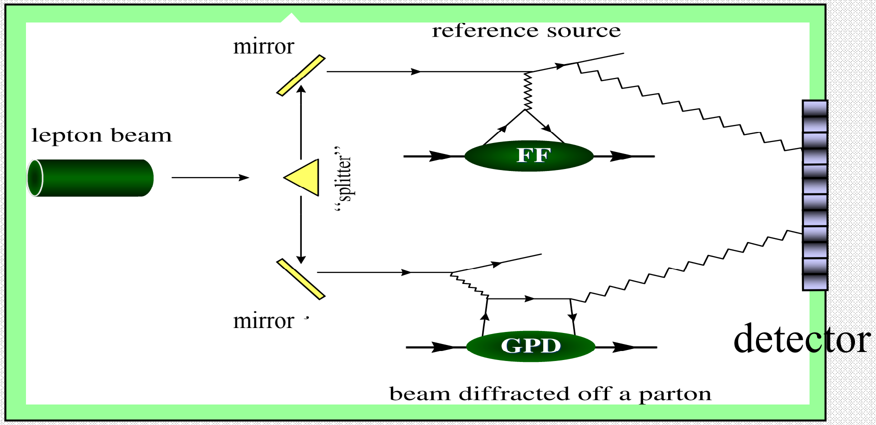
From Holography to Tomography

A. Belitsky, B. Mueller, NPA711 (2002) 118



An Apple

A Proton



By varying the energy and momentum transfer to the proton we probe its interior and generate tomographic images of the proton ("femto tomography").

Burkert

Impact parameter dependent PDFs

- define state that is localized in $\perp$ position:

$$\left|p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
$\mathbf{R}_\perp \equiv \frac{1}{p^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp = \mathbf{0}_\perp\right|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

GPDs $\longleftrightarrow$ $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ nucleon-helicity nonflip GPDs can be related to distribution of partons in $\perp$ plane

$$
\begin{aligned}
q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\
\Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2),
\end{aligned}
$$

- **no rel. corrections** to this result! (Galilean subgroup of $\perp$ boosts)
- $q(x, \mathbf{b}_\perp)$ has **probabilistic interpretation**, e.g.

$$
\begin{aligned}
q(x, \mathbf{b}_\perp) \geq |\Delta q(x, \mathbf{b}_\perp)| \geq 0 \quad &\text{for} \quad x > 0 \\
q(x, \mathbf{b}_\perp) \leq |\Delta q(x, \mathbf{b}_\perp)| \leq 0 \quad &\text{for} \quad x < 0
\end{aligned}
$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow | \bar{q}(0)\, \gamma^+ q(x^-) | P,\uparrow \rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow | \bar{q}(0)\, \gamma^+ q(x^-) | P,\downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle.$

$\hookrightarrow$ unpolarized quark distribution for this state:

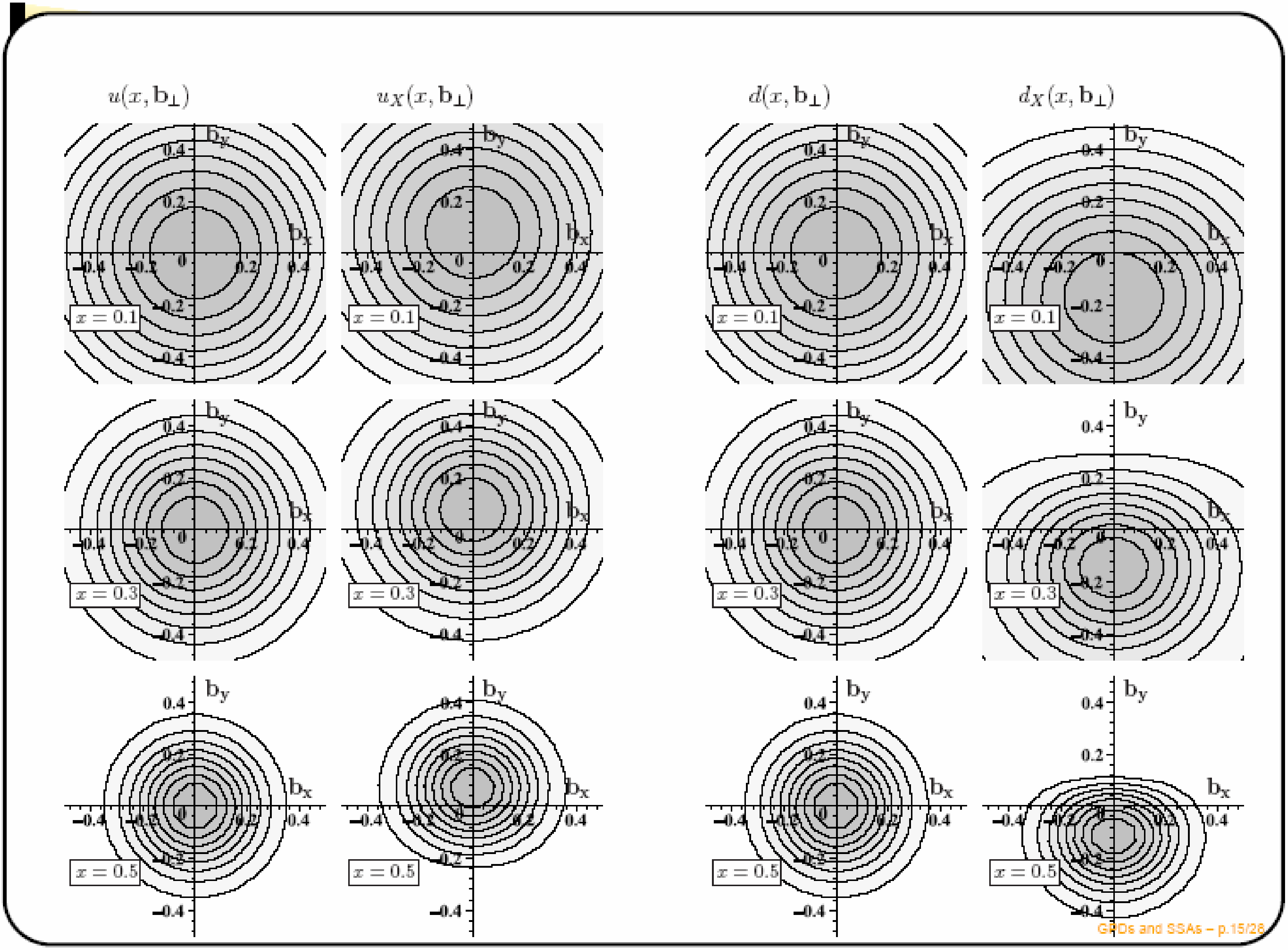
$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$E_u(x,0,-\mathbf{\Delta}_\perp^2) = \frac{\kappa_u^p}{2} H_u(x,0,-\mathbf{\Delta}_\perp^2)$$

$$E_d(x,0,-\mathbf{\Delta}_\perp^2) = \kappa_d^p H_d(x,0,-\mathbf{\Delta}_\perp^2)$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

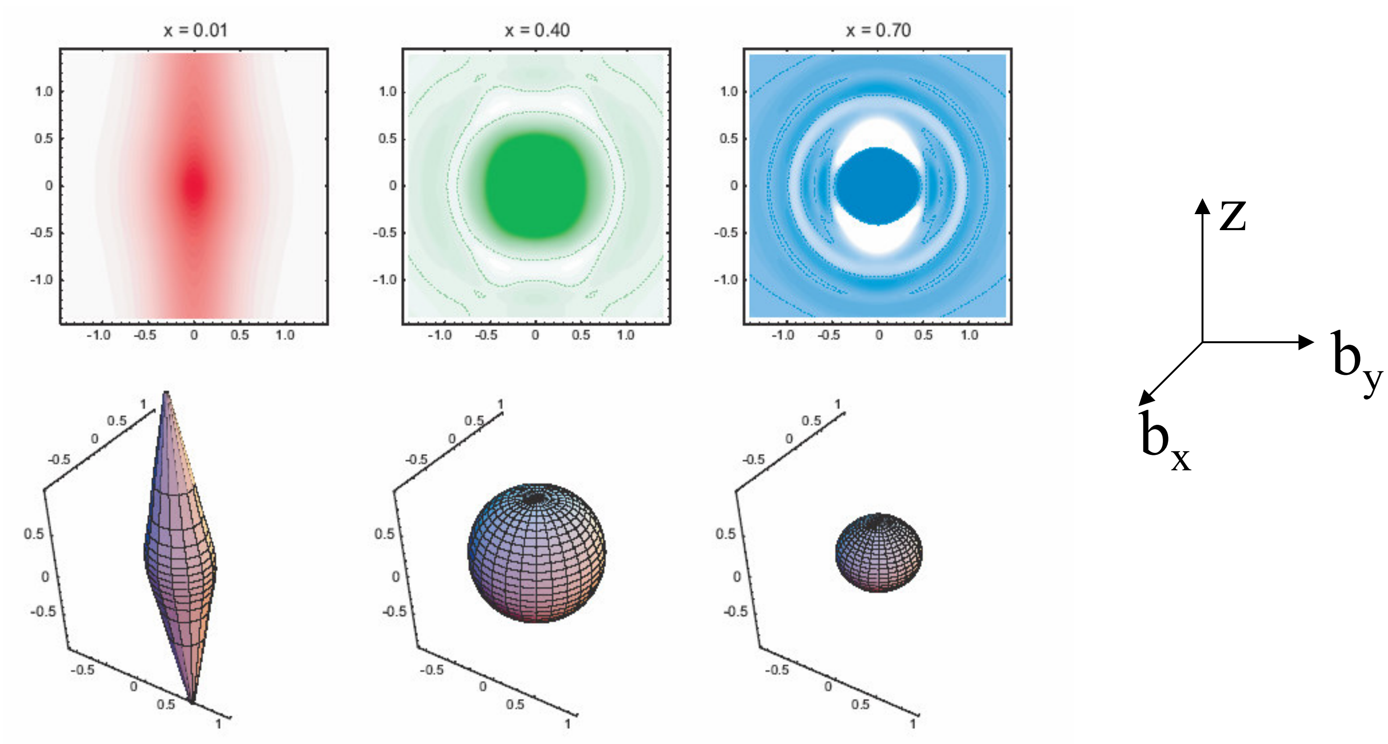


Burkardt

$$\lim_{x \to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

Imaging quarks at fixed Feynman-x

- For every choice of x, one can use the Wigner distributions to picture the nucleon in 3-space; **quantum phase-space tomography!**



X. Ji

GPDs ON A LATTICE

$$\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} = \overline{q}\,\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n\}}\,q$$

$\longrightarrow$ Generalised Form Factors

$$\langle p', s'|\mathcal{O}^{\{\mu_1\cdots\mu_n\}}(\Delta)|p, s\rangle =$$

$$\bar{u}(p', s')\gamma^{\{\mu_1}u(p, s)\sum_{i=0}^{\frac{n-1}{2}} A_{qn,2i}(t)\Delta^{\mu_2}\ldots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\ldots\overline{p}^{\mu_n\}}$$

$$+\,\bar{u}(p', s')\frac{i\sigma^{\{\mu_1\nu}\Delta_\nu}{2m}u(p, s)\sum_{i=0}^{\frac{n-1}{2}} B_{qn,2i}(t)\Delta^{\mu_2}\ldots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\ldots\overline{p}^{\mu_n\}}$$

$$+\,C_{qn}(t)\,\frac{1}{m}\bar{u}(p', s')u(p, s)\Delta^{\mu_1}\ldots\Delta^{\mu_n}|_{\text{n even}}$$

Zanotti

$$A_{10}^{q}(Q^2) = F_1^{q}(Q^2)$$

$$B_{10}^{q}(Q^2) = F_2^{q}(Q^2)$$

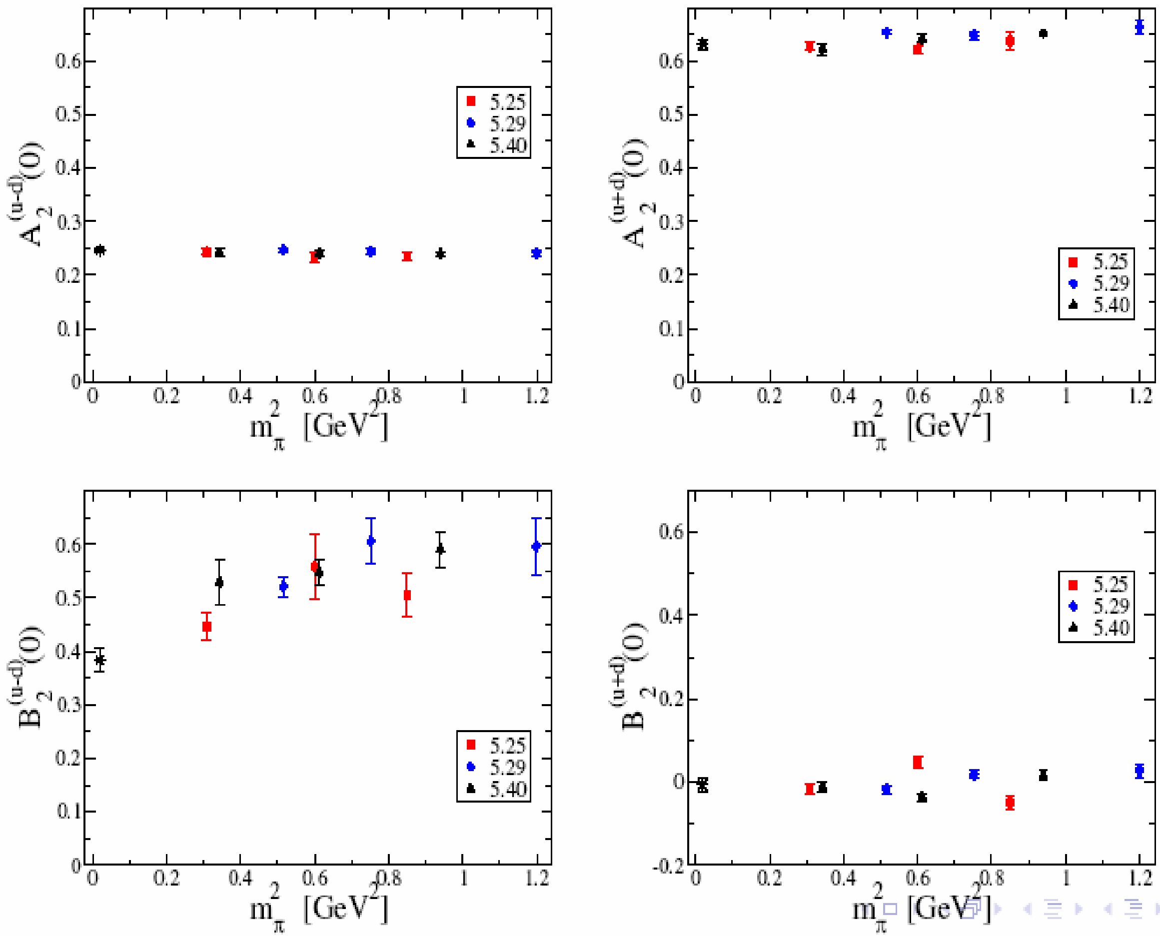
$$\tilde{A}_{10}^{q}(Q^2) = G_A^{q}(Q^2)$$

$$\tilde{B}_{10}^{q}(Q^2) = G_P^{q}(Q^2)$$

$$J^{q} = \tfrac{1}{2}\left(A_{20}^{q}(0) + B_{20}^{q}(0)\right)$$

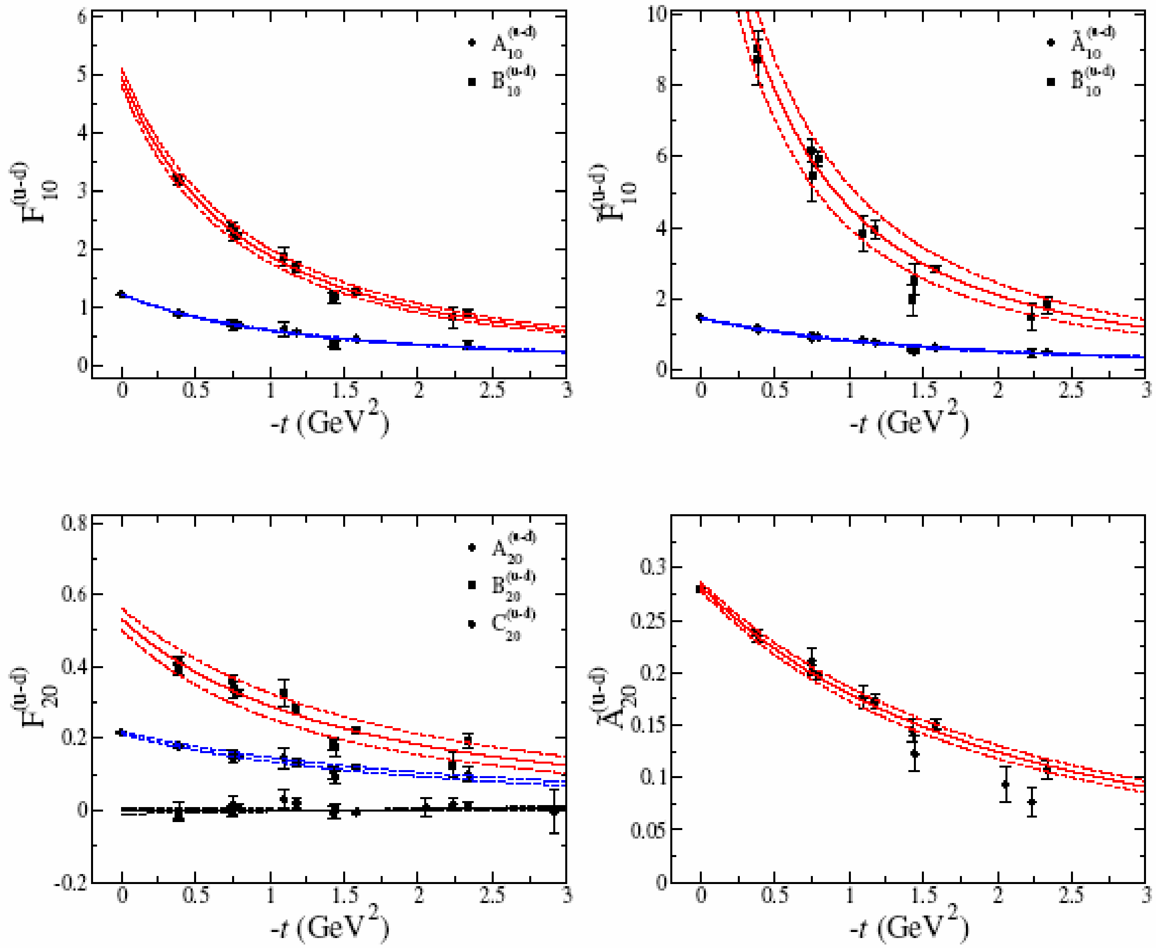
$$\tfrac{1}{2}\Sigma^{q} = \tilde{A}_{10}^{q}(0)$$

Angular Momentum $J^q = L^q + S^q = \frac{1}{2}(A_2^q + B_2^q)$, $(\overline{\text{MS}}\,4\ \text{GeV}^2)$



Zanotti

Generalised Form Factors, $(m_\pi \approx 950\text{MeV})$



Zanotti

Summary of LHPC hadron structure program

- Long term program to compute all $n \leq 4$ GFF's in dynamical lattice QCD.
- Current pion masses $m_\pi \approx 350 - 750$ MeV and lattice spacing $a \approx \frac{1}{8}$ fm.
- Status of the calculation

Operators	Matrix elements	Operator renorm.	GFF extraction	Analysis
$\overline{q}\Gamma_\mu q$	Done!	Done!	Almost done	Starting
$\overline{q}\Gamma_{(\mu}D_{\nu)}q$	Done!	Done!	Almost done	Starting
$\overline{q}\Gamma_{(\mu}D_\nu D_{\rho)}q$	Done!	Done!	Almost done	Starting
$\overline{q}\Gamma_{(\mu}D_\nu D_\rho D_{\sigma)}q$	Not yet	Done!	Not yet	Not yet

- Only isovector flavor combinations for GFF's in this round.
- Finite perturbative renormalization needed to quote results in $\overline{\text{MS}}$ scheme.

$$\langle P'S' | \mathcal{O}_\Gamma^{\mu_1 \cdots \mu_n} | PS \rangle_{\overline{\text{MS}}} = Z \, \langle P'S' | \mathcal{O}_\Gamma^{\mu_1 \cdots \mu_n} | PS \rangle_{\text{latt}}$$

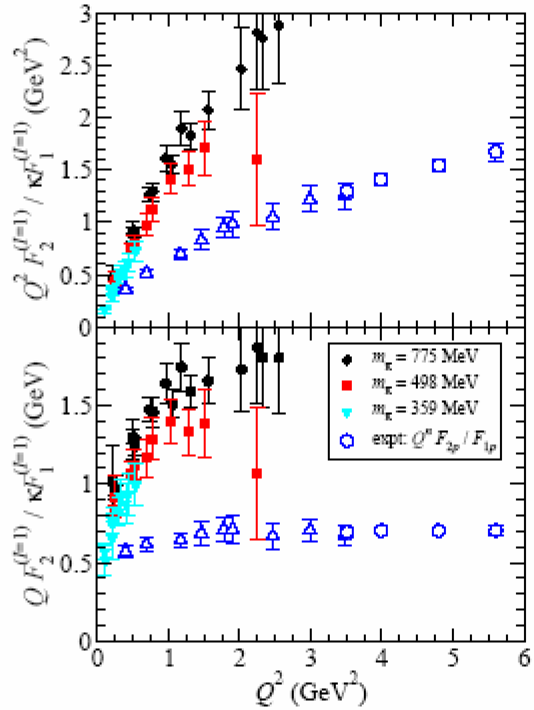
- Lighter pion masses $m_\pi \approx 250 - 350$ MeV finished by next year.

Fleming

Nucleon F_2/F_1 on the Lattice (I)

PRELIMINARY

- Only $I = 1$ form factors computed so far to avoid disconnected diagrams. $F_1^{I=1} = F_{1p} - F_{1n}$ but F_{1n}, F_{2n} not known accurately for $Q^2 \gtrsim 1\ \mathrm{GeV}^2$.
- Our normalization is $F_2(Q^2) \to \kappa$ as $Q^2 \to 0$.

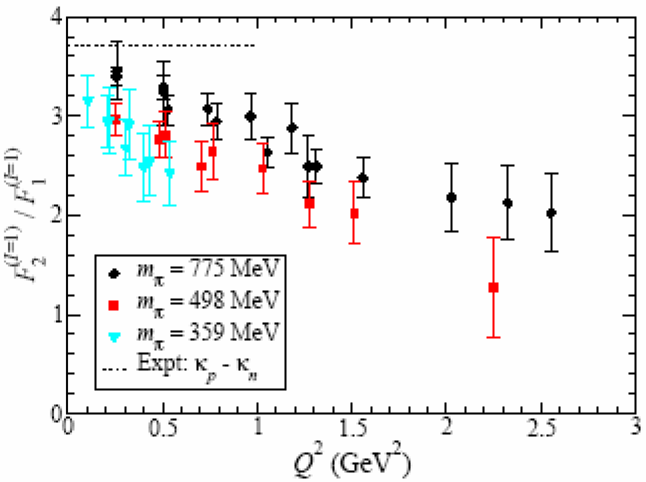


Fleming

Nucleon F_2/F_1 on the Lattice (II)

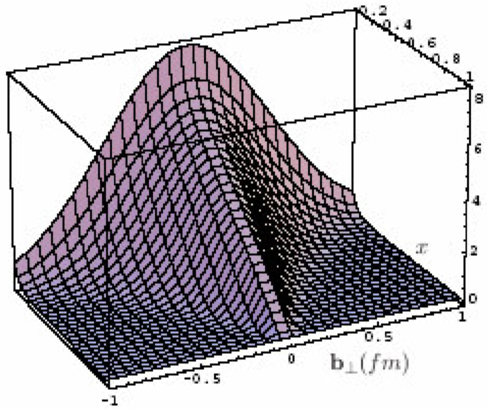
PRELIMINARY

- $F_2^{I=1}/F_1^{I=1} \to \kappa_p - \kappa_n$ as $Q^2 \to 0$.
- PDG: $\kappa_p = 1.792847351(28)$
- PDG: $\kappa_n = -0.91304273(45)$
- So, comparison of $I = 1$ with $p - n$ could be OK with proper chiral extrapolation.



Fleming

Transverse quark distributions



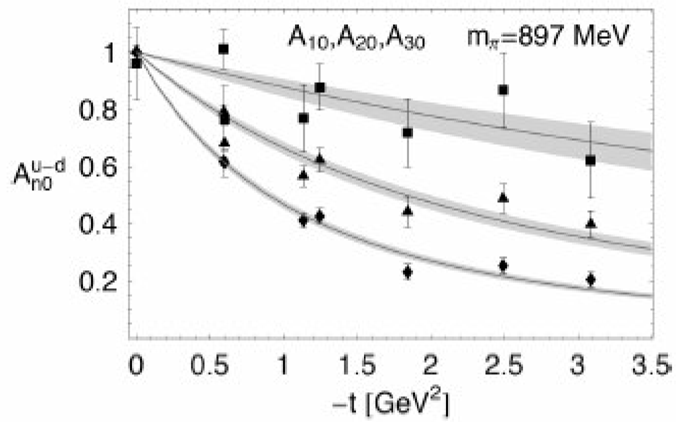
M. Burkardt hep-ph/0207047

$$A^q_{n0}(-\Delta^2_\perp) = \int d^2b_\perp e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \int_{-1}^{1} x^{n-1} q(x,\mathbf{b}_\perp)$$

$$\langle b^2_\perp \rangle^q_{(n)} = -4\frac{A^{q\prime}_{n0}(0)}{A^q_{n0}(0)}$$

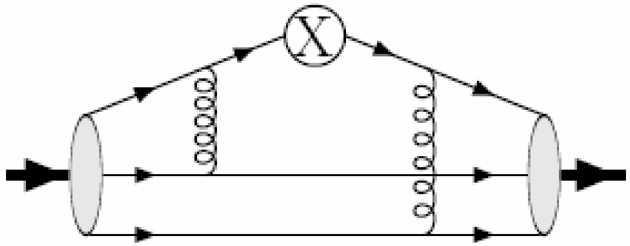
$$\lim_{x\to 1} q(x,\mathbf{b}_\perp) \propto \delta(b^2_\perp)$$

- Higher moments A_{n0} weight $x \sim 1$.
- Slope of A^q_{n0} decreases as n increases.
- Slope of $A^{u-d}_{10}(0) =$ -0.93(4) $(\text{GeV})^2$.
- Slope of $A^{u-d}_{30}(0) =$ -0.13(3) $(\text{GeV})^2$.
- Will this continue at light pion masses?

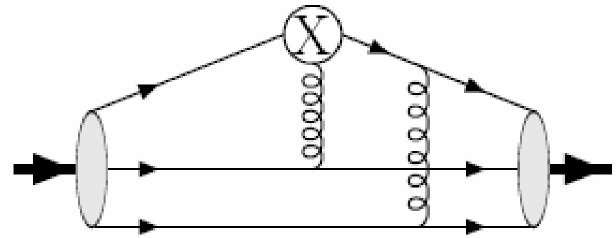


D. Renner (LHPC/SESAM)

Fleming



(1)



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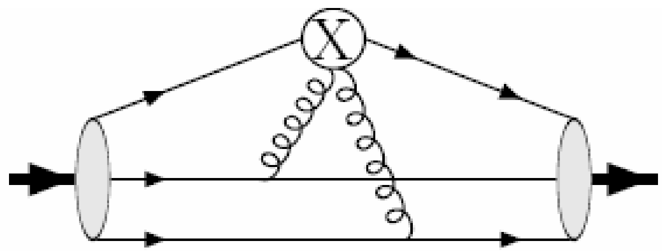


(3)

$$H_q(x,\xi,t) \;=\; \int [dx][dy]\Phi_3^*(y_1,y_2,y_3)\Phi_3(x_1,x_2,x_3)T_{Hq}(x_i,y_i,x,\xi,t)\ ,$$

hoodbhoy

GPDs - Experimental Aspects

- DVCS measured at HERA (at H1 and Zeus)
- DVCS measured at JLab (fixed target,CLAS)
- DVCS planned at COMPASS, CERN

- DVMP measured at HERA
- DVMP measured at JLab
- DVMP measured (old data, 2002) at COMPASS

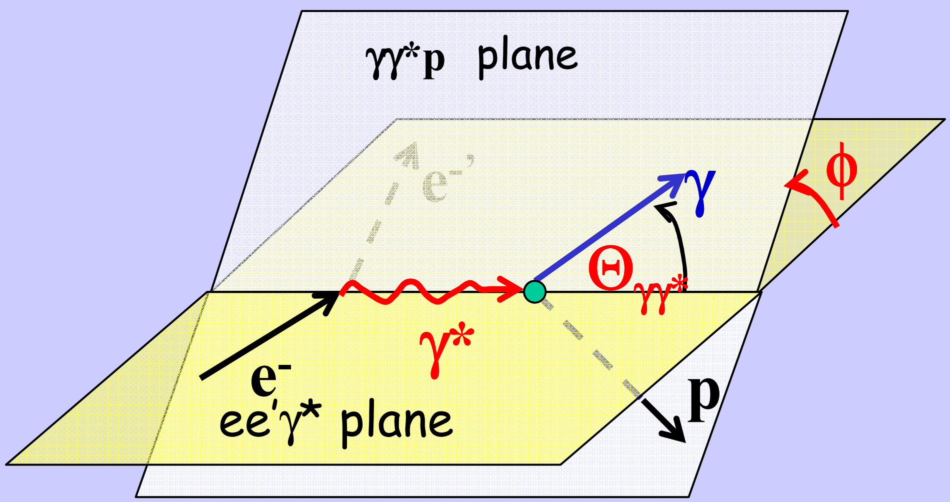
- DDVCS planned at JLab

Some Generalities

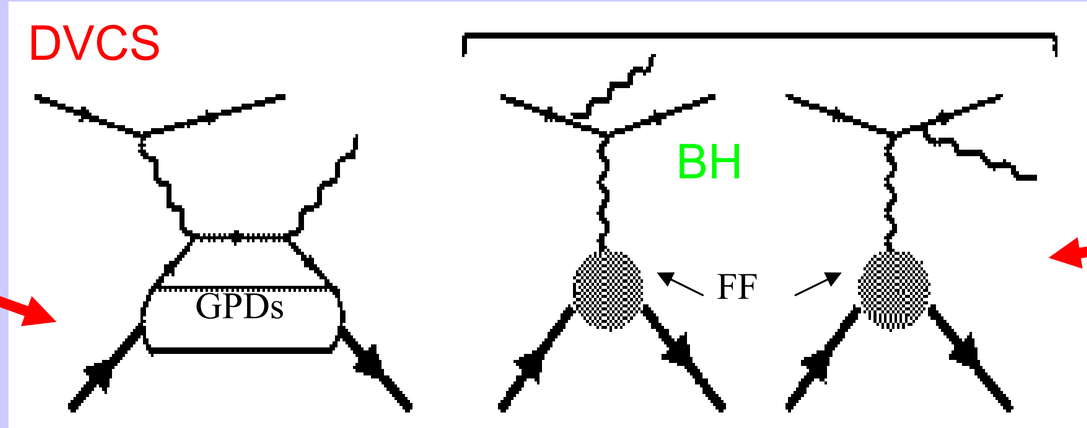
$$\frac{1}{x-\xi+i\varepsilon}=P\left(\frac{1}{x-\xi}\right)-i\pi\delta(x-\xi)$$

$$\Rightarrow \mathrm{Im}\{F\}=\pi\sum e_q^2\left\{F^q(\xi,\xi,t,Q^2)\,\mathrm{m}F^q(-\xi,\xi,t,Q^2)\right\}$$

$$\mathrm{Re}\{F\}=-\sum e_q^2 P\int_{-1}^{+1} dx F^q(x,\xi,t,Q^2)\left\{\frac{1}{x-\xi}\pm\frac{1}{x+\xi}\right\}$$



small signal



big noise

$$A_{LU}(\phi) = \frac{d\overrightarrow{\sigma}(\phi) - d\overleftarrow{\sigma}(\phi)}{d\overrightarrow{\sigma}(\phi) + d\overleftarrow{\sigma}(\phi)}$$ (Beam Spin Asymmetry, BSA)

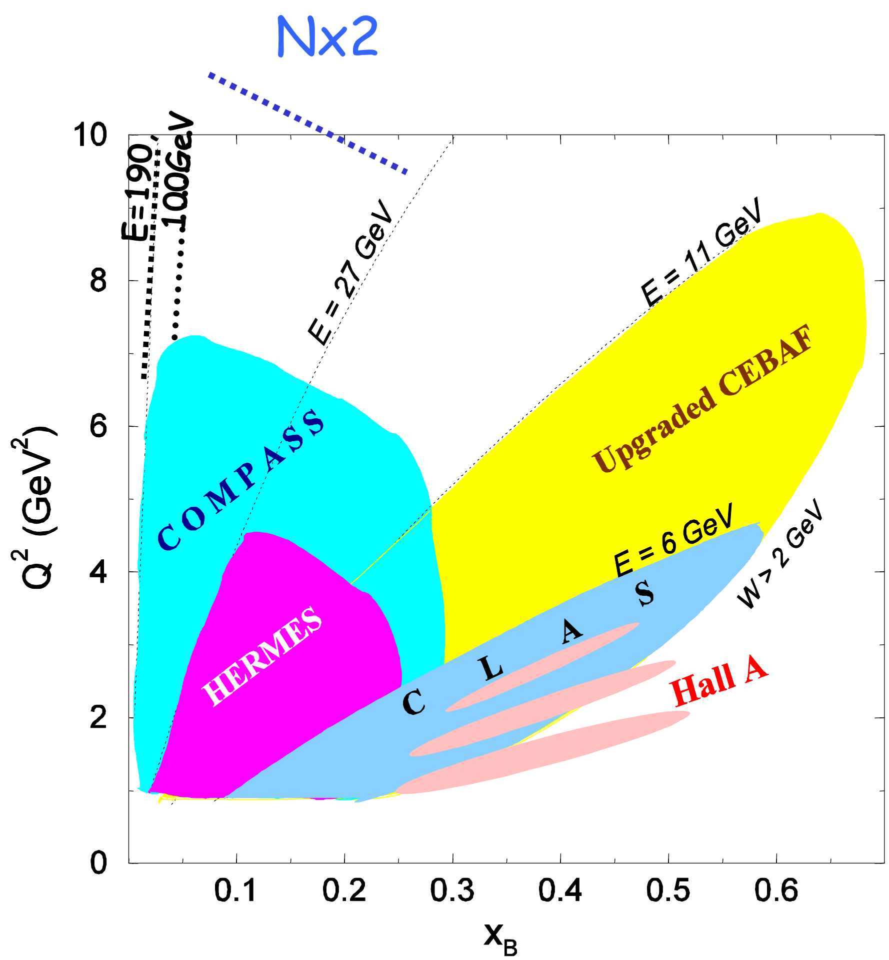
$$A_{C}(\phi) = \frac{d\sigma^{+}(\phi) - d\sigma^{-}(\phi)}{d\sigma^{+}(\phi) + d\sigma^{-}(\phi)}$$ (Beam Charge Asymmetry, BCA)

find that:

$A_{LU}(\phi) \propto \mathrm{Im}(\tilde{M}) \sin\phi$ and $A_{C}(\phi) \propto \mathrm{Re}(\tilde{M}) \cos\phi$

where: $$\tilde{M} = \frac{\sqrt{t_0 - t}}{2m}\left[F_1 \mathrm{H} + \xi(F_1 + F_2)\tilde{\mathrm{H}} - \frac{t}{4m^2}\mathrm{E}\right]$$

Kinematical domain



Collider :

H1 & ZEUS 0.0001<x<0.01

Fixed target :

JLAB	6-11GeV	SSA,BCA?
HERMES	27 GeV	SSA,BCA

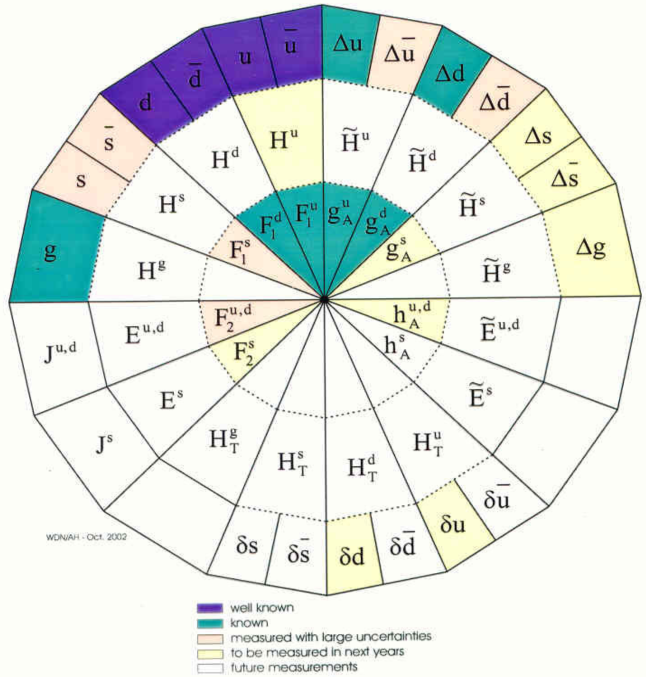
COMPASS could provide data on :

- Cross section (190 GeV)
- BCA (100 GeV)
- Wide Q^2 and x_{bj} ranges

Limitation due to luminosity

Burtin

Exp. Status on Parton Distr.'s



(cf. W.-D. N., hep-ex/0210409)
---- 0503010

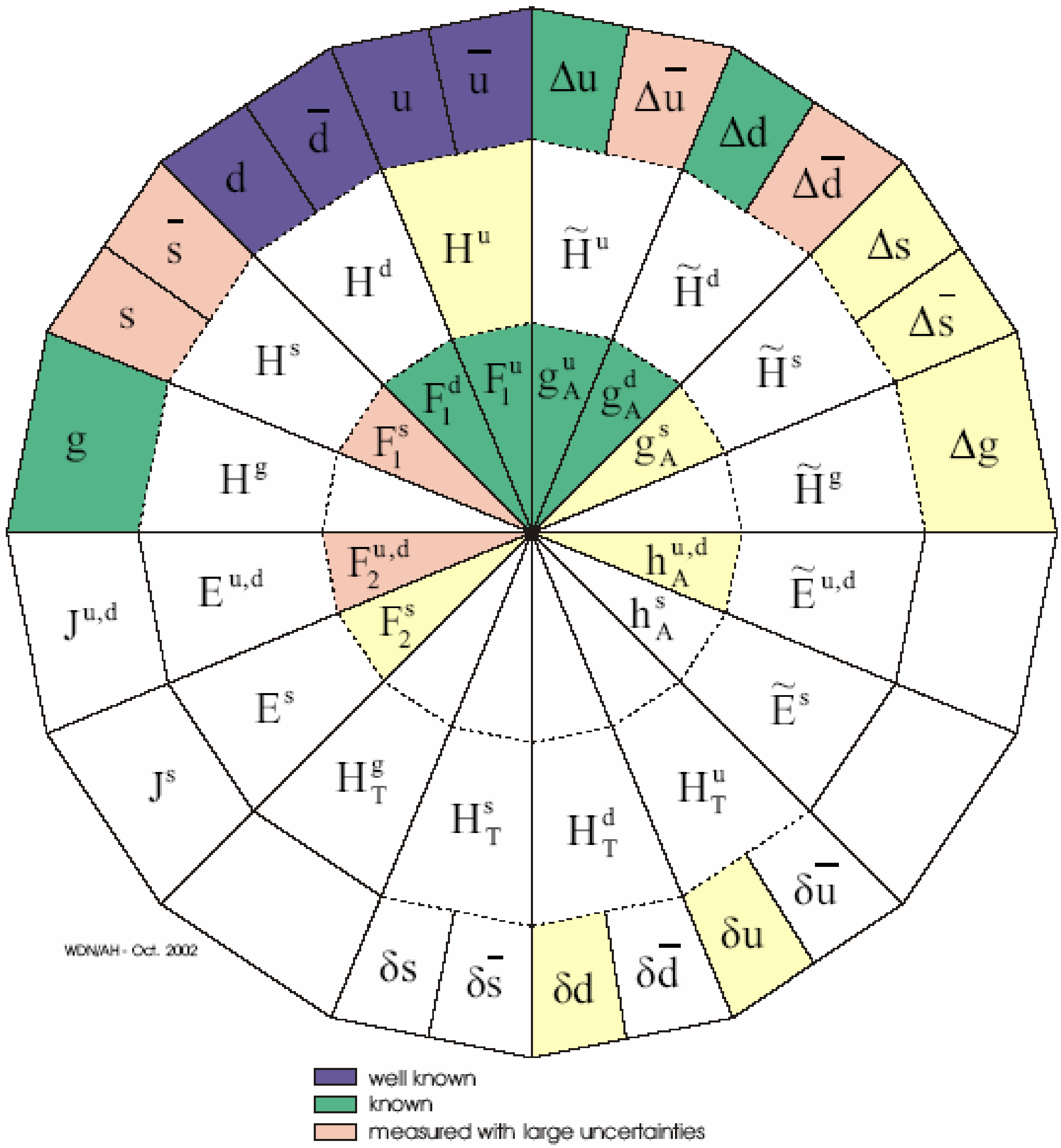
Generalized Parton Distributions:

$H^q, \tilde{H}^q, E^q, \tilde{E}^q$ chirally-even quark GPDs
$H_T^q, \tilde{H}_T^q, E_T^q, \tilde{E}_T^q$ chirally-odd quark GPDs

Forward Parton Distributions:

$q(x, Q^2)$ quark number density distribution (f_1^q)
$\Delta q(x, Q^2)$ quark helicity distribution (g_1^q)
$\delta q(x, Q^2)$ quark transversity distribution (h_1^q)

Nowak


$\bar{u}$
u
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d
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s
g
Δu
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Δd
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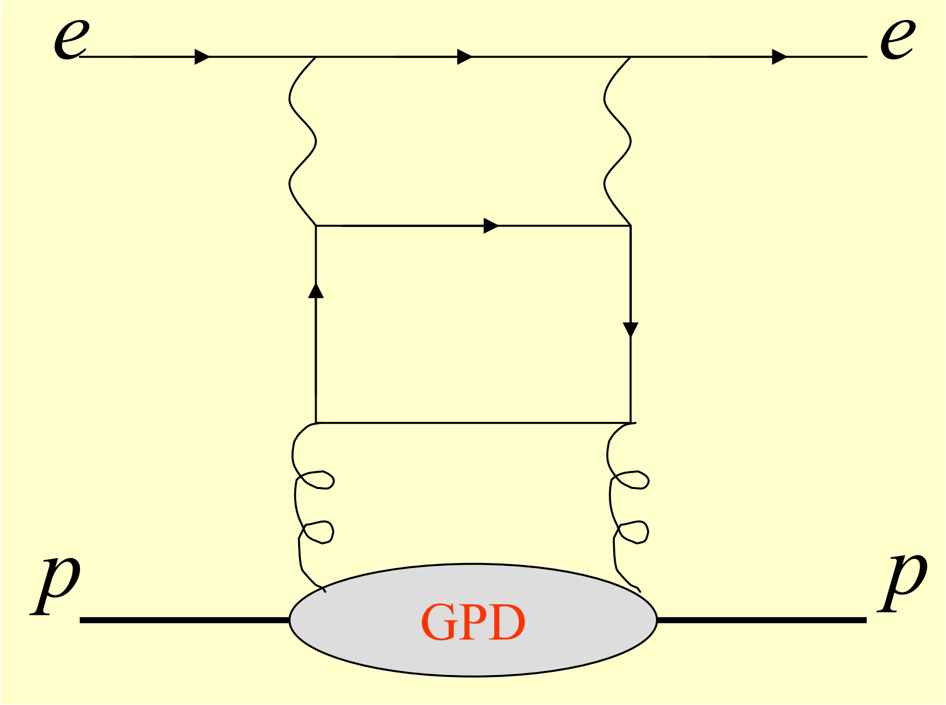
Δs
$\Delta\bar{s}$
Δg
H^u
H^d
H^s
H^g
$\tilde{H}^u$
$\tilde{H}^d$
$\tilde{H}^s$
$\tilde{H}^g$
F_1^u
F_1^d
F_1^s
g_A^u
g_A^d
g_A^s
$F_2^{u,d}$
F_2^s
$h_A^{u,d}$
h_A^s
$E^{u,d}$
E^s
$\tilde{E}^{u,d}$
$\tilde{E}^s$
$J^{u,d}$
J^s
H_T^g
H_T^s
H_T^d
H_T^u
δs
$\delta\bar{s}$
δd
$\delta\bar{d}$
δu
$\delta\bar{u}$
WDN/AH - Oct. 2002
well known
known
measured with large uncertainties

Nowak



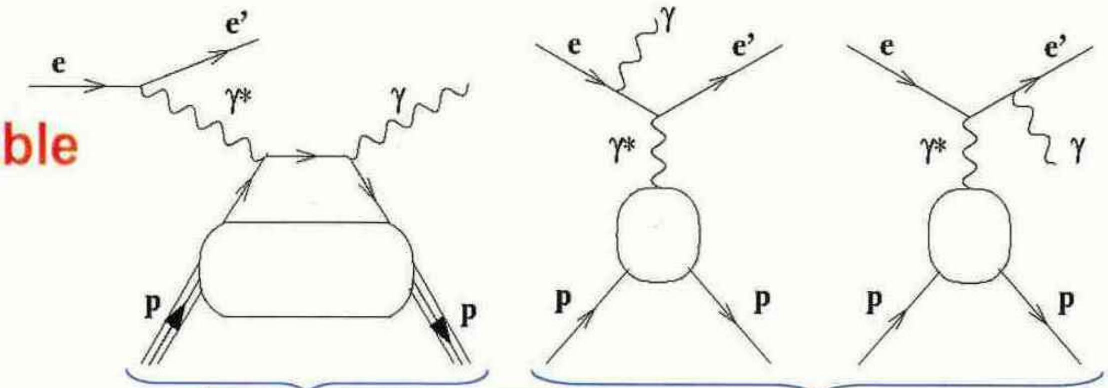
Helicity-flip GPDs

P. Hoodbhoy and X. Ji, PR D 58 (1998) 054006

$$\frac{1}{x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'S'|F^{(\mu\alpha}(-\frac{\lambda}{2}n)F^{\nu\beta)}(\frac{\lambda}{2}n)|PS\rangle$$
$$= H_{Tg}(x,\xi)\bar{U}(P'S')\frac{\bar{P}^{([\mu}i\Delta^{\alpha]}\sigma^{\nu\beta)}}{M}U(PS)$$
$$+\quad E_{Tg}(x,\xi)\bar{U}(P'S')\frac{\bar{P}^{([\mu}\Delta^{\alpha]}}{M}\frac{\gamma^{[\nu}\Delta^{\beta])}}{M}U(PS)+\ldots\;.$$

DVCS & **B**ethe-**H**eitler (BH)

indistinguishable final state



DVCS & Bethe-Heitler

$$\tau = \left|\tau_{BH}\right|^2 + \left|\tau_{DVCS}\right|^2 + \underbrace{\tau_{DVCS}\tau^*_{BH} + \tau^*_{DVCS}\tau_{BH}}_{I}$$

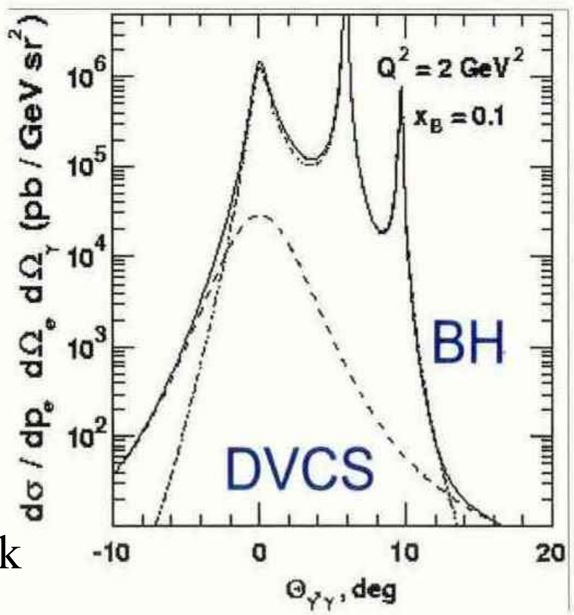
one measures interference of two processes

but BH is calculable in QED

DVCS is suppressed in respect to BH

@ HERMES

@ HERMES



Nowak

Hermes: DVCS on transversely polarized target

projections to appear soon on hep-ph → EPJC

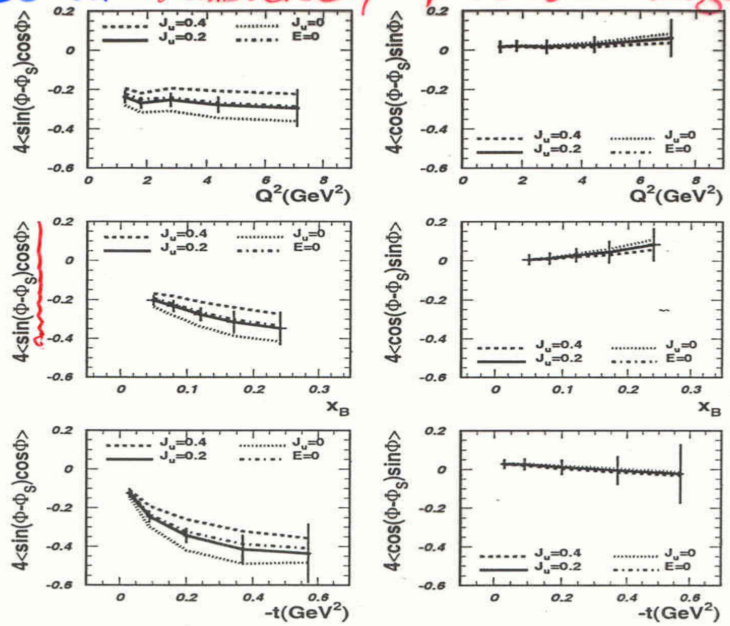


Fig. 5. Expected DVCS asymmetries $A_{UT}^{\sin(\phi-\phi_S)\cos\phi}$ and $A_{UT}^{\cos(\phi-\phi_S)\sin\phi}$ in the Regge ansatz for $b_{val} = 1$, $b_{sea} = 1$, $J_u = 0.4$ $(0.2, 0.0)$, $J_d = 0.0$. $E = 0$ denotes zero effective contribution from the quark GPDs E_q. The calculations are done at the average kinematic values as listed in Tab. 1. Projected statistical errors are shown.

projections for HERMES (2003 - 2005) 8 M DIS = 0.2 fb^{-1}

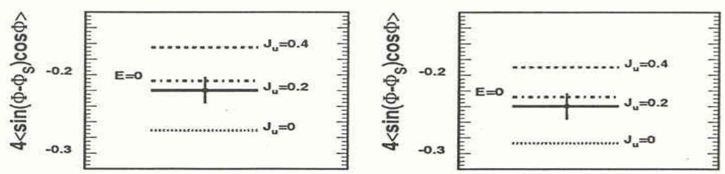


Fig. 6. Expected DVCS asymmetry $A_{UT}^{\sin(\phi-\phi_S)\cos\phi}$ with $b_{val} = 1$ and $b_{sea} = \infty$ (left panel) or $b_{sea} = 1$ (right panel), $J_u = 0.4$ $(0.2, 0.0)$, $J_d = 0.0$ in the Regge ansatz at the average kinematics of the full measurement. $E = 0$ denotes zero effective contribution from the GPDs E_q. The projected statistical error for 8 million DIS events is shown. The systematic error is expected to not exceed the statistical one.

⇒ promising sensitivity!

Nowak

HERMES (2 fb^{-1}):

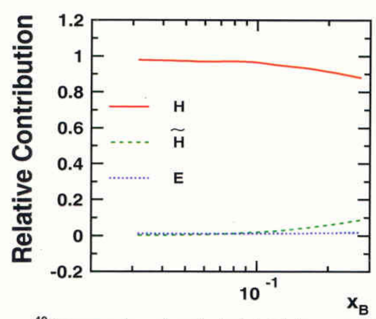
Im$\mathcal{H}$ Measurement in 2006 ? *

Lepton helicity asymmetry: $A_{LU}^{sin\phi} \approx \mathcal{C}_{unp}^{\mathcal{I}}/\mathcal{C}_{unp}^{DVCS}$ with

$$\mathcal{C}_{\rm unp}^{\rm DVCS} = \frac{1}{(2-x_{\rm B})^2}\Big\{4(1-x_{\rm B})\left(\mathcal{H}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{H}}^*\right) - x_{\rm B}^2\Big(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}$$

$$- \left(x_{\rm B}^2 + (2-x_{\rm B})^2\frac{t}{4M^2}\right)\mathcal{E}\mathcal{E}^* - x_{\rm B}^2\frac{t}{4M^2}\widetilde{\mathcal{E}}\widetilde{\mathcal{E}}^*\Big\}.$$

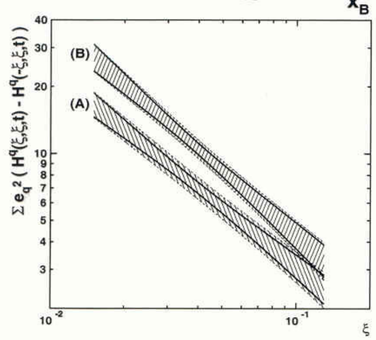
$$\mathcal{C}_{\rm unp}^{\mathcal{I}} = F_1\mathcal{H} + \frac{x_{\rm B}}{2-x_{\rm B}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}$$



At $-t < 0.15$ GeV^2:

Relative contribution of GPD H dominates

$\Rightarrow$ Asymmetry $A_{LU}^{sin\phi}$ mainly depending on Im$\mathcal{H}$



Extraction of Im$\mathcal{H}$ possible:

$\Leftarrow$ Two different bands for different GPD param.'s

$\Leftarrow$ Solid line: 1σ stat. errors

$\Leftarrow$ Dashed line: syst. extraction uncertainty added

*) Projections: V. Korotkov, W.-D. N., NPA 711, 175c, (2002)

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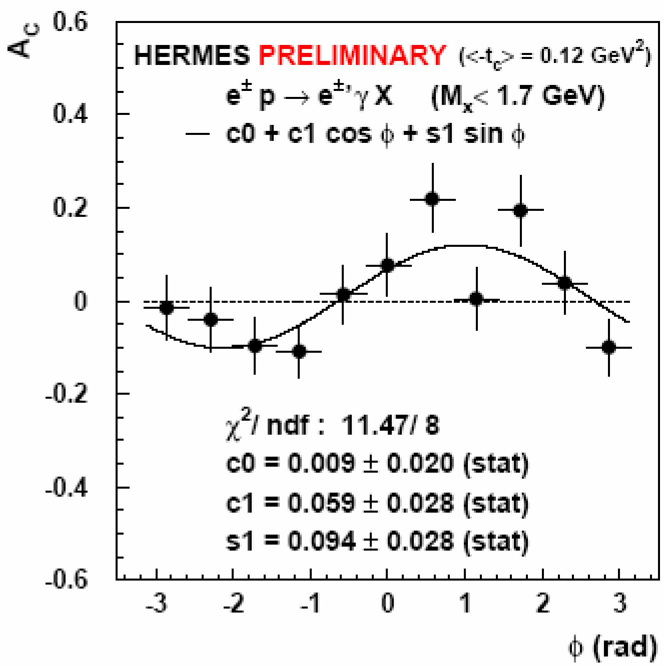
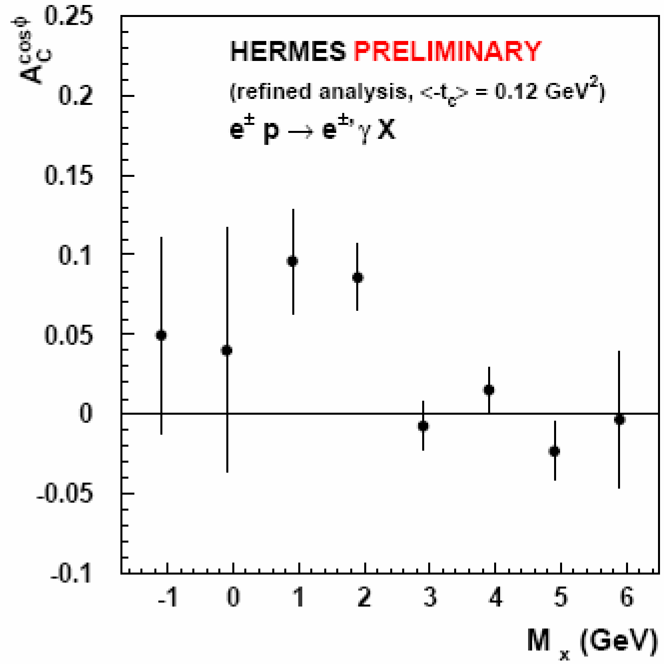
Beam–Charge Asymmetry (BCA)

$$_{\mathrm{C}}(\phi) = \frac{N^+(\phi)-N^-(\phi)}{N^+(\phi)+N^-(\phi)} \propto I \propto \pm\left(c_0^I + \sum_{n=1}^{3} c_n^I \cos(n\phi) + \lambda \sum_{n=1}^{2} s_n^I \sin(n\phi)\right)$$

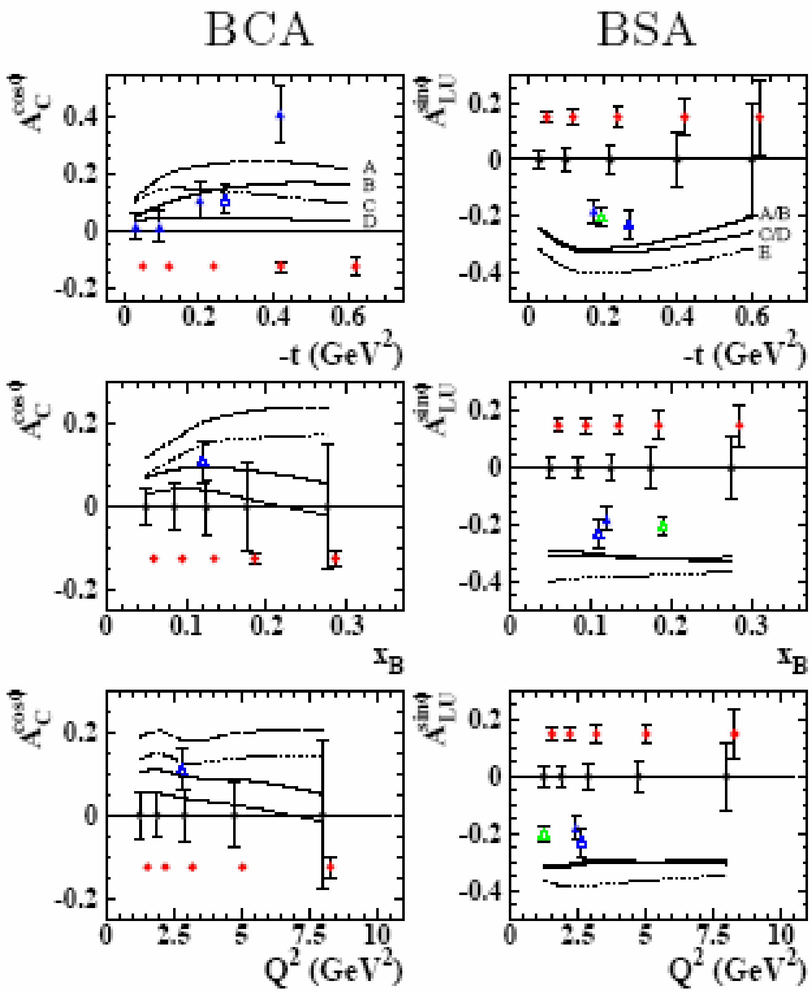




A_{C} in exclusive bin: Expected $\cos(\phi)$ dependence $\Rightarrow \mathrm{Re}\, M_{unp}^{1,1}$
$\sin\phi$ due to polarized beam

$\cos(\phi)$–Moments zero at higher missing mass

The GPD H, Summary and Outlook



△: HERMES prelim./published

△: CLAS, PRL, 2001 ($\times -1$)

● Hydrogen data (1996-2000), Analysis almost completed

● BCA: $1fb^{-1}\,e^+$ and $1fb^{-1}\,e^-$

● BSA: $1fb^{-1}\,e^+$, Pol. = 40%

(exp. 2006/2007 Recoil data)

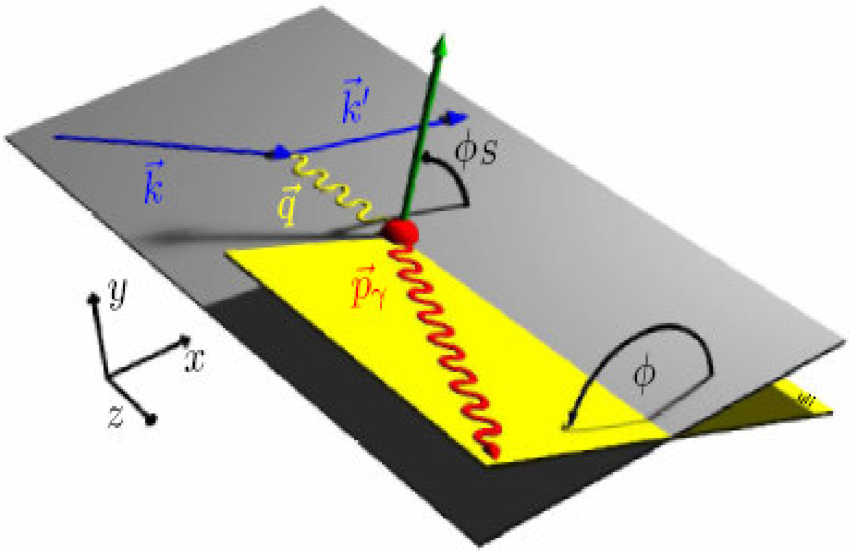
BCA: high sensitivity to t-dependence (fact./Regge) and D-term

BSA: highest sensitivity to b_s parameter in profile function

Possibility to "map out" GPD H^u in the final two HERA years.

WHAT ABOUT THE GDP E ?

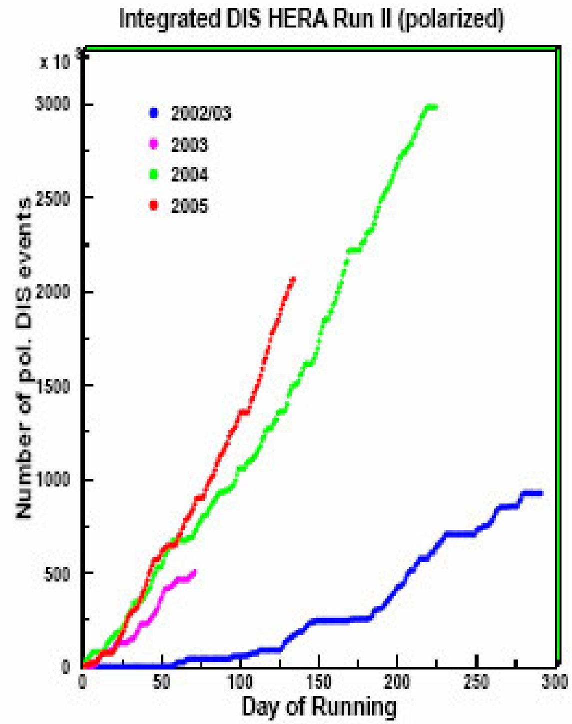
A_{UT}: UNPOLARIZED BEAM,
TRANSVERSELY POL. TARGET



$$A_{UT}^{\sin(\phi-\phi_s)\cos\phi} \sim \frac{-t}{4M_p}(F_2 H_1 - F_1 E_1)$$

$$A_{UT}^{\cos(\phi-\phi_s)\sin\phi} \to \frac{-t}{4M_p}(F_2 \widetilde{H}_1 - \xi F_1 \widetilde{E}_1)$$

DATA TAKING WITH TRANSVERSE HYDROGEN TARGET IN PROGRESS ...
≈ 6 MILLION ON TAPE



≈ 8 MIO EXPECTED IN TOTAL
(NOVEMBER 2005)

Ellinghaus

GPD	Reaction	Obs.	Expt		Status	
$H(\pm\xi,\xi,t)$	ep→epγ (DVCS)	BSA	CLAS	4.2 GeV	Published PRL	*From*
			CLAS	4.8 GeV	Preliminary	*$ep \to epX$*
			CLAS	5.75 GeV	Preliminary	
		(+ σ)	Hall A	5.75 GeV	Fall 04	*Dedicated*
			CLAS	5.75 GeV	Spring 05	*set-up*
$\widetilde{H}(\pm\xi,\xi,t)$	**ep→epγ (DVCS)**	TSA	CLAS	5.65 GeV	Preliminary	
$E(\pm\xi,\xi,t)$	**e(n)→enγ (DVCS)**	BSA	Hall A	5.75 GeV	Fall 04	
$(u+d)$	**ed→edγ (DVCS)**	BSA	CLAS	5.4 GeV	under analysis	
$H(\|x\|<\xi,\xi,t)$	**ep→epe$^+$e$^-$ (DDVCS)**	BSA	CLAS	5.75 GeV	under analysis	
$\int_x H,E \quad (u+d)$	**ep→epρ**	σ_L	CLAS	4.2 GeV	Published PLB	
			CLAS	5.75 GeV	under analysis	
$\int_x H,E \quad (2u-d)$	**ep→epω**	(σ_L)	CLAS	5.75 GeV	Accepted EPJA	

+ other meson production channels π, η, Φ under analyses in the three Halls.

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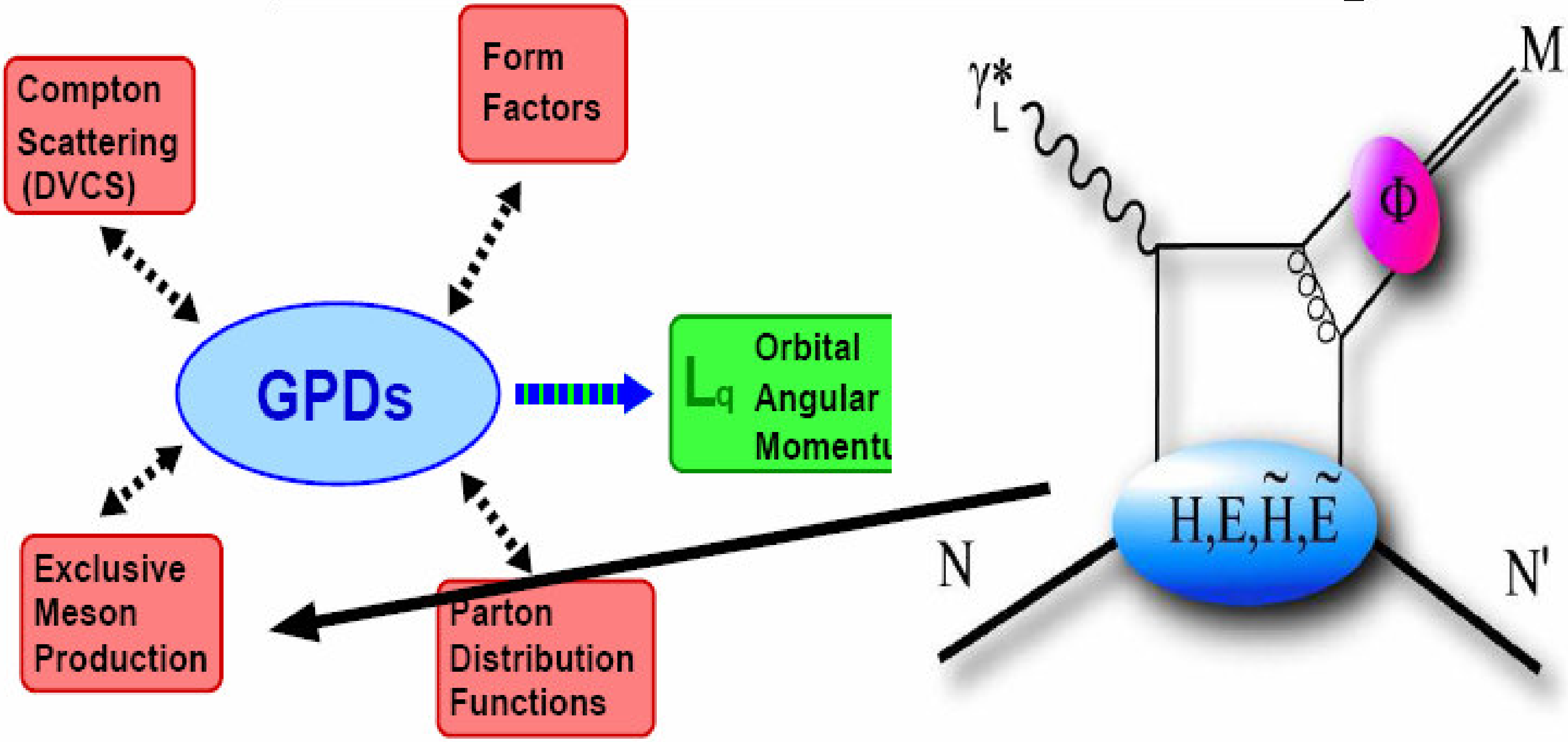
Exclusive Reactions & GPDs

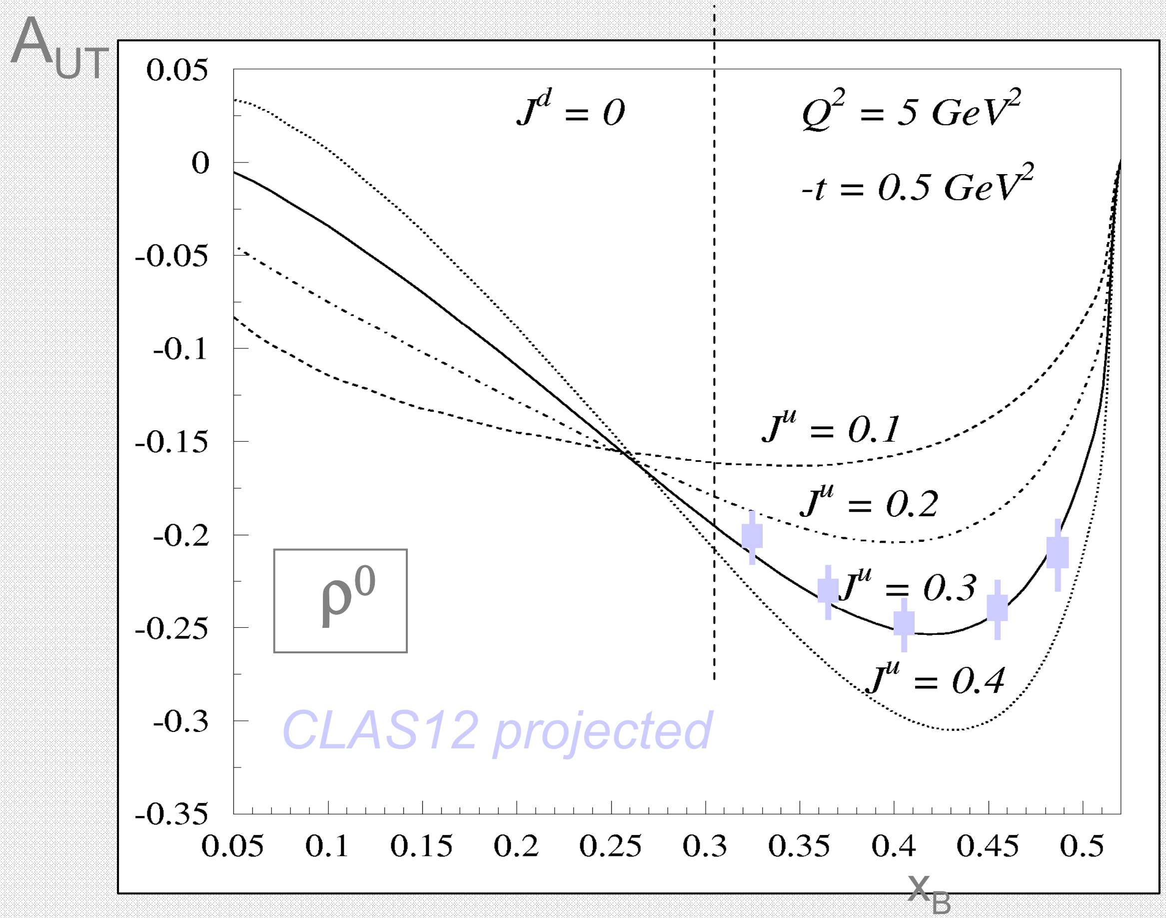


☞ Quantum numbers of final meson state select different $GPDs$

- Pseudoscalar mesons $(\pi, \eta ...)$: $\tilde{H}, \tilde{E}$
- Vector mesons $(\rho, \omega, \phi ...)$: H, E (flavour singlet)
- f-meson family $(f_0, f_2, ...)$: H, E (flavour non-singlet)

Exclusive ρ production on transverse target

$A_{UT} \sim \mathrm{Im}(AB^*)$



ρ^0: $A \sim 2H^u + H^d$, $B \sim 2E^u + E^d$

ρ^+: $A \sim H^u - H^d$, $B \sim E^u - E^d$

Asymmetry depends linearly on the GPD **E** in Ji's sum rule.

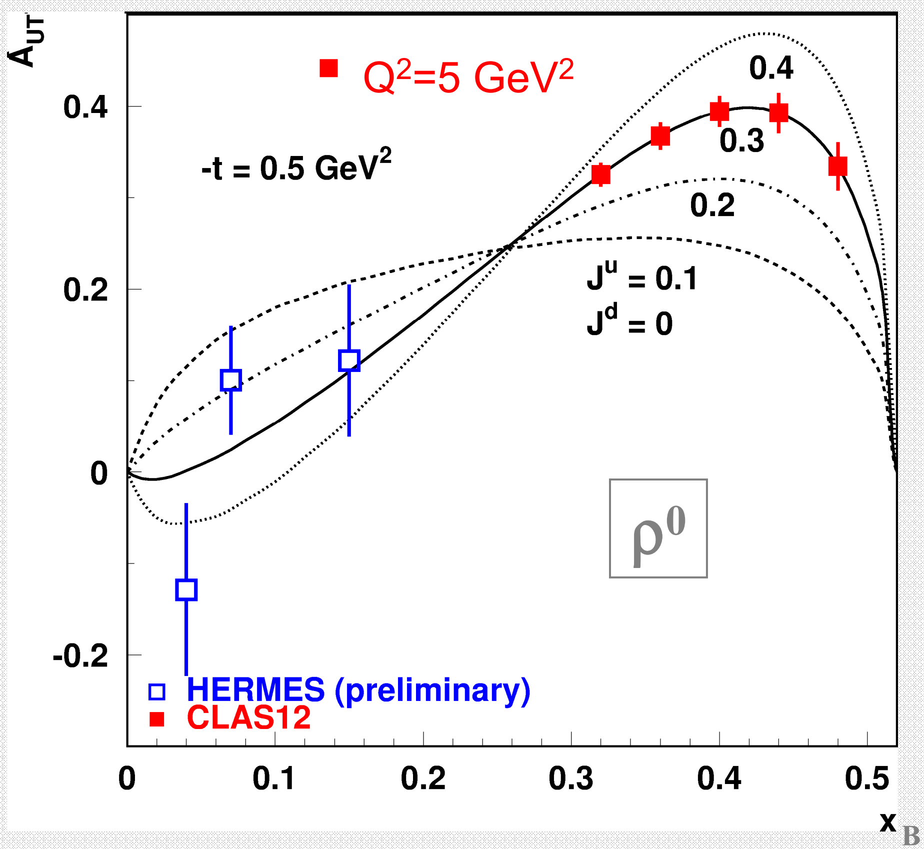
ρ^0 **and** ρ^+ measurements allow separation of E^u, E^d

K. Goeke, M.V. Polyakov, M. Vanderhaeghen, 2001

Burkert

Exclusive ρ^0 production on transverse target

$$A_{UT} = -\frac{2\Delta_\perp(\mathrm{Im}(AB^*))/\pi}{|A|^2(1-\xi^2) - |B|^2(\xi^2+t/4m^2) - \mathrm{Re}(AB^*)2\xi^2}$$



ρ^0

$A \sim 2H^u + H^d$

$B \sim 2E^u + E^d$

E^u, E^d needed for angular momentum sum rule.

K. Goeke, M.V. Polyakov, M. Vanderhaeghen, 2001

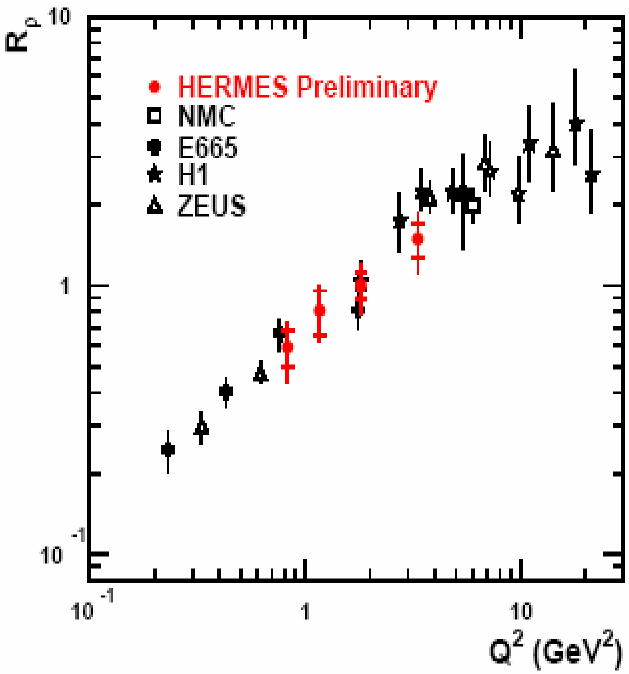
Burkert

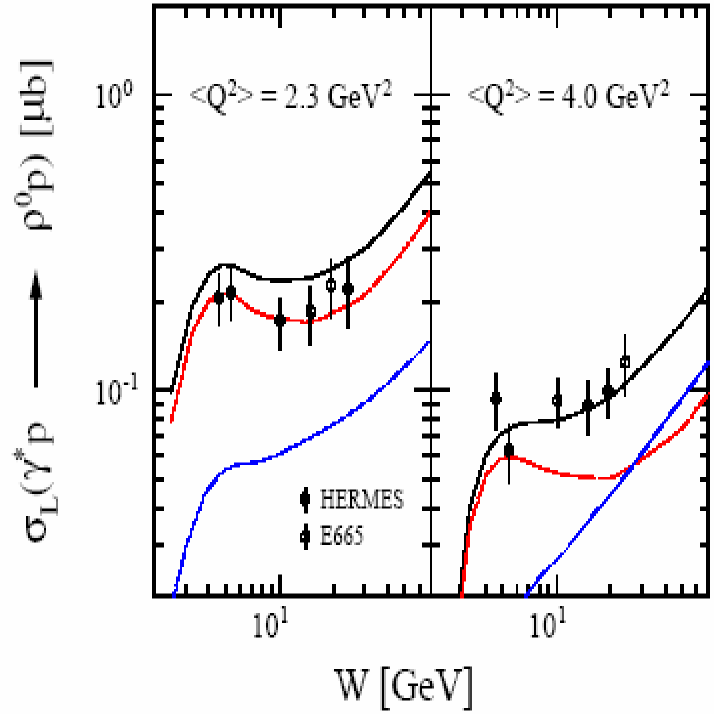
Hard Exclusive ρ^o Production

$\gamma_L^\star p \longrightarrow p\rho^o$

Measurement of the cross-section σ_L

$$R = \frac{\sigma_L}{\sigma_T} = \frac{1}{\epsilon}\frac{r_{00}^{04}}{1-r_{00}^{04}}$$





- GPD calculations in terms of H & E

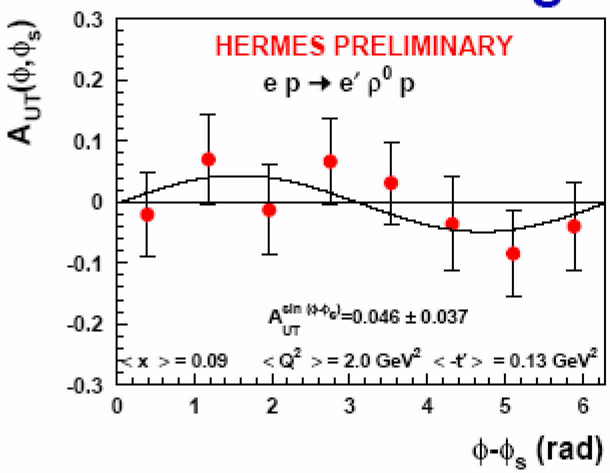
  - Vanderhaeghen, Guichon & Guidal -

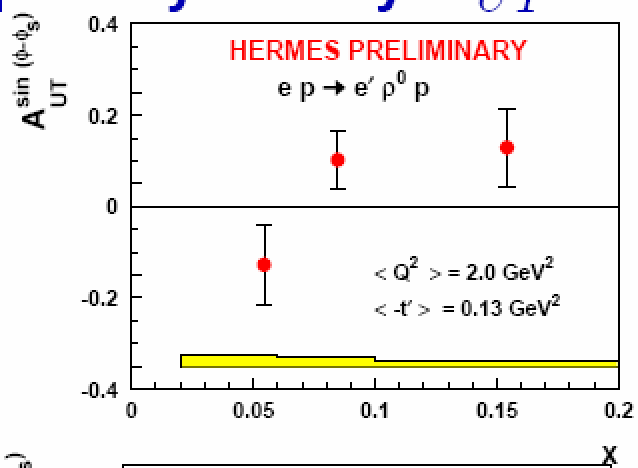
- $q\bar{q}$-exchange
- gluon-exchange

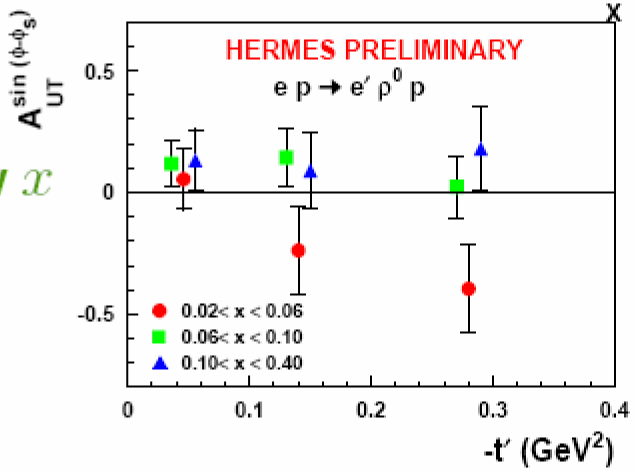
} mechanisms considered

Hard Exclusive ρ^o Production

Transverse Target Spin Asymmetry A_{UT}







- No σ_L separation yet!
- Indication of t-depend. at low x

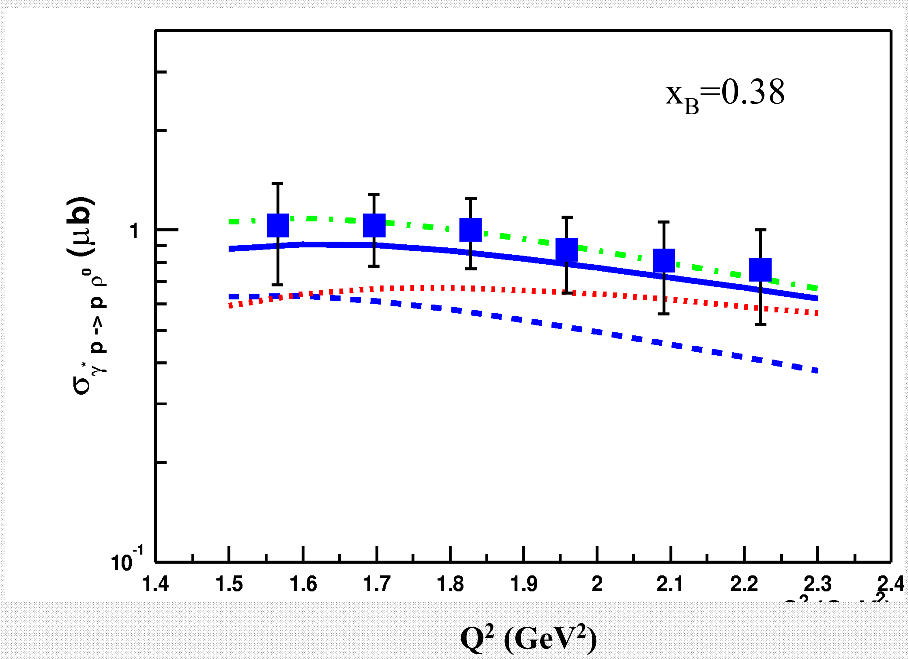
☞ possibly doubled statistics at end 2005

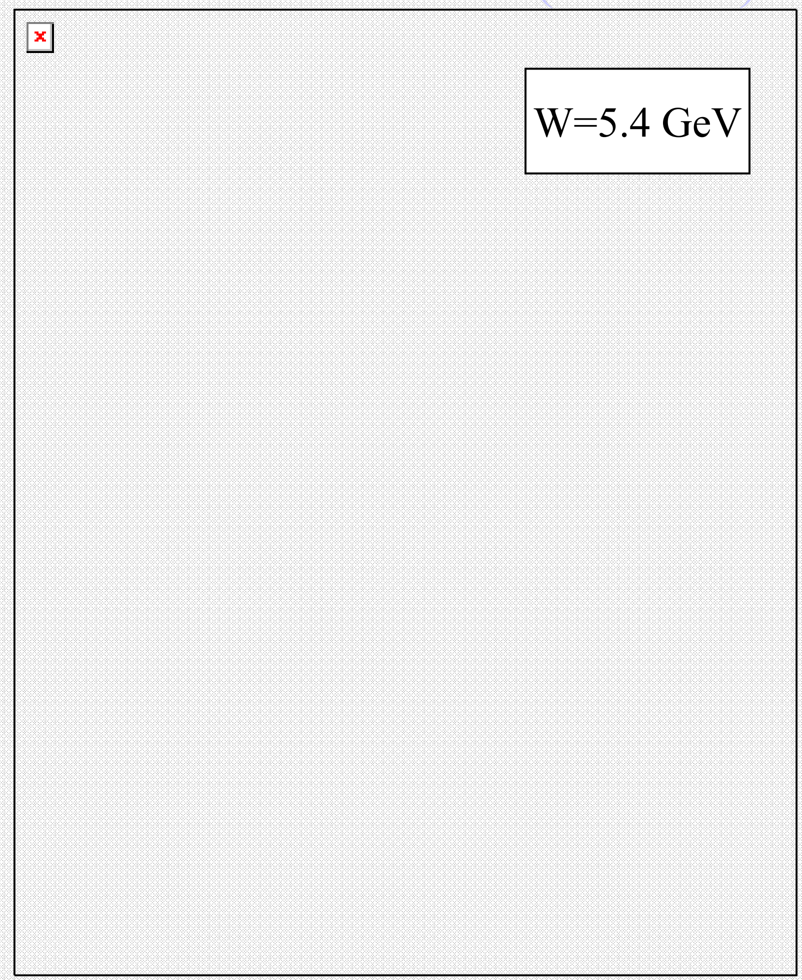
☞ σ_L/σ_T separation possible

Exclusive ep ⟶ epρ^0_L production

HERMES (27GeV)

CLAS (4.3 GeV)





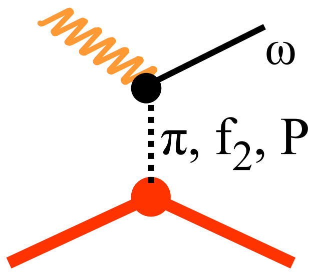
GPD formalism approximately describes CLAS and HERMES data $Q^2 > 2$ GeV^2

Burkert

Deeply virtual meson production

Meson and Pomeron (or two-gluon) exchange …

(Photoproduction)

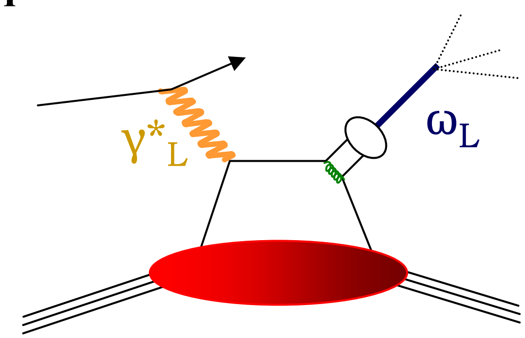


ρ^0	(σ), f_2, P
ω	π, f_2, P
Φ	P

… or scattering at the quark level ?

Flavor sensitivity of DVMP on the proton:

ρ^0	2u+d, 9g/4
ω	2u-d, 3g/4
Φ	s, g
ρ^+	u-d



$$\frac{d\sigma_L}{dt} \propto \frac{1}{Q^4}\left[\frac{\alpha_S}{Q}\sum\iint\frac{\psi_M(z)}{z}\frac{1}{x\pm\xi\mp i\varepsilon}(aH+bE)(x,\xi,t)dxdz\right]^2 \propto \frac{f(\xi,t)}{Q^6}$$

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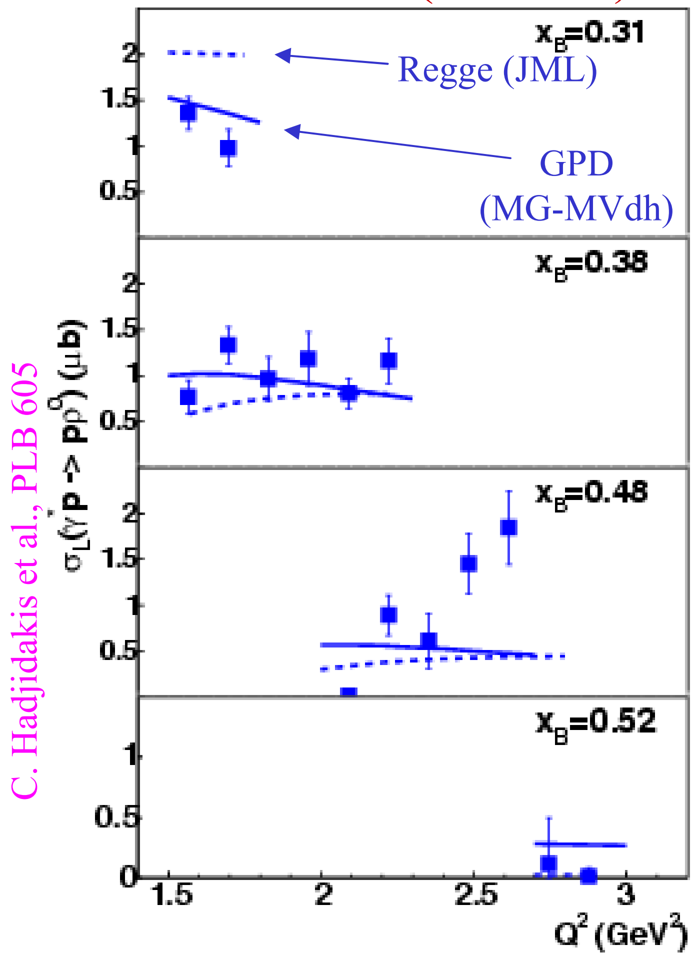
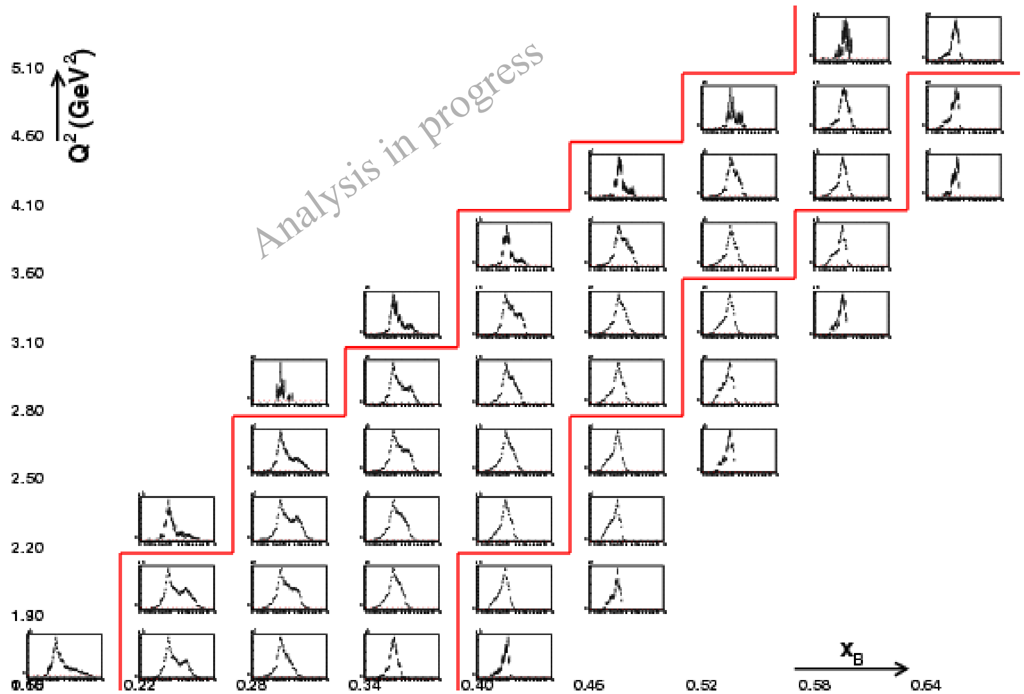
Exclusive ρ meson production: $ep \rightarrow ep\rho$

CLAS (4.2 GeV)



GPD formalism (beyond leading order) describes approximately data for x_B<0.4, Q^2 >1.5 GeV^2

CLAS (5.75 GeV)



Two-pion invariant mass spectra

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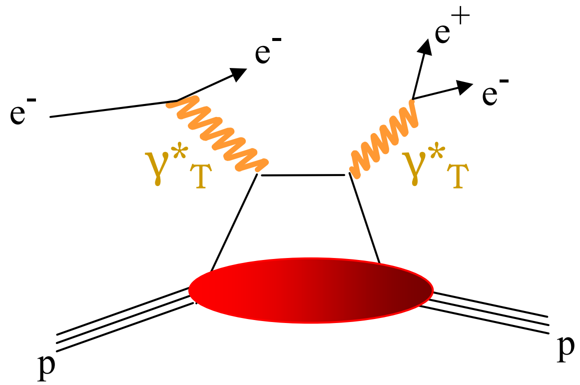
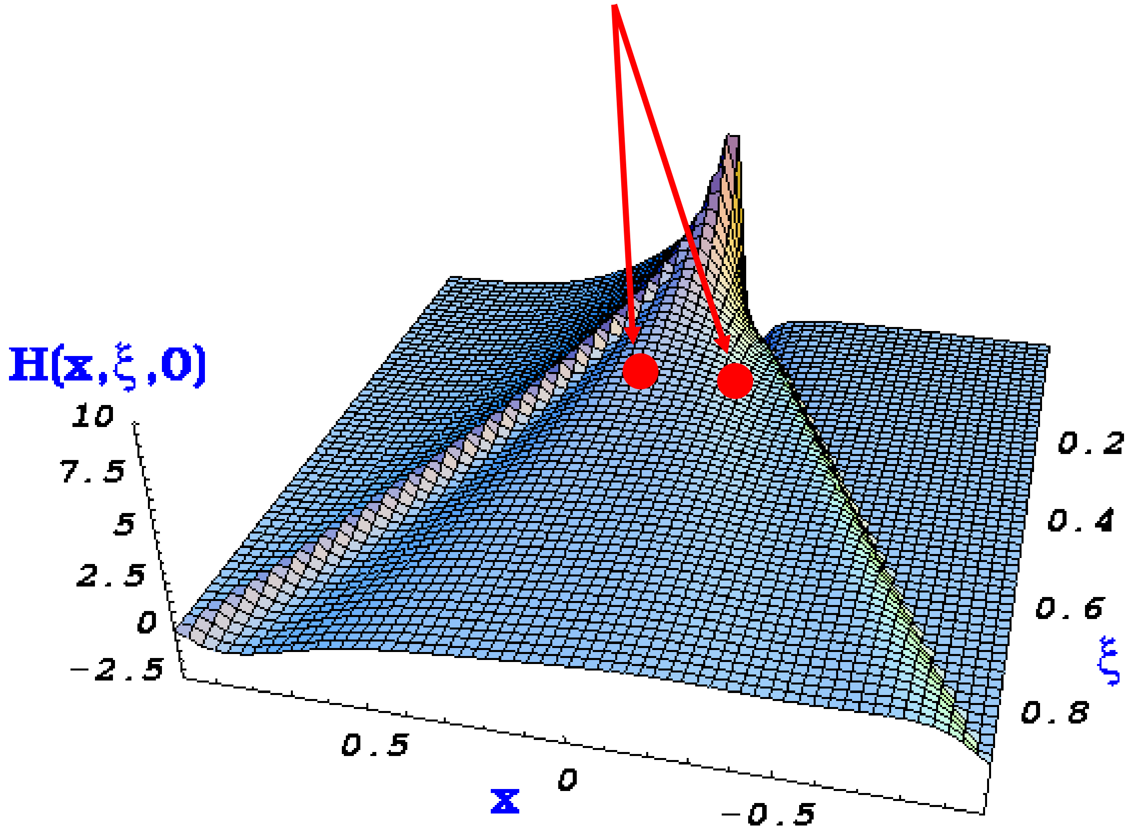
DDVCS
(Double Deeply Virtual Compton Scattering)

DDVCS-BH interference generates a **beam spin asymmetry** sensitive to

$$\mathrm{Im}\, T^{DDVCS} \sim H(\pm x(\xi, q'), \xi, t) + \mathrm{K}$$



The (continuously varying) virtuality of the outgoing photon allows to "tune" the kinematical point (x,ξ,t) at which the GPDs are sampled (with |**x** | < ξ).



M. Guidal & M. Vanderhaeghen, PRL 90
A. V. Belitsky & D. Müller, PRL 90

M.Garcon

DDVCS: first observation of ep → epe$^+$e$^-$

* **<u>Positrons identified</u>** among large background of positive pions

* **<u>ep→epe$^+$e$^-$ cleanly selected</u>** (mostly) through missing mass ep→epe$^+$X

* Φ distribution of outgoing γ* and <u>beam spin asymmetry</u> extracted
(integrated over γ* virtuality)

but…

<u>A problem for both experiment and theory</u>:

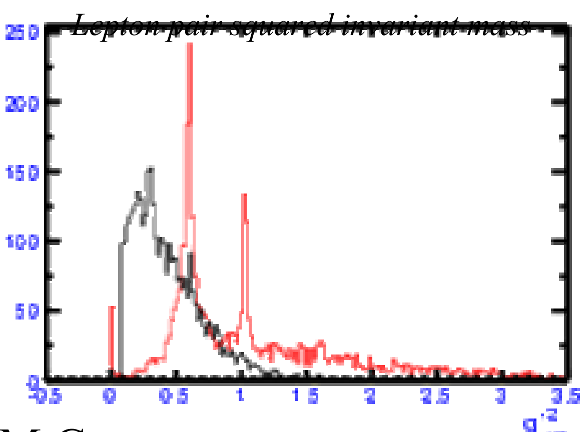
* 2 electrons in the final state → *antisymmetrisation* was not included in calculations,

→ define domain of validity for *exchange diagram*.

* data analysis was performed assuming two different hypotheses

either detected electron = scattered electron

or detected electron belongs to lepton pair from γ*



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Hyp. 2 seems the most valid

→ **quasi-real photoproduction of vector mesons**

GPD CHALLENGES

- Goal: map out the full dependence on x, ξ, t, Q^2
- Develop models consistent with known forward distributions, form factors, polynomiality constraints, positivity,…
- More lattice moments, smaller pion masses, towards unquenched QCD,…
- Launch a world-wide program for analyzing GPDs perhaps along the lines of CTEQ for PDFs.
- High energy, high luminosity is needed to map out GPDs in deeply virtual exclusive processes such as DDVCS (JLab with 12GeV unique).