

# String Compactification with magnetic fluxes

- In this talk — we discuss how — by turning on gauge fluxes
- — which couple to the endpoints of open strings
- one can obtain stabilization of — closed string moduli
- This is done by analyzing — supersymmetry constraints — and — RR tadpole conditions
- — stabilization of complex and Kahler moduli — studied in a  $T^6/Z_2$  orientifold
- based on work with — I. Antoniadis and T. Maillard, hep-th/0505260.
- also, with S. Mukhopadhyay and K. Ray (in preparation)
- to elaborate —  $D3$ -branes normally used — in this compactification — replaced by  $D9$ 's — with magnetic fluxes along six compactified directions
- we show — this is a consistent compactification — in the process the moduli are fixed.

# Plan

- short introduction
- Analyze Supersymmetry of the brane – on magnetized tori – in some detail
- Present solutions of the supersymmetry conditions
- Discuss tadpole cancellation conditions
- some explicit models

# a short introduction

- Discussions on moduli stabilization in IIB string theory – one generally uses — closed string 3-form fluxes – along the six compactified directions.
- The fluxes generate a potential in four dimensions — a potential for the geocentric moduli —as well as the axion-dilaton fields — and lead to their stabilization – upon minimization
- In the implementation process – there are restrictions:
- The primitivity condition –  $J \wedge G = 0$  : ( $J$  – Kahler form,  $G$ : (imaginary self-dual (2, 1) - form flux) – can fix some of the Kahler moduli as well – but never all.
- In fact – this condition is trivial for CY's
- In the present talk – we discuss a different procedure for stabilizing the moduli
- This is achieved – by turning on fluxes of the worldvolume gauge fields on the brane
- and demanding that the magnetic field that is turned on preserves  $N = 1$  supersymmetry after compactification to four dimensions.

# magnetic field

- Another reason – D-branes with fluxes – generally introduced – is to obtain stabilized models – with chiral fermions
- The spectrum – Landau energy levels – harmonic oscillator term + a term proportional to spin – (oscillator frequency given by the magnetic field) –
- interplay between the two terms leaves one chirality of fermion massless, — the other becomes massive – but pairs up with the opposite chirality from a massive level (process repeats at all levels).
- In string theory – work of Bachas, 1995, Blumenhagen, Gorlich, Kors, Lust: 2000, Blumenhagen, Lust and Taylor: 2003, Cascalas and Uranga: 2003 etc.
- known that – by turning on constant fluxes - one generates non-commutativity – such magnetized tori – also known as –noncommutative tori
- Now – start discussion of supersymmetry property of the magnetized branes.

# Part-I: Supersymmetry

- Start with the string action. The worldsheet action, in NSR formulation is given by:

$$(1) \quad S = I_0 + I_1 + I_2$$

$$(2) \quad I_0 = -\frac{1}{4\pi\alpha'} \int d\tau \int_0^\pi d\sigma (\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^a \partial_\alpha \psi_\mu)$$

$$(3) \quad I_1 = - \int d\tau \left( q_L F_{ij} [X^i \partial_\tau X^j - \frac{i}{2} \bar{\psi}^i \rho^0 \psi_j] \right)_{\sigma=0}$$

$$(4) \quad I_2 = - \int d\tau \left( q_R F_{ij} [X^i \partial_\tau X^j - \frac{i}{2} \bar{\psi}^i \rho^0 \psi_j] \right)_{\sigma=\pi}$$

- $\rho^a$ : Dirac matrices on the worldsheet, one also sets:  $\alpha' = \frac{1}{2}$ . Also:  
 $i, j = 4, 5, \dots, 9$

# D-brane supersymmetry

- discussion on supersymmetry below following:  
Berkooz, Douglas, Leigh 1996, Balasubramanian, Leigh 1996, Witten 2000, Marino, Minasian, Moore, Strominger 1999
- Now we start to discuss the supersymmetry of D-branes. For a non-magnetized Dp-brane, the supersymmetry conditions are:  
$$\epsilon_L = \Gamma_0 \Gamma_1 \dots \Gamma_p \epsilon_R$$
where  $\epsilon_L$  and  $\epsilon_R$  are two spinors, both of +ve chirality in ten-dimensions – one coming from the left-sector of string theory and another from the right sector
- Next question — what is the supersymmetry preserved by a magnetized Dp-brane.
- First consider fluxes turned on along  $T^2$  only
- On a two dimensional torus  $F_{ij}$  has only a single nonzero component identified with  $F_{ij} = H \epsilon_{ij}$
- To understand supersymmetry — we study the boundary conditions
- For worldsheet fermions  $\psi$ , we recall, before the magnetic field – turned on  
$$\psi_L = \psi_R|_{\sigma=0}$$

# spinor transformation

- However — now one has – by defining  $b = \pi q_L H$ :

$$\psi_R = \frac{1+ib}{1-ib} \psi_L |_{\sigma=0}$$

- then by using the relation:  $b = \tan\theta$  – one obtains

$$\psi_R = e^{2i\theta} \psi_L |$$

- or in real notation the boundary condition of worldsheet fermions changes to:

$$\psi_R^4 = \cos 2\theta \psi_L^4 - \sin 2\theta \psi_L^5 |$$

$$\psi_R^5 = \cos 2\theta \psi_L^5 - \sin 2\theta \psi_L^4 |$$

- In other words, there is a rotation in the left-sector in directions  $\psi_L^4$  and  $\psi_L^5$ , with respect to the non-magnetized case.

- simply note that for the rotation of vectors by an angle  $2\theta$ , as above, in

$X^4 - X^5$  space – spinors transformation by:

$$\epsilon_L \rightarrow e^{\theta \gamma^{45}} \epsilon_L$$

# supersymmetry of the brane

- As a result, in general the D-brane supersymmetry condition now has a form  $\epsilon_L = \Gamma_0 \Gamma_1 \dots \Gamma_p \rho(F) \epsilon_R$  with  $\rho(F)$  giving the rotation of the spinors.
- For magnetic fields which can be of block diagonal form along three  $T^2$ 's of the compactified six dimensional space, we have:  $\rho \equiv e^{\theta_1 \gamma^{12} + \theta_2 \gamma^{34} + \theta_3 \gamma^{56}}$
- We will come back to a general form of  $\rho$  little later. At the moment – let us analyze some simple cases.
- The question relevant to us: When a magnetized D-brane, say  $D5$ ,  $D7$  or  $D9$  will have the same supersymmetry as that of the  $D3$  brane.
- This is because – we are studying  $T^6/Z_2$  orientifold model where we necessarily have the  $O3$  planes – having same susy property – as  $D3$ 's.
- first example (D5-D3): without magnetic field  
Susy of the  $D3$ :  $\epsilon_L = \Gamma_0 \dots \Gamma_3 \epsilon_R$   
susy of  $D5$ :  $\epsilon_L = \Gamma_0 \dots \Gamma_5 \epsilon_R$
- Both can be consistent only if  $\Gamma_4 \Gamma_5 \epsilon_R = \epsilon_R$
- This is not possible as  $\Gamma_{45}$  has only imaginary eigen values. In this case, the situation does not change much in the presence of magnetic field



## D3-D5/D3-D7

- In the case of magnetic field – condition translates into;
- $\Gamma^{45} . e^{\theta \Gamma^{45}}$  – having e.values  $\pm 1$
- for this to happen,  $\theta = \pm \frac{\pi}{2}$
- However from the relation  $b = \tan \theta$  – we learn that —this corresponds to infinite magnetic field.
- In other words the range of  $\theta$  is restricted from  $-\pi/2 < \theta < \pi/2$
- On the other hand — it is possible for a magnetized  $D7$  to preserve the same supersymmetry — as that of an ordinary  $D3$
- for this to happen – we obtain a condition – exactly in the same way :  
 $\pm \theta_1 \pm \theta_2 = 0$
- where  $\theta_1$  and  $\theta_2$  – the spinor rotations – associated with magnetic fields — in directions,  $x^4, x^5$  and  $x^6, x^7$ , the directions that are — transverse to the  $D3$ , but are —the longitudinal directions of  $D7$ .
- Note also the relation:  $b_i = \tan \theta_i$  between the magnetic field and the spinor rotation angle  $\theta$

# self-duality

- This implies – for a  $D7$  compactification on  $T^4$  – the magnetic fields have to satisfy a relation
- $b_1 = \pm b_2$ ,
- or written in a covariant notation:
- $F_{ij} = \pm \epsilon_{ijkl} F_{kl}$
- In other words, the magnetic fields are either self-dual or anti-self dual – instanton configurations in 4d (Euclidean gauge theory).
- This equation – written in a complex coordinate:  $z^i = x^i + iy^i$ ,  $i = 1, 2$
- a notation we will use below, by identifying directions  $x^4, x^6, x^8$  with  $x^i$ 's and directions  $x^5, x^7, x^9$  with  $y^i$ 's (for  $i = 1, 2, 3$ ).
- In this complex notation: the self-duality condition becomes
- $F_{(2,0)} = 0$
- for  $T^6$  – we have a similar equation – which we will use for the moduli stabilization.

# $D9$ on $T^6$

- This is the case which will be of most interest to us, — as mentioned
- Let us review the situation again, starting with the non-magnetized case. — We have the  $D3$  brane supersymmetry:  
$$\epsilon_L = \Gamma_0 \Gamma_1 \dots \Gamma_3 \epsilon_R$$
- and the  $D9$ -brane supersymmetry:  $\epsilon_L = \Gamma_0 \Gamma_1 \dots \Gamma_9 \epsilon_R$
- For both of these to be consistent, one will have to have:  $\Gamma_4 \dots \Gamma_9 \epsilon_R = \epsilon_R$  which is not possible, as  $\Gamma_4 \dots \Gamma_9$  has only imaginary eigen value.
- So, no  $D9$  can be put together with  $D3$  to produce a supersymmetric system.

## $D9$ on $T^6$ with magnetic field

- The situation changes when magnetic fields are turned on along the compactified directions of  $D9$
- As a result, the supersymmetry condition now becomes  
$$\Gamma_4 \dots \Gamma_9 e^{\theta_1 \Gamma^{45} + \theta_2 \Gamma^{67} + \theta_3 \Gamma^{89}} \epsilon_R = \epsilon_R$$
- and leads to the condition:  $\pm \theta_1 \pm \theta_2 \pm \theta_3 = \frac{\pi}{2}$  where — of course — we have turned on magnetic field components only along three factorized  $T^2$ 's.

# spinor rotation matrix

- We now obtain supersymmetry condition – for a general (constant) magnetic flux on  $T^6$ . – For this we write down the spinor rotation matrix for a general background metric and gauge flux
- First – restricting to the – internal six dimensional space – with a metric  $g_{ij} = \delta_{ij}$  – we can write –  

$$\rho(F) = \frac{1}{\sqrt{\det(1+F)}} \text{EXP.} \left[ -\frac{1}{2} F_{ij} \Gamma^{ij} \right]$$
- where notation: ‘Exp.’ – stands for an exponential expansion – with complete antisymmetrization – in indices of  $F_{ij}$ . As a result, the expansion is always finite.
- Now we discuss the general situation with the  $D9$  branes. The condition we analyze is:

$$(5) \quad \Gamma^{4\dots 9} \frac{1}{\sqrt{\det(1+F)}} \text{EXP} \left( -\frac{1}{2} F_{ij} \Gamma^{ij} \right) \epsilon_R = \epsilon_R$$

- also for general  $G$  – we make a change –

$$\frac{1}{\sqrt{\det(1+F)}} \rightarrow \frac{\sqrt{\det G}}{\sqrt{\det(G+F)}}$$

# covariant form

- The supersymmetry is written as:

$$(6) \quad \frac{1}{\sqrt{\det(1+F)}} \Gamma^{4\dots 9} \left( 1 - F_{ij} \Gamma^{ij} + \frac{1}{2} F_{[ij} F_{kl]} \Gamma^{ijkl} - \frac{1}{3!} F_{[ij} F_{kl} F_{mn]} \Gamma^{ijklmn} \right) \epsilon_R = \epsilon_R$$

- Moreover, this eqn. can be written in a covariant form (not keeping track of factors) as:

$$(7) \quad \frac{\sqrt{\det G}}{\sqrt{\det(1+F)}} \left( \epsilon^{ijklmn} \Gamma^{ijklmn} + \epsilon^{ijklmn} F_{ij} \Gamma^{klmn} + \frac{1}{2} \epsilon^{ijklmn} F_{[ij} F_{kl]} \Gamma^{mn} + \frac{1}{3!} \epsilon^{ijklmn} F_{[ij} F_{kl} F_{mn]} \Gamma^{ijklmn} \right) \epsilon_R = \epsilon_R$$

- Then using property of spinors:  $\Gamma_{m\bar{n}} \epsilon = i J_{m\bar{n}} \epsilon$  etc., (with  $J$ : Kahler form) where  $\epsilon$  is the covariantly constant spinor on the particular space one is talking about. In our case they are  $\epsilon_L$  and  $\epsilon_R$ , or more precisely their decompositions in terms of a 4d and a 6d spinor.

# constraint on Kahler moduli

● One obtains:

$$(8) \quad \frac{\sqrt{\det G}}{\sqrt{\det(G+F)}} [-iJ \wedge J \wedge J - J \wedge J \wedge F + iJ \wedge F \wedge F + F \wedge F \wedge F] = V_6$$

where  $V_6$  is the six dimensional volume element. It comes in the process of changing the equation in components to that in terms of Kahler form.

● in writing this form – a number of terms are dropped – only  $F_{i\bar{j}}$  (in complex notation) – are kept. This amounts to using – (a condition mentioned earlier):

$$(9) \quad F_{(2,0)} = 0$$

● In a compact form one finally writes:

$$(10) \quad (iJ + F)^3 = e^{i\theta} \frac{\sqrt{\det(G+F)}}{\sqrt{\det G}} V_6$$

# supersymmetry: final form

- for us:  $\theta = 0$ , corresponding to  $D3$  brane supersymmetry. on the other hand:  $\theta = \frac{\pi}{2}$  if one wants a  $D9$  brane supersymmetry.
- These two cases will correspond to IIB on  $T^6/\Omega(-)^{FL}R$ , with  $R : (X^5, \dots, X^9) \rightarrow -(X^5, \dots, X^9)$  or IIB/ $\Omega$  on  $T^6$
- one can similarly analyze other situations. Condition we have derived – can also be written as:

$$(11) \quad e^{-i\theta} (iJ + F)^3 = \frac{\sqrt{(G + F)}}{\sqrt{\det G}} V_6$$

- which further implies, (since RHS is a real quantity):

$$\text{Im}[e^{-i\theta} (iJ + F)^3] = 0$$

- In our case, as  $\theta = 0$ , we therefore have:

$$J \wedge J \wedge J - J \wedge F \wedge F = 0$$

- the real part of the above condition:

$$(12) \quad \text{Re}[e^{-i\theta} (iJ + F)^3] = \frac{\sqrt{(G + F)}}{\sqrt{\det G}} V_6$$

# positivity

- which also implies:

$$(13) \quad \int \operatorname{Re}[e^{-i\theta}(iJ + F)^3] = \int \sqrt{(G + F)}$$

can also be verified. We have already done this for the  $2 \times 2$  case.

- Now since BI action:

$$(14) \quad \begin{aligned} V_{DBI} &= \frac{\mu_9}{g_s} \int_{M_{10}} \sqrt{(G + F)} = \frac{\mu_9}{g_s} \int_{T^6} \sqrt{(G + F)} \int_{M^4} \sqrt{g} \\ &= \frac{\mu_9}{g_s} \int_{T^6} \operatorname{Re}[e^{-i\theta}(iJ + F)^3] \int_{M^4} \sqrt{g} \end{aligned}$$

- This implies a condition:

$$(15) \quad \operatorname{Re}[e^{-i\theta}(iJ + F)^3] > 0$$



# positivity

- condition can be seen from the requirement of the right sign for KE of the 4d gauge field. For  $\theta = 0$  we then obtain:

$$(16) \quad F \wedge F \wedge F - J \wedge J \wedge F > 0$$

## summary: part-I

- To summarize: the key supersymmetry conditions for us are:

$$(17) \quad F_{(2,0)} = 0$$

$$(18) \quad J \wedge J \wedge J - J \wedge F \wedge F = 0$$

$$(19) \quad F \wedge F \wedge F - J \wedge J \wedge F > 0$$

- Above discussion: following Marino, Minasian, Moore, Strominger:  
hep-th/0011206

# Part-II

- 1. Fixing Complex Structure Moduli
- Ref: hep-th/0505260;  
earlier work: Antoniadis and Maillard: hep-th/0412008 (for type I on  $T^6$ ).
- We now show how the condition:  $F_{(2,0)}^a = 0$ , for a set of brane-stacks – denoted by index  $a$  – fixes the complex structure moduli
- For this, – first we introduce a set of brane-stacks which have fluxes of various types – meaning – having different components of  $F$  turned on, – with different magnitudes.
- The brane-stacks are being labeled by an index ‘ $a$ ’
- The complex structure matrix (in this case a  $3 \times 3$  matrix), appears in this condition through the definition  $z^i = x^i + \tau y^i$ ,

# torus : $T^6$

- the six dimensional torus of for us will be defined by periodic coordinates  $x^i$ ,  $y_i$  ( $i = 1, 2, 3$ ):  $x^i = x^i + 1$ ,  $y_i = y_i + 1$ .
- the orientation defined by:  
$$\int dx^1 \wedge dy_1 \wedge dx^2 \wedge dy_2 \wedge dx^3 \wedge dy_3 = 1$$
- the complex structure can be defined by choosing complex coordinate:  
 $z^i = x^i + \tau^{ij} y_j$ , with  $\tau^{ij}$  being  $3 \times 3$  complex matrix — implying 9 complex components.

## torus – contd.

- one can also define a basis for cohomology  $H^3(T^6, Z)$ , with a symplectic structure:  $\alpha_0, \alpha_{ij}, \beta^0, \beta^{ij}: (i, j = 1, 2, 3)$  with:  $\alpha_0 = dx^1 \wedge dx^2 \wedge dx^3$  etc.  
 $\int_{T^6} \alpha_A \wedge \beta^B = -\delta_A^B.$
- Furthermore  $H^3(T^6, Z)$  can be decomposed into (3, 0), (2, 1), (1, 2) and (0, 3) forms. In this decomposition,  $\omega = dz^1 \wedge z^2 \wedge dz^3$  is the unique (3, 0) form.
- The complex structure  $\tau$  is then also identified as a period of  $\Omega$  and can be identified from a relation:  
$$\Omega = \tau^a \alpha_a - \mathcal{G}_b \beta^b$$
- in our case, we again get their identification with  $\tau^{ij}$ 's.
- complex structure and Kahler moduli – also parameterized by the set of (2, 1)  $(\delta g_{ij})$  and (1, 1)  $(\delta g_{i\bar{j}})$  deformations of the metric. In particular, Kahler forms  $J$  are given as:  
$$J = i\delta g_{i\bar{j}} dz^i \wedge \bar{z}^j$$

# fixing $\tau^{ij}$ 's

- $F_{(2,0)} = 0$  are now used to fix  $\tau^{ij}$ 's.
- In addition we also use the fact that flux components  $F_{x^i x^j}^a, F_{y^i y^j}^a, F_{x^i y^j}^a$  are rationally quantized:
  - $q_a F_{ij}^a \equiv 2\pi p_{ij}^a = 2\pi \frac{m_{ij}^a}{n_{ij}^a}$
  - precise form of  $n_{ij}^a$  – will be clear later on – using the mapping between the worldvol. to space-time.
  - Using the definition of  $p^a$ 's given here, the complex structure matrix,  $\tau$  satisfies the equation:
    - (20) 
$$F_{(2,0)}^a = 0 \rightarrow \tau^T p_{xx}^a \tau - \tau^T p_{xy}^a - p_{yx}^a \tau + p_{yy}^a = 0,$$
  - Then by specifying  $p_{xx}^a, p_{xy}^a$  etc., for set of branes, one aims to fix  $\tau$ 's.

# fixing $\tau$ 's

- one can show that the off-diagonal components of  $\tau$  can be forced to be zero, by taking appropriate fluxes  $p_{x^i y^j}$ ,  $p_{x^i x^j}$  etc. along various brane-stacks:

$$(21) \quad \tau^{12} = \tau^{13} = \tau^{21} = \tau^{23} = \tau^{31} = \tau^{32} = 0.$$

- For diagonal components of  $\tau$  one can obtain:

$$(22) \quad \frac{\tau^{11}}{\tau^{22}} = \frac{p_{x^2 y^1}^1}{p_{x^1 y^2}^1} \equiv K_1, \quad \frac{\tau^{22}}{\tau^{33}} = \frac{p_{x^3 y^2}^2}{p_{x^2 y^3}^2} \equiv K_2, \quad \frac{\tau^{33}}{\tau^{11}} = \frac{p_{x^1 y^3}^3}{p_{x^3 y^1}^3} \equiv K_3,$$

and

$$(23) \quad \tau^{11} \tau^{22} = -\frac{p_{y^1 y^2}^4}{p_{x^1 x^2}^4} \equiv -K_4, \quad \tau^{22} \tau^{33} = -\frac{p_{y^2 y^3}^5}{p_{x^2 x^3}^5} \equiv -K_5, \quad \tau^{33} \tau^{11} = -\frac{p_{y^3 y^1}^6}{p_{x^3 x^1}^6} \equiv -K_6$$

- with solution given as:

$$(24) \quad \tau^{11} = i\sqrt{K_1 K_4}, \quad \tau^{22} = i\sqrt{\frac{K_4}{K_1}}, \quad \tau^{33} = i\sqrt{K_1 K_4 K_3}.$$

## contd.

- We therefore see that by specifying fluxes,  $p^a$ 's along a set of brane - stacks, one can fix the complex structure moduli.
- To stabilize the Kahler moduli one makes use of the conditions:

$$(25) \quad J \wedge J \wedge J - J \wedge F^a \wedge F^a = 0$$

with the constraint:

$$(26) \quad F^a \wedge F^a \wedge F^a - J \wedge J \wedge F^a > 0$$

where  $a$  is denoting the brane-stack.

- Without going into detail, we mention that by specifying fluxes, as mentioned above for the complex-structure stabilization
- One can obtain solution for  $J$ 's: the off-diagonal Kahler moduli are zero:  $J_{i\bar{j}} = 0$  and
- Diagonal components of  $J_{i\bar{j}}$  are stabilized to the string scale
- This is possible for many combination of fluxes and branes.
- However, one needs to satisfy an additional constraint for building any model.

# Part-III: RR Tadpoles

- Before giving explicit model we discuss the RR-tadpole cancellations
- Constant fluxes generate RR charges, corresponding to lower dimensional branes. Cancellation of all these charges - implies the worldvolume theory free of anomaly.
- The amount of charge that is generated can be seen by looking at the WZ couplings of the brane. The total action is given by

$$(27) \quad I = V_{DBI} + V_{WZ}$$

- We have already looked at  $V_{DBI}$ .  $V_{WZ}$  has a general form (using that RR forms are even or odd under the orientifolding  $\Omega(-)^{FL}$ ):

$$(28) \quad V_{WZ} = \mu_9 \sum_a N_a \int_{M^{10}} (C_4 \wedge F^a \wedge F^a \wedge F^a + C_8 \wedge F^a)$$

$N_a$ 's are the number of branes in a stack with fluxes  $F_{ij}^a$ .

- implies new contributions to the 3-brane and 7-brane tadpoles.



# tadpole

- Using – Jacobi matrix – giving map from the worldvolume – to space-time (Bianchi and Trevigne, 2005):  $W_A^i = \frac{\partial X^i}{\partial \sigma_A}$

- The tadpole cancellation conditions are read from:

$$(29) \quad \sum_a N_a W_a F^a \wedge F^a \wedge F^a + N = 16$$

and

$$(30) \quad \sum_a N_a W_a F^a = 0$$

where  $N$  is the number of ordinary  $D3$ -branes which one can also put to cancel the 3-brane tadpole and  $W_a = \det W$ ,  $N_a$ : no. branes in  $a$ 'th stack.

# tadpole-contd.

- We choose the matrix  $W_A^i$  to be diagonal:

$$(31) \quad W = \text{diag.}(n_1, n_2, n_3; n_1, n_2, n_3)$$

where the first three directions correspond to the x directions and the last three to the y directions.

toy model

- We work with stacks, 1, 1'. 2, 2'.
- 1', 2' defined as: off-diagonal component of fluxes are of opposite sign w.r.t. the branes: 1, 2.
- As a result there are no 7-brane off-diagonal tadpoles left. the diagonal one: computed below
- We make the following choices for brane-1 for  $n_i$ 's:

$$(32) \quad n_1^{(1)} = 2, \quad n_2^{(1)} = 1, \quad n_3^{(1)} = 3, \quad m_{23}^{(1)} = 1, \quad m_{x^3 y^3}^{(1)} = 1$$

- for brane 1', the sign of the off-diagonal  $m_{ij}$  is opposite. For brane-2 we have:

$$(33) \quad n_1^{(2)} = 1, \quad n_2^{(2)} = 2, \quad n_3^{(2)} = 1, \quad m_{13}^{(2)} = 1, \quad m_{12}^{(2)} = 1, \quad m_{x^3 y^3}^{(2)} = -1$$

# tadpole-contd.

- they correspond to flux:

$$(34) \quad p_{23}^{(1)} = \frac{1}{3}, \quad p_{12}^{(2)} = \frac{1}{2}, \quad p_{13}^{(2)} = 1$$

- 7-brane tadpole contributions are:

$$(35) \quad (q^7)_{x^3 y^3}^{(1)} + (q^7)_{x^3 y^3}^{(1')} = 2(n_1^{(1)} n_2^{(1)})^2 m_{x^3 y^3}^{(1)}$$

$$(36) \quad (q^7)_{x^3 y^3}^{(2)} + (q^7)_{x^3 y^3}^{(2')} = 2(n_1^{(2)} n_2^{(2)})^2 m_{x^3 y^3}^{(2)}$$

- We now write down the 3-brane tadpole contribution. First, it is zero for the stacks 1 and 1'. For the stack-2 on the other hand, using the diagonal form of  $W$  we get:

# tadpole-contd.



$$(37) \quad \begin{aligned} (q^3)^{(3)} &= -W_3 p_3^{(3)} (p_{12}^{(3)})^2 \\ &= -(m_{12}^{(3)})^2 m_{x^3 y^3}^{(3)} \end{aligned}$$



where we have used relations such as:

$$(38) \quad p_{12} = \frac{m_{12}}{n_1 n_2}$$



Then using the above results in the 7-brane tadpoles above, we obtain:

$$(39) \quad (q^7)_{x^3 y^3}^{tot} = 8(m_{x^3 y^3}^{(2)} + m_{x^3 y^3}^{(3)}) = 0$$



Also: for the total 3-brane tadpole:

$$(40) \quad (q^3)_{x^3 y^3}^{tot} = -2 \cdot (-1) (m_{12}^{(3)})^2 = 2$$



we have therefore shown that using branes 1, 1', 2, 2' one can obtain a consistent model with all the 7-brane tadpoles cancelled and 3-brane tadpole = 2.

# comments

- However, this model corresponds to the case – when one of the diagonal  $J$ :  $J_{x^2 y^2}$  is negative.
- We then – turn on antisymmetric tensor  $B$ : — their values are constrained to be 0 or  $\frac{1}{2}$
- We choose:  $B_{x^1 x^2} = B_{y^1 y^2} = \frac{1}{2}$  — and show;  $G + B$  is positive definite — implying a well-defined worldsheet theory
- 3-brane tadpoles are to be saturated to 4 rather than 16 (due to the presence of – exotic orientifold planes)
- The model satisfies this condition —
- Several explicit models along this line – constructed with partial moduli stabilizations

## contd.

- It is difficult to obtain stabilization in  $T^6/Z_2$  orientifold model without  $B$  – as generated 7-brane tadpoles are to cancel among themselves.– consistent with supersymmetry and other requirements — however, no no-go theorem yet.
- In another work in progress, with S. Mukhopadhyay and K. Ray (in progress) – we used non-abelian gauge fluxes
- it seems possible to solve supersymmetry and tadpole cancellation conditions to get all (diagonal)  $J$ 's positive.

# other issues

- **RR Moduli:** There are 15 RR moduli, 9 of them complexify the Kahler forms to form the chiral multiplet in four dimensions. they are all absorbed by 9 gauge bosons on the branes, as one fixes the Kahler moduli. The remaining 6 RR moduli, together with 6 complex components of the metric, form the complex structure  $\tau$  which has nine components – and get fixed by the mechanism mentioned.
- **large radius:** The solution obtained above: all the radii are fixed at the string scale, — as identified in this solution – due to the factorized form – of the solution in products of three  $T^2$ 's:  $\tau_{ii}$  are given by the ratio of the radii and  $J_i$  by the product.
- however – possible to obtain solution with large radii, by scaling the windings  $n_i$  while maintaining the tadpole conditions.

## axion-dilaton moduli

- Unlike the stabilization using 3-form fluxes, the axion-dilaton modulus does not appear – when using magnetic fluxes on D-branes
- not surprising – since the string construction and action we are using, does not allow  $\phi$  to vary.

# final

- One way to stabilize them – combine this mechanism with the one using close string fluxes
- However, possible(?) to generalize the Dp-Dp' system to the one involving the  $(p, q)$  - branes which carry  $SL(2, Z)$  S-duality charge – and get constraints on them??