



# $O(\alpha_s)$ RADIATIVE CORRECTIONS TO POLARIZED TOP QUARK DECAY INTO A CHARGED HIGGS

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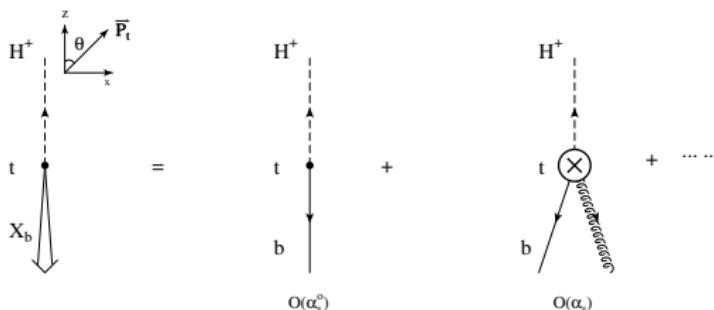
# INTRODUCTION

Charged Higgs occur

- in the 2-Higgs doublet SM,
  - in the MSSM (naturally).
- 
- Mass of the charged Higgs is not known.  
Particle Data Group:  $m_{H^\pm} > 79.3 \text{ GeV. (CL 95\%)}$
- 
- Possible decay processes:  
 $t \rightarrow H^+ + b$       if       $m_H < m_t - m_b$   
 $H^- \rightarrow \bar{t} + b$  or  $H^+ \rightarrow t + \bar{b}$       if       $m_H > m_t + m_b$

# THE GENERAL ANGULAR DECAY DISTRIBUTION

The decay  $t(\uparrow) \rightarrow b + H^+$ :



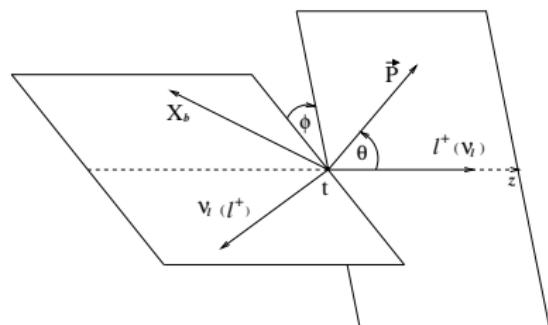
$$\frac{d\Gamma}{d \cos \theta} = \frac{1}{2} (\Gamma + P_t \Gamma^P \cos \theta)$$

asymmetry parameter  $\alpha_H := \frac{\Gamma^P}{\Gamma}$

## ANOTHER EXAMPLE OF ANGULAR DECAY DISTRIBUTIONS

The angular decay distribution in the decay  $t(\uparrow) \rightarrow b + W^+(\ell^+ \nu)$ :

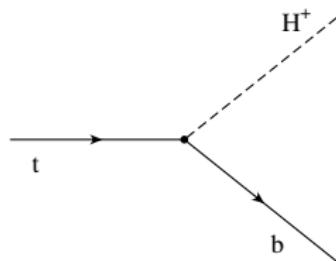
$$\frac{d\Gamma}{dx_{\ell,\nu} d\cos\theta d\phi} = \frac{1}{4\pi} \left[ \frac{d\Gamma_A}{dx_{\ell,\nu}} + P \left( \frac{d\Gamma_B}{dx_{\ell,\nu}} \cos\theta + \frac{d\Gamma_C}{dx_{\ell,\nu}} \sin\theta \cos\phi \right) \right]$$



$$\vec{P} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$

$$x_{\ell,\nu} = \frac{2E_{\ell,\nu}}{m_t}$$

# BORN TERM RESULTS



Structure of tbH-vertex:  $(a + b\gamma_5)$ : ( $\tan \beta = \frac{v_1}{v_2}$ )

**model 1:**  $a = \frac{g_w}{2\sqrt{2}m_W} V_{tb}(m_t - m_b) \cot \beta,$

$$b = \frac{g_w}{2\sqrt{2}m_W} V_{tb}(m_t + m_b) \cot \beta,$$

**model 2:**  $a = \frac{g_w}{2\sqrt{2}m_W} V_{tb}(m_t \cot \beta + m_b \tan \beta),$

$$b = \frac{g_w}{2\sqrt{2}m_W} V_{tb}(m_t \cot \beta - m_b \tan \beta).$$

In the leading-order calculation we define following scaled masses

$$y := \frac{m_{H^+}}{m_t}, \quad \epsilon := \frac{m_b}{m_t}$$

and scaled kinetic variables

$$\begin{aligned} p_0 &:= \frac{1}{2}(1 + \epsilon^2 - y^2) \\ p_3 &:= \frac{1}{2}\sqrt{\lambda(1, \epsilon^2, y^2)} \end{aligned}$$

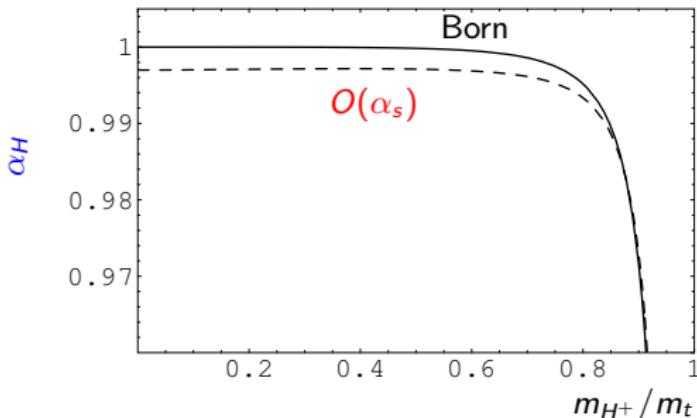
where  $\lambda(a, b, c) := a^2 + b^2 + c^2 - 2(ab + bc + ca)$  (Källén function).

**The Born amplitude is:**  $\mathcal{M}_0 = \bar{u}_b(a\mathbb{1} + b\gamma_5)u_t$

## TOTAL RATES

$$\Gamma_{Born} = \frac{p_3}{16\pi m_t^3} \left\{ 2p_0(a^2 + b^2) + 2\epsilon(a^2 - b^2) \right\}$$

$$\Gamma_{Born}^P = \frac{p_3}{16\pi m_t^3} \left\{ 4ab p_3 \right\}.$$

**Figure 1**

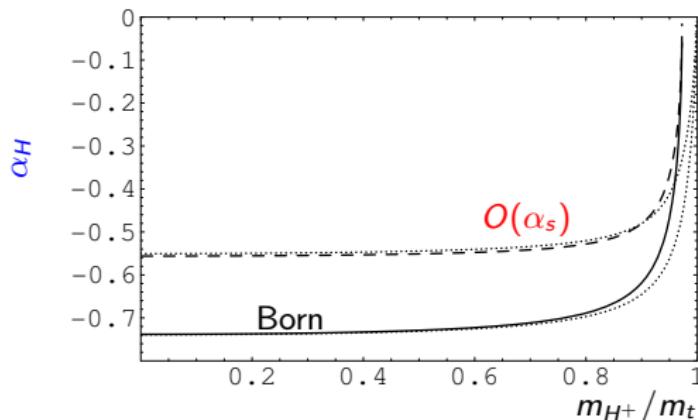
Asymmetry parameter  $\alpha_H$  for model 1:

$m_b = 4.8 \text{ GeV}$ ,  $m_t = 175 \text{ GeV}$ ,

Born term: full line,

$O(\alpha_s)$ : dashed line.

( $\tan \beta = 10$ )

**Figure 2**

Asymmetry parameter  $\alpha_H$  for model 2:

$m_b = 4.8 \text{ GeV}$ ,  $m_t = 175 \text{ GeV}$ ,

Born term: full line,

$O(\alpha_s)$ : dashed line.

( $\tan \beta = 10$ )

# NEXT-TO-LEADING ORDER CORRECTIONS

- The virtual corrections
- The real corrections

# VARIABLES IN THE NLO CALCULATIONS

Scaled masses:

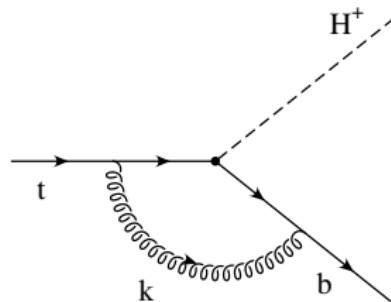
$$y := \frac{m_{H^+}}{m_t}, \quad \epsilon := \frac{m_b}{m_t} \quad \Lambda := \frac{m_g}{m_t}$$

Scaled kinetic variables

$$\begin{array}{ll} p_0 := \frac{1}{2}(1 + \epsilon^2 - y^2), & w_0 := \frac{1}{2}(1 - \epsilon^2 + y^2), \\ p_3 := \frac{1}{2}\sqrt{\lambda(1, \epsilon^2, y^2)}, & w_3 := p_3, \\ p_{\pm} := p_0 \pm p_3, & w_{\pm} := w_0 \pm w_3, \\ Y_p := \frac{1}{2} \ln \frac{p_+}{p_-}. & Y_w := \frac{1}{2} \ln \frac{w_+}{w_-}. \end{array}$$

where  $\lambda(a, b, c) := a^2 + b^2 + c^2 - 2(ab + bc + ca)$  (Källén function)

# VIRTUAL CORRECTIONS



$$\begin{aligned} \bar{u}_b \int \frac{d^D k}{(2\pi)^4} \left( \frac{-ig_s \lambda^a}{2} \gamma_\mu \right) \frac{i}{(\not{p}_b - \not{k}) - m_b} (a + b\gamma_5) \frac{i}{(\not{p}_t - \not{k}) - m_t} \left( \gamma_\nu \frac{-ig_s \lambda^b}{2} \right) u_t (-i\delta_{ab} \frac{g^{\mu\nu}}{k^2 - m_g^2}) \\ = -ig_s^2 C_F \bar{u}_b \int \frac{d^D k}{(2\pi)^4} \frac{\gamma_\mu (\not{p}_b - \not{k} + m_b)(a + b\gamma_5)(\not{p}_t - \not{k} + m_t)\gamma^\mu}{[(p_b - k)^2 - m_b^2][(p_t - k)^2 - m_t^2][k^2 - m_g^2]} u_t \end{aligned}$$

*numerator* :  $4(p_t \cdot p_b) - 2\not{p}_t \not{k} + 2\not{p}_b \not{k} - Dk^2$  with  $D = 4 - 2\omega$

*denominator* :  $[(p_b - k)^2 - m_b^2][(p_t - k)^2 - m_t^2][k^2 - m_g^2] := D_b \cdot D_t \cdot D_g$

# TYPES OF INTEGRALS IN THE LOOP

$$\int \frac{d^D k}{(2\pi)^4} \frac{1}{D_b \cdot D_t \cdot D_g} := \underbrace{C_0}_{IR-div.}$$

$$\int \frac{d^D k}{(2\pi)^4} \frac{k^\mu}{D_b \cdot D_t \cdot D_g} := I_3^\mu = C_1 p_t^\mu + C_2 p_b^\mu$$

$$\begin{aligned} \int \frac{d^D k}{(2\pi)^4} \frac{k^2}{D_b \cdot D_t \cdot D_g} &= \int \frac{d^D k}{(2\pi)^4} \frac{k^2 - m_g^2 + m_g^2}{D_b \cdot D_t \cdot D_g} \\ &= \int \frac{d^D k}{(2\pi)^4} \frac{1}{D_b \cdot D_t} + m_g^2 \int \frac{d^D k}{(2\pi)^4} \frac{1}{D_b \cdot D_t \cdot D_g} \\ &:= \underbrace{B_0}_{UV-div.} + m_g^2 C_0 \end{aligned}$$

$C_0$  → scalar three – point one – loop integral

$B_0$  → scalar two – point one – loop integral

$I_3^\mu = C_1 p_t^\mu + C_2 p_b^\mu$  → tensor three – point one – loop integral,  
calculated using Passarino – Veltman method

$$I_3(m_H^2, m_t^2, m_b^2, m_g^2)$$

$$\begin{aligned}
&= -\frac{i}{16\pi^2} \frac{1}{2\sqrt{\lambda(m_H^2, m_t^2, m_b^2)}} \left[ \ln \left( \frac{m_t^2 + m_b^2 - m_H^2 + \sqrt{\lambda(m_H^2, m_t^2, m_b^2)}}{m_t^2 + m_b^2 - m_H^2 - \sqrt{\lambda(m_H^2, m_t^2, m_b^2)}} \right) \ln \left( \frac{m_t m_b}{m_g^2} \right) \right. \\
&\quad + \ln \left( \frac{m_t^2 + m_b^2 - m_H^2 + \sqrt{\lambda(m_H^2, m_t^2, m_b^2)}}{2m_t^2} \right) \ln \left( \frac{(m_t^2 - m_b^2 + \sqrt{\lambda(m_H^2, m_t^2, m_b^2)})^2 - (m_H^2)^2}{(m_t^2 - m_b^2 - \sqrt{\lambda(m_H^2, m_t^2, m_b^2)})^2 - (m_H^2)^2} \right) \\
&\quad \left. - 2\text{Li}_2 \left( \frac{2\sqrt{\lambda(m_H^2, m_t^2, m_b^2)}}{m_t^2 - m_b^2 + m_H^2 + \sqrt{\lambda(m_H^2, m_t^2, m_b^2)}} \right) + 2\text{Li}_2 \left( \frac{2\sqrt{\lambda(m_H^2, m_t^2, m_b^2)}}{m_t^2 - m_b^2 - m_H^2 + \sqrt{\lambda(m_H^2, m_t^2, m_b^2)}} \right) \right]
\end{aligned}$$

# LOOP INTEGRALS

$$\begin{aligned}
 C_1 &= -\frac{1}{m_t^2 y^2} \left\{ \ln \epsilon + \frac{1}{2p_3} (1 - \epsilon^2 - y^2) \right\}, \\
 C_2 &= \frac{1}{m_t^2 y^2} \left\{ \ln \epsilon + \frac{1}{2p_3} (1 - \epsilon^2 + y^2) \right\}, \\
 B_0 &= \frac{1}{\omega} + \gamma_E + \ln \left( \frac{4\pi\mu^2}{m_t^2} \right) + 2 + \frac{2p_3}{y^2} Y_p + \frac{1 - \epsilon^2 + y^2}{y^2} \ln \epsilon, \\
 C_0 &= -\frac{1}{4m_t^2 p_3} \left\{ (\ln \epsilon - 2 \ln \Lambda) Y_p + 2(Y_p + \ln \epsilon)(Y_w + Y_p) \right. \\
 &\quad \left. - 2\text{Li}_2 \left( 1 - \frac{w_-}{w_+} \right) + 2\text{Li}_2 \left( 1 - \frac{w_- p_-}{w_+ p_+} \right) \right\}.
 \end{aligned}$$

## RENORMALIZED AMPLITUDE OF THE VIRTUAL CORRECTIONS

$$\mathcal{M}_{loop} = \bar{u}_b \left\{ \left( a\mathbb{1} + b\gamma_5 \right) \Lambda_1 + a\Lambda_2 + \delta\Lambda \right\} u_t$$

$$\begin{aligned}\Lambda_1 &= \frac{\alpha_s C_F m_t^2}{2\pi} \left\{ 2p_0 C_0 + (2p_0 + (1 + \epsilon)) C_1 + (2p_0 + (1 + \epsilon)) C_2 + 2B_0 - 1 \right\}, \\ \Lambda_2 &= \frac{\alpha_s C_F}{\pi} \left\{ -\epsilon(C_1 + C_2) \right\},\end{aligned}$$

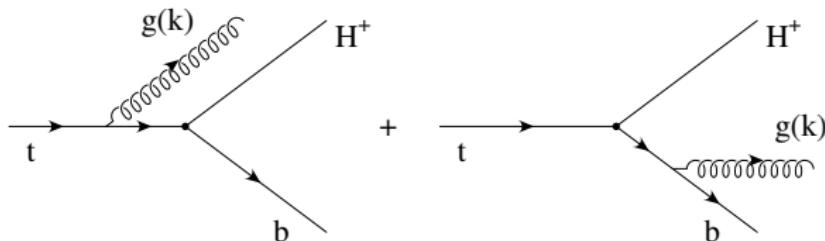
Renormalization gives:

$$\begin{aligned}\delta\Lambda &= (a + b) \frac{\mathbf{1} + \gamma_5}{2} \left( \frac{1}{2}(Z_2^t - 1) + \frac{1}{2}(Z_2^b - 1) - \frac{\delta m_t}{m_t} \right) + \\ &\quad + (a - b) \frac{\mathbf{1} - \gamma_5}{2} \left( \frac{1}{2}(Z_2^t - 1) + \frac{1}{2}(Z_2^b - 1) - \frac{\delta m_b}{m_b} \right).\end{aligned}$$

# THE VIRTUAL ONE-LOOP CORRECTIONS

$$\begin{aligned}
 \Gamma_{loop} &= \Gamma_{Born} \left( Z_2^t - 1 + Z_2^b - 1 - \frac{\delta m_t}{m_t} - \frac{\delta m_b}{m_b} + 2\Lambda_1 \right) + \\
 &+ \frac{4m_t p_3}{16\pi} \left( 2a^2 \Lambda_2(p_0 + \epsilon) - 2ab p_0 \left( \frac{\delta m_t}{m_t} - \frac{\delta m_b}{m_b} \right) \right), \\
 \Gamma_{loop}^P &= \Gamma_{Born}^P \left( Z_2^t - 1 + Z_2^b - 1 - \frac{\delta m_t}{m_t} - \frac{\delta m_b}{m_b} + 2\Lambda_1 \right) + \\
 &+ \frac{4m_t p_3^2}{16\pi} \left( 2ab \Lambda_2 - (a^2 + b^2) \left( \frac{\delta m_t}{m_t} - \frac{\delta m_b}{m_b} \right) \right).
 \end{aligned}$$

# REAL EMISSIONS



$$\begin{aligned} \mathcal{M}_{tree} &= g_s \frac{\lambda^a}{2} \bar{u}_b \left\{ \frac{2p_t^\sigma - \not{k}\gamma^\sigma}{2k \cdot p_t} - \frac{2p_b^\sigma + \gamma^\sigma \not{k}}{2k \cdot p_b} \right\} (a\mathbf{1} + b\gamma_5) u_t \varepsilon_\sigma^*(k) \\ &= g_s \frac{\lambda^a}{2} \bar{u}_b \left\{ \left( \frac{2p_t^\sigma}{2k \cdot p_t} - \frac{2p_b^\sigma}{2k \cdot p_b} \right) + \left( \frac{-\not{k}\gamma^\sigma}{2k \cdot p_t} - \frac{\gamma^\sigma \not{k}}{2k \cdot p_b} \right) \right\} (a\mathbf{1} + b\gamma_5) u_t \varepsilon_\sigma^*(k) \end{aligned}$$

# CONVERGENT AND DIVERGENT PART OF THE TREE

$$|\mathcal{M}_{tree}|^2 = |\mathcal{M}_{tree}^{con}|^2 + |\widetilde{\mathcal{M}}_0|^2 |\mathcal{M}|_{SGF}^2$$

where the universal soft-gluon factor  $|\mathcal{M}|_{SGF}^2$  is given by

$$|\mathcal{M}|_{SGF}^2 = -\frac{\alpha_s}{4\pi} C_F \left\{ \frac{m_t^2}{(k \cdot p_t)^2} + \frac{m_b^2}{(k \cdot p_b)^2} - 2 \frac{p_b \cdot p_t}{(k \cdot p_b)(k \cdot p_t)} \right\}$$

## Difficulty!

Divergent integration  $|\widetilde{\mathcal{M}}_0|^2 |\mathcal{M}|_{SGF}^2$  is very difficult because of  $m_g \neq 0$ .

## Solution

Integration  $(|\widetilde{\mathcal{M}}_0|^2 - |\mathcal{M}_0|^2) |\mathcal{M}|_{SGF}^2$  is convergent and simpler.

- $|\widetilde{\mathcal{M}}_0|^2$ : Born term structure with **three body** kinematics,  $p_t = p_b + p_H + k$ ,
- $|\mathcal{M}_0|^2$ : Born term structure with **two body** kinematics,  $p_t = p_b + p_H$ .

$$|\mathcal{M}_{tree}|^2 = \left\{ |\mathcal{M}_{tree}^{con}|^2 + (|\widetilde{\mathcal{M}}_0|^2 - |\mathcal{M}_0|^2) |\mathcal{M}|_{SGF}^2 \right\} + |\mathcal{M}_0|^2 |\mathcal{M}|_{SGF}^2$$

# PHASE SPACE INTEGRATION WITHOUT FACTORIZATION

$$\begin{aligned}
 \Gamma &= \frac{1}{2m_t} \frac{1}{(2\pi)^5} \int dR_3(p_t; p_H, p_b, k) |M_{tree}|^2 \\
 &= \frac{1}{2m_t} \frac{1}{(2\pi)^5} \int \frac{d^3 \vec{p}_H}{2E_H} \frac{d^3 \vec{p}_b}{2E_b} \frac{d^3 \vec{k}}{2E_g} \delta^4(p_t - p_b - p_H - k) |M_{tree}|^2 \\
 &= \frac{1}{8m_t} \frac{1}{(2\pi)^3} \int_{E_H^{min}}^{E_H^{max}} dE_H \int_{k_0^{min}}^{k_0^{max}} dk_0 |M_{tree}|^2
 \end{aligned}$$

with the integration limits

$$\begin{aligned}
 k_0^{max} &= \frac{(m_t - E_H)(m_t^2 - m_b^2 + m_H^2 + m_g^2 - 2m_t E_H)}{2(m_t^2 + m_H^2 - 2m_t E_H)} \\
 &\quad + \frac{\sqrt{(E_H^2 - m_H^2)(m_t^2 + m_H^2 - 2m_t E_H - (m_b + m_g)^2)(m_t^2 + m_H^2 - 2m_t E_H - (m_b - m_g)^2)}}{2(m_t^2 + m_H^2 - 2m_t E_H)} \\
 k_0^{min} &= \frac{(m_t - E_H)(m_t^2 - m_b^2 + m_H^2 + m_g^2 - 2m_t E_H)}{2(m_t^2 + m_H^2 - 2m_t E_H)}
 \end{aligned}$$

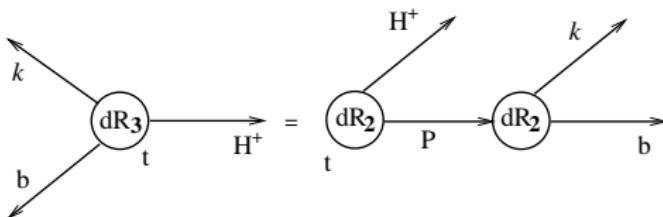
$$\begin{aligned} E_H^{max} &= \frac{m_t^2 + m_H^2 - (m_b + m_g)^2}{2m_t} \\ E_H^{min} &= m_H \end{aligned}$$

In the  $m_g \rightarrow 0$  the integration limits simplifies to

$$\begin{aligned} k_0^{max} &= \frac{(m_t - E_H + \sqrt{E_H^2 - m_H^2})(m_t^2 - m_b^2 + m_H^2 + 2m_t E_H)}{2(m_t^2 + m_H^2 - 2m_t E_H)} \\ k_0^{min} &= \frac{(m_t - E_H - \sqrt{E_H^2 - m_H^2})(m_t^2 - m_b^2 + m_H^2 + 2m_t E_H)}{2(m_t^2 + m_H^2 - 2m_t E_H)} \end{aligned}$$

$$\begin{aligned} E_H^{max} &= \frac{m_t^2 + m_H^2 - m_b^2}{2m_t} \\ E_H^{min} &= m_H \end{aligned}$$

# ADVANTAGES OF FACTORIZING THE PHASE SPACE

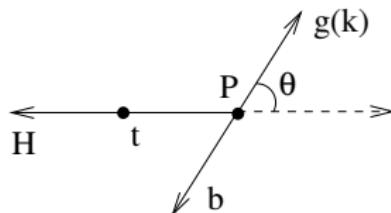


$$\begin{aligned}
 dR_3(p_t; p_H, p_b, k) &= \int \frac{d^3 \vec{p}_H}{2E_H} \frac{d^3 \vec{p}_b}{2E_b} \frac{d^3 \vec{k}}{2E_g} \delta^4(p_t - p_b - p_H - k) \\
 &= dP^2 dR_2(p_t; p_H, P) dR_2(P; p_b, k) \\
 &= m_t^2 \cancel{dz} \underbrace{dR_2(p_t; p_H, P)}_{(1)} \underbrace{dR_2(P; p_b, k)}_{(2)}
 \end{aligned}$$

$$\text{with } z = \frac{P^2}{m_t^2} = \frac{(p_b + k)^2}{m_t^2}.$$

Integration ① is trivial:  $\int dR_2(p_t; p_H, P) = \frac{\sqrt{\lambda(1, y^2, z)}}{8} 4\pi$

Integration ② is done in P-rest frame:



$$\begin{aligned} & \int dR_2(P; p_b, k) \\ &= \int \frac{d^3 p_b}{2E_b} d^4 k \delta(k^2 - m_g^2) \delta^4(P - p_b - k) \\ &= \int \frac{\pi p_g^*}{2\sqrt{z}} d\cos\theta \end{aligned}$$

$$\left| \begin{array}{c} |M_{tree}|^2 \\ (p_t \cdot P, P \cdot k, p_t \cdot k) \\ \left\{ \frac{1+z-y^2}{2}, \frac{z+\Lambda^2-\epsilon^2}{2}, \frac{(PG)(QP)}{2P^2} + \frac{(PG)\sqrt{\lambda(z, y^2, \Lambda^2)}}{2P^2} \cos\theta \right\} \end{array} \right.$$

with  $p_g^* = \frac{\sqrt{\lambda(z, y^2, \Lambda^2)}}{2\sqrt{z}}$  is the modulus of gluon momentum in P-rest frame.

THE CONLUSION IS

$$\int dR_2(P; p_b, k) |M_{tree}|^2 \rightarrow \int d\cos\theta (p_t \cdot k)^n$$

# $\int dz$ INTEGRATION

$$R(n) := \int_{\epsilon^2}^{(1-y)^2} \frac{dz}{(z - \epsilon^2)\sqrt{\lambda'^n}},$$

$$R(m, n) := \int_{\epsilon^2}^{(1-y)^2} \frac{z^m dz}{\sqrt{\lambda'^n}},$$

$$S(n) := \int_{\epsilon^2}^{(1-y)^2} \frac{1}{(z - \epsilon^2)\sqrt{\lambda'^n}} \ln \left( \frac{1 - y^2 + z + \sqrt{\lambda'}}{1 - y^2 + z - \sqrt{\lambda'}} \right) dz,$$

$$S(m, n) := \int_{\epsilon^2}^{(1-y)^2} \frac{z^m}{\sqrt{\lambda'^n}} \ln \left( \frac{1 - y^2 + z + \sqrt{\lambda'}}{1 - y^2 + z - \sqrt{\lambda'}} \right) dz.$$

with  $\lambda' := \lambda(1, z, y^2)$ .

# SOFT GLUON INTEGRATION

$$\begin{aligned}
 \Delta_{SGF} &= \int dz \int d\cos\theta |M|_{SGF}^2 \\
 &= 4(1 - \frac{p_0 Y_p}{p_3})(\ln[(1-y)^2 - \epsilon^2] - \ln\Lambda - \ln\epsilon \\
 &\quad \frac{2p_0}{p_3} \left\{ 2\text{Li}_2\left(1 - \frac{1-y}{p_+}\right) - 2\text{Li}_2\left(1 - \frac{1-y}{p_-}\right) \right. \\
 &\quad \left. - \text{Li}_2\left(1 - \frac{p_-}{p_+}\right) \right\} - 2 + Y_p(Y_p + 1)
 \end{aligned}$$

# TREE RESULTS

$$\Gamma_{tree} = -\frac{1}{4\pi m_t} \left[ \frac{\alpha_s}{4\pi} C_F m_t^2 (a^2 + b^2) \left\{ \frac{3}{4} R(0, -1) + \frac{\epsilon^2(1-y^2)}{4} R(-2, -2) \right. \right. \\ \left. \left. - \frac{1-y^2+3\epsilon^2}{4} + \frac{1}{2}\epsilon^2 S(0, 0) - \frac{1}{2}S(1, 0) \right\} + PS_2^{-1} \Gamma_{Born} \Delta_{SGF} \right],$$

$$\Gamma_{tree}^P = -\frac{1}{4\pi m_t} \left[ \frac{\alpha_s}{4\pi} C_F m_t^2 2ab \left\{ -4p_3 R(-1) + 4p_3^2 R(0) + \frac{1}{4}(1-y^2)^2 \epsilon^2 R(-2, 0) \right. \right. \\ \left. \left. - \frac{1}{4}((1-y^2)^2 + 2(3-y^2)\epsilon^2) R(-1, 0) + \frac{1}{4}(2+10y^2-\epsilon^2) R(0, 0) + \frac{7}{4}R(1, 0) \right. \right. \\ \left. \left. - +(1-y^2+\epsilon^2) 2p_3 S(0) - (1-y^2+\epsilon^2) 2p_3^2 S(1) + 2p_3 S(0, 0) \right. \right. \\ \left. \left. + \frac{1}{2}(4y^2(1-y^2) + (7+5y^2)\epsilon^2 - 2\epsilon^4) S(0, 1) - \frac{1}{2}(3-3y^2+\epsilon^2) S(1, 1) \right. \right. \\ \left. \left. - \frac{1}{2}S(2, 1) + PS_2^{-1} \Gamma_{Born}^P \Delta_{SGF} \right\} \right],$$

where  $PS_2 = \frac{p_3}{16\pi m_t}$  is the two- body phase space.

# TOTAL QCD NLO RESULT

$$\begin{aligned}
 \text{Total result} &= \text{Loop} + \text{Tree} \\
 &= \text{Loop} + (\text{Tree div.} + \text{Tree conv.}) \\
 &= \underbrace{(\text{Loop} + \text{Treediv.})}_{\text{cancellation of } \ln m_g \text{ div.}} + \text{Tree conv.}
 \end{aligned}$$

$$\begin{aligned}
 \Gamma^P = & \frac{\alpha_s C_F}{8\pi^2} m_t \left\{ -3(a^2 + b^2)p_3^2 \ln(\epsilon) \right. \\
 & + ab \left( \frac{1}{4}(-11 + 28y) - 16y^2 - 8y^3 + 7y^4 + \epsilon^2(4 + 8y - 14y^2) + 7\epsilon^4 \right. \\
 & + (2 - 9y^2 + y^4 - \epsilon^2(4 + 3y^2) + 2\epsilon^4) \frac{p_3}{y^2} Y_p + \frac{8p_3^2 w_0}{y^2} \ln(\epsilon) \\
 & + 8p_3^2 \ln\left(\frac{1-y}{(1-y)^2 - \epsilon^2}\right) + (3 - 3y^2 + 2\epsilon^2(4 + y^2) - 2\epsilon^4) \ln\left(\frac{1-y}{\epsilon}\right) \\
 & + 4p_0 p_3 (2\text{Li}_2(1 - \frac{1-y}{p_-}) - 2\text{Li}_2(1 - \frac{1-y}{p_+}) - \text{Li}_2(w_-) + \text{Li}_2(w_-) + 2 \ln\left(\frac{(1-y^2) - \epsilon^2}{\epsilon^2}\right) Y_p) \\
 & \left. \left. - (2 + y^2 - \epsilon^2(3 + 2y^2) + \epsilon^4)(2\text{Li}_2(y) - \text{Li}_2(w_-) - \text{Li}_2(w_-)) \right\} \right.
 \end{aligned}$$

- Our unpolarized results agree with A.Czarnecki and S.Davidson (1993).

# SUMMARY OF THE NLO CALCULATION TECHNIQUES

Important points:

- Three-point one-loop integration  $C_0$
- Soft gluon factor integration

Time consuming calculations:

- Phase space integration of the real emission
- Collecting virtual and real corrections and simplifying

Cancellation of divergencies:

- UV-divergencies are regulated by dimensional regularization.  
They cancel in the loop.
- IR-divergencies are regulated by a gluon mass. Both loop and tree have IR-divergencies but they cancel in the sum.
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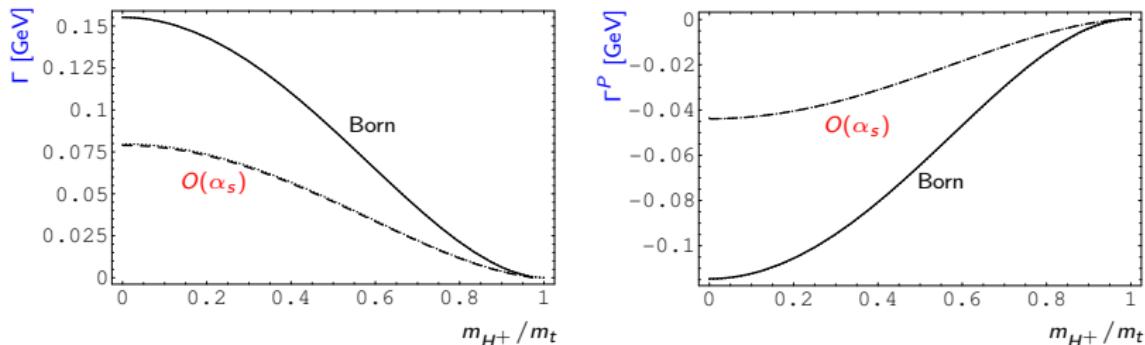
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**Figure 3**

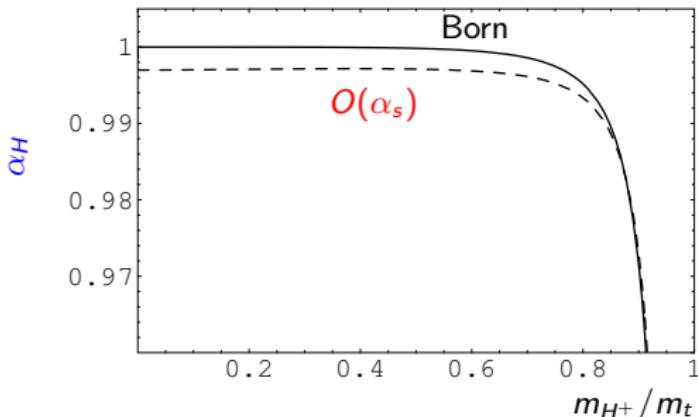
Unpolarized ( on the left) and polarized (on the right) decay rate for model 2:  
 $m_b = 4.8$  GeV,  $m_t = 175$  GeV,

Born term: full line,

$O(\alpha_s)$ : dashed line.

The barely visible dotted lines show the corresponding  $m_b \rightarrow 0$  curves.

( $\tan \beta = 10$ )

**Figure 1**

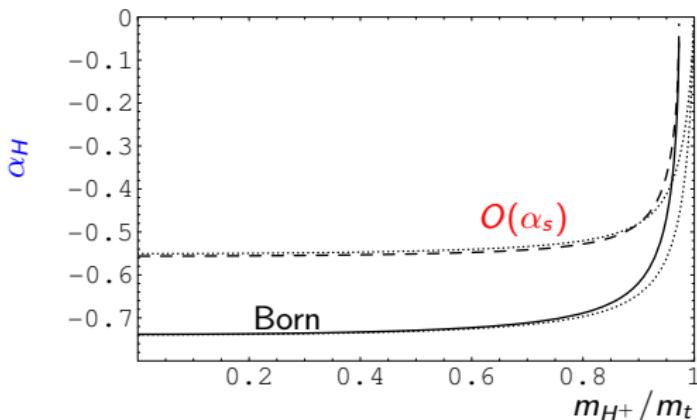
Asymmetry parameter  $\alpha_H$  for model 1:

$m_b = 4.8 \text{ GeV}$ ,  $m_t = 175 \text{ GeV}$ ,

Born term: full line,

$O(\alpha_s)$ : dashed line.

( $\tan \beta = 10$ )

**Figure 2**

Asymmetry parameter  $\alpha_H$  for model 2:

$m_b = 4.8 \text{ GeV}$ ,  $m_t = 175 \text{ GeV}$ ,

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( $\tan \beta = 10$ )

# CONCLUSIONS

- Radiative corrections to the unpolarized, polarized rates and the asymmetry parameter can become quite large (depending on the model and the value of  $\tan \beta$ ).
- We present a compact expression for the unpolarized and polarized rates which can be employed to scan the predictions of the 2HDM parameter space.

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