Two Gas-Puff Staged Pinch Dynamics with Finite-β Effects

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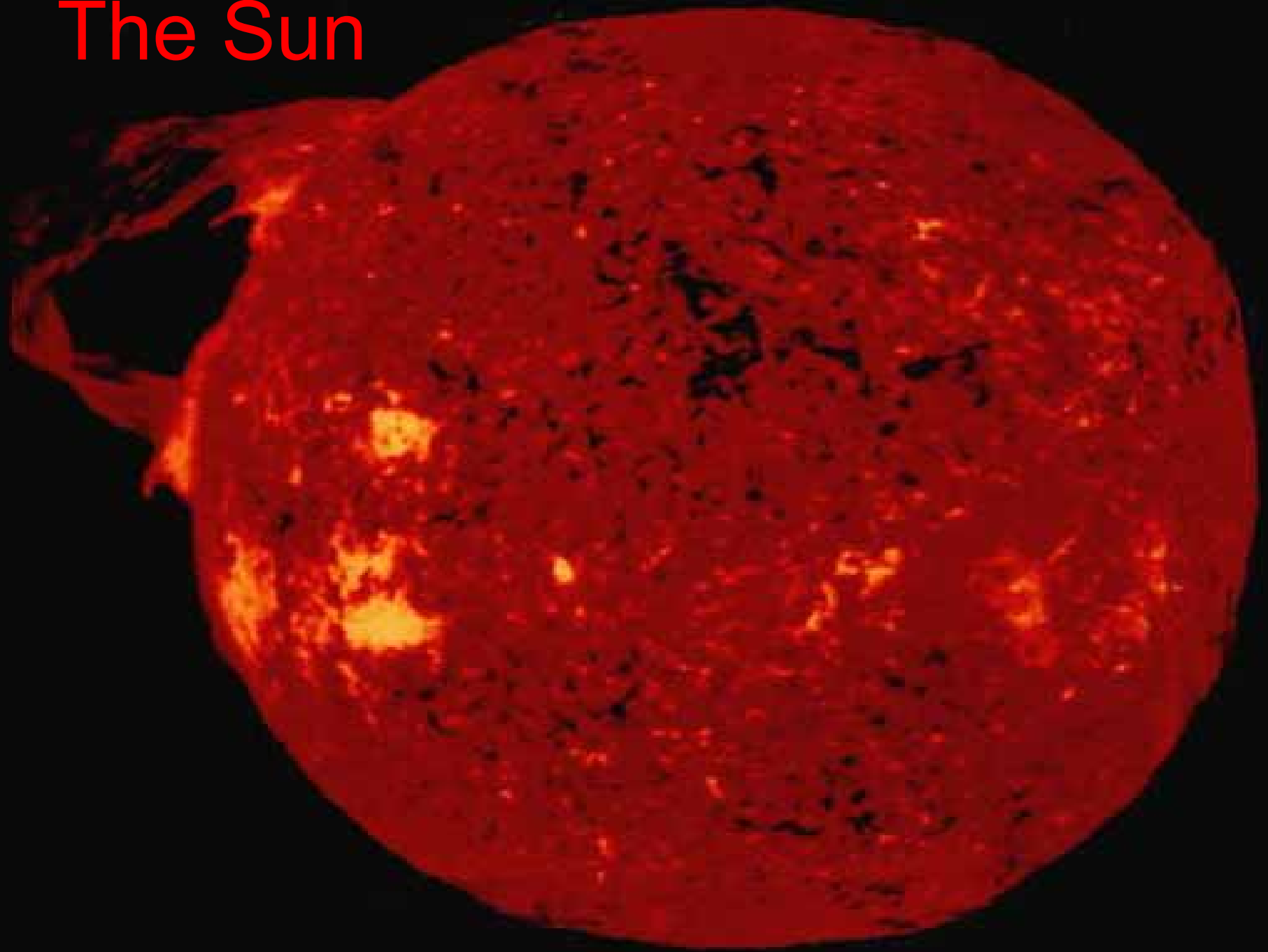Quaid-i-Azam University,
Islamabad

Introduction

"Every time you look up at the sky, every one of those points of light is a reminder that *fusion power is extractable from hydrogen* and other light elements"
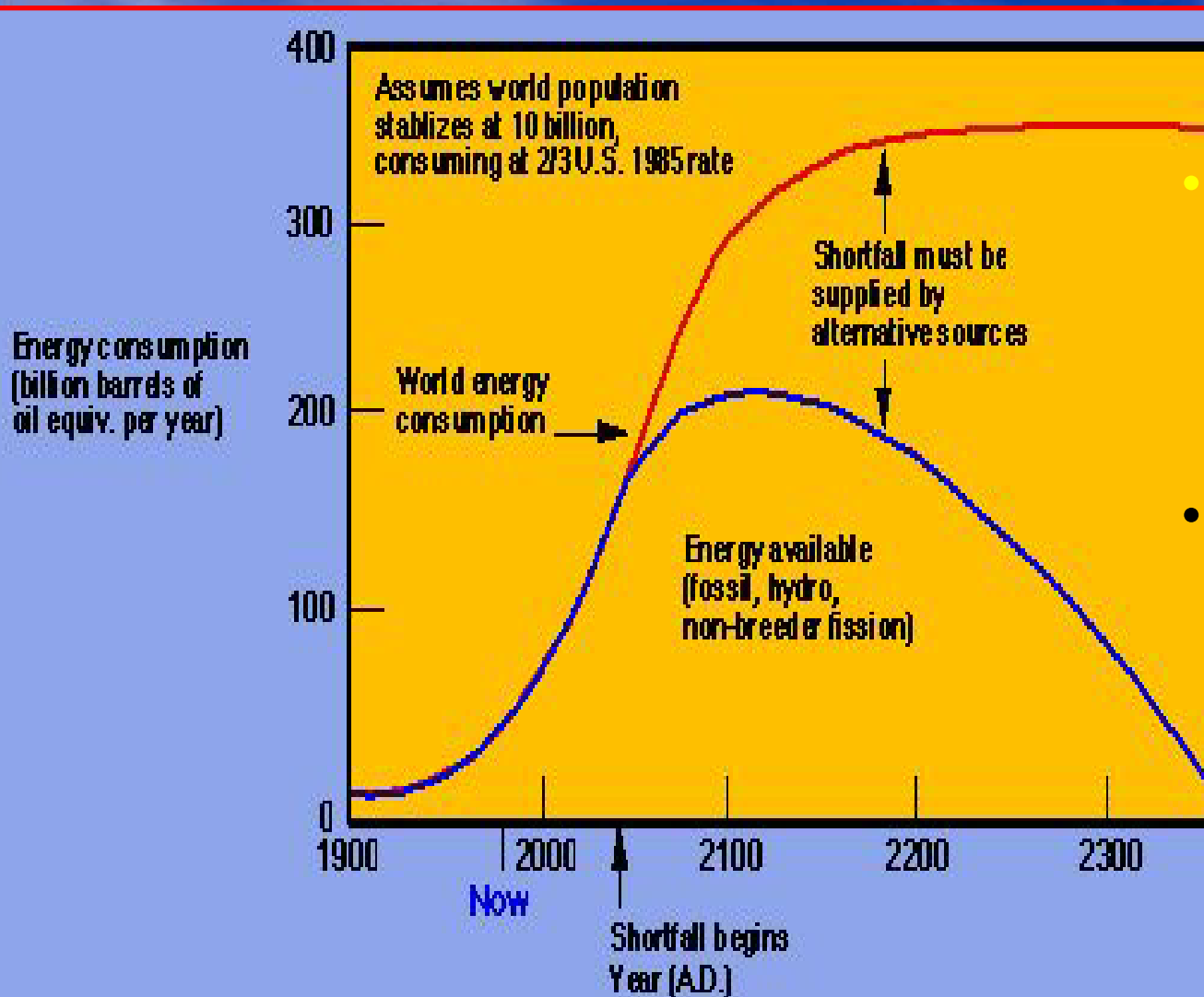
-Carl Sagan, 1991

The Sun

No More Fossil Fuel?
Need For New Energy Sources

Energy consumption (billion barrels of oil equiv. per year)

400

Assumes world population stabilizes at 10 billion, consuming at 2/3 U.S. 1985 rate

300

Shortfall must be supplied by alternative sources

World energy consumption

200

Energy available (fossil, hydro, non-breeder fission)

100

0

1900   2000   2100   2200   2300

Now

Shortfall begins
Year (A.D.)

- *If we continue to burn fossil fuels for energy, they will only last another few hundred years.*
- This means that an energy shortfall could occur within the **next fifty years**.

Nuclear Power

- Clean
- No CO_2
- No immediate pollution problems with waste disposal
- Safety concerns

Fusion Advantages

- Abundant fuel, available to all nations
  - Deuterium and lithium easily available for thousands of years
- Environmental Advantages
  - No carbon emissions, short-lived radioactivity
- Modest land usage
  - Compact relative to solar, wind and biomass
- Can't blow up
- Can produce electricity and hydrogen

Fusion Disadvantages

- *Huge research and development costs*
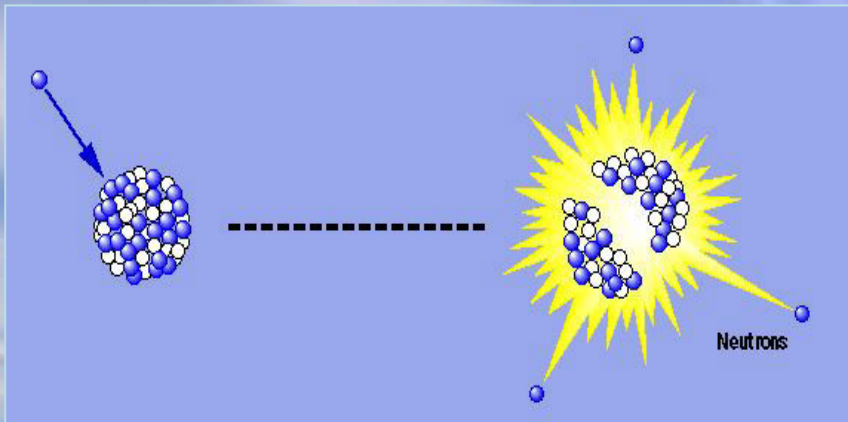- Radioactivity
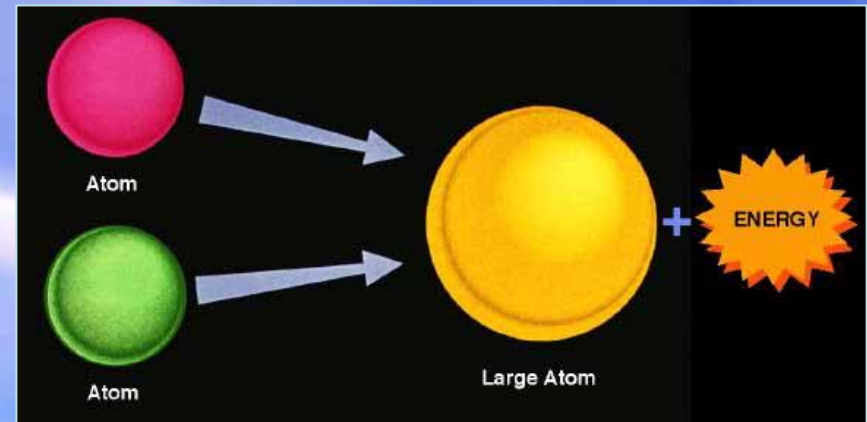
Background

Fusion Basics

Nuclear Power

- Nuclear fission
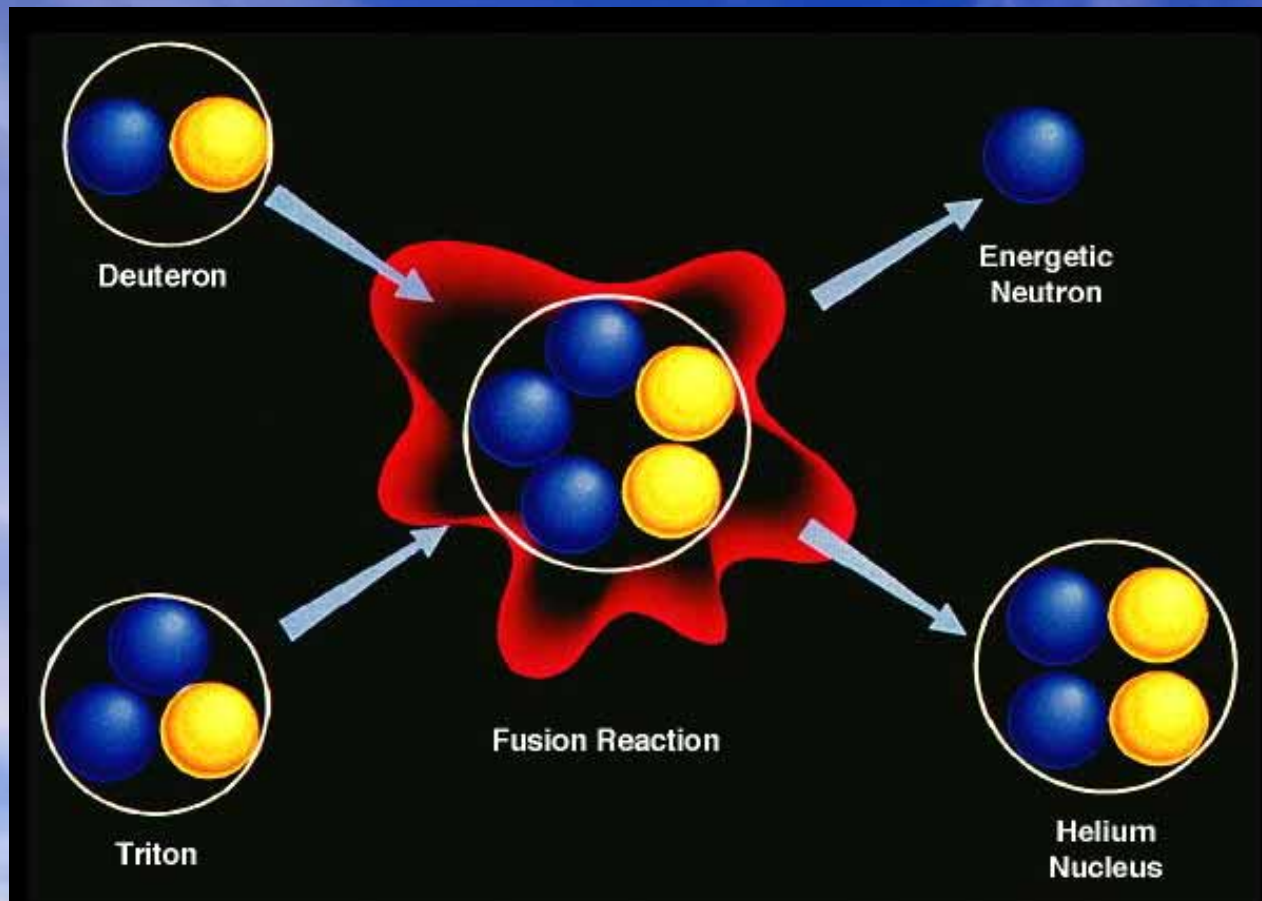   - Where heavy atoms, such as uranium, are split apart releasing energy that holds the atom together
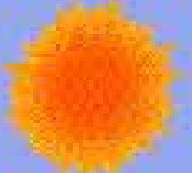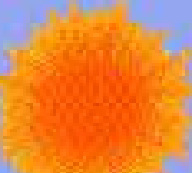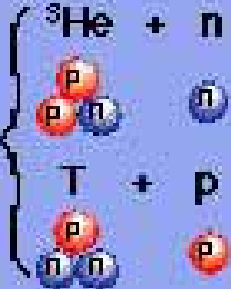
- Nuclear fusion
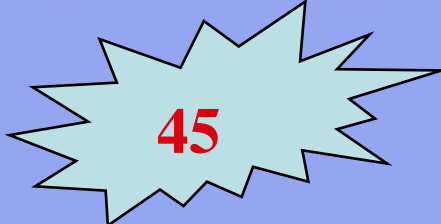   - Where light atoms, such as hydrogen, are joined together to release energy

Neutrons

Atom

Atom

Large Atom

ENERGY

Fusion Reaction with D-T

$$_1D^2 + {}_1T^3 => {}_2He^4 + {}_0n^1$$

Ideal Ignition Temperatures

Reaction		Ignition Temperature		Output Energy
Fuel	Product	(millions of °C)	(keV)	(keV)
D + T	^{4}He + n	45	4	17,600
D + ^{3}He	^{4}He + p	350	30	18,300
D + D	^{3}He + n	400	35	~4,000
	T + p	400	35	~4,000

Plasma Confinement & Heating
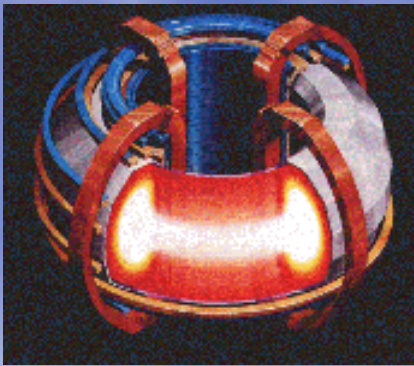
Tokamak Schematic
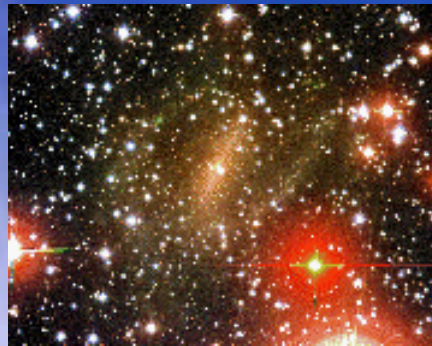
Stars & Galaxies
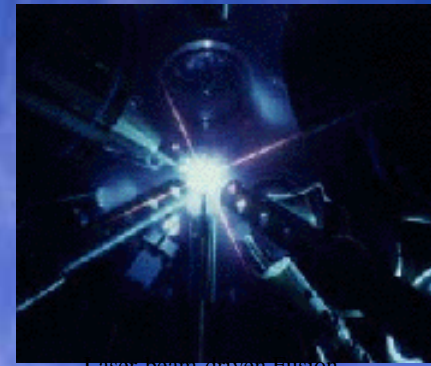
Laser-beam-driven Fusion

Magnetic

- Electromagnetic Waves
- Ohmic Heating (by electric currents)
- Neutral Particle Beams (atomic hydrogen)
- Compression (by magnetic fields)
- Fusion Reactions (primarily D+T)

Gravity

- Compression (gravity)
- Fusion Reactions (such as the p-p chain)

Inertial

- Compression (implosion driven by laser or ion beams, or by X-rays from laser or ion beams)
- Fusion Reactions (primarily D+T)

Inertial Confinement Fusion
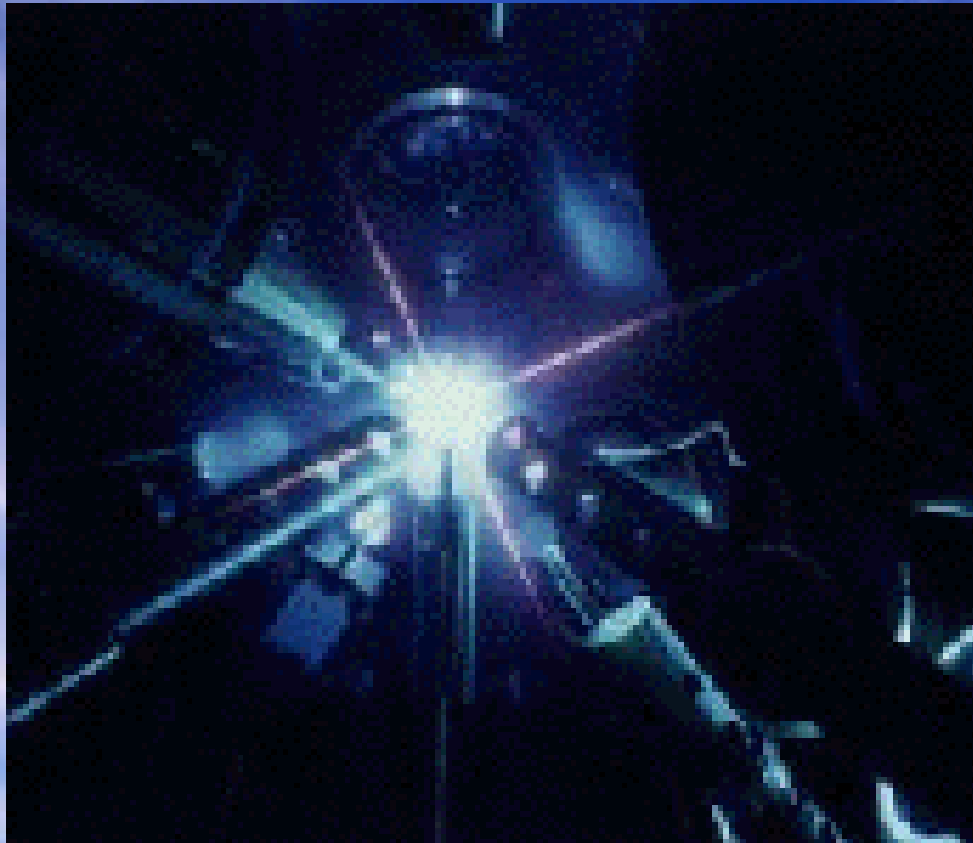
Fusion By Magnetic Confinement

Toroidal Magnetic Confinement of Plasma

Z-Pinch Device
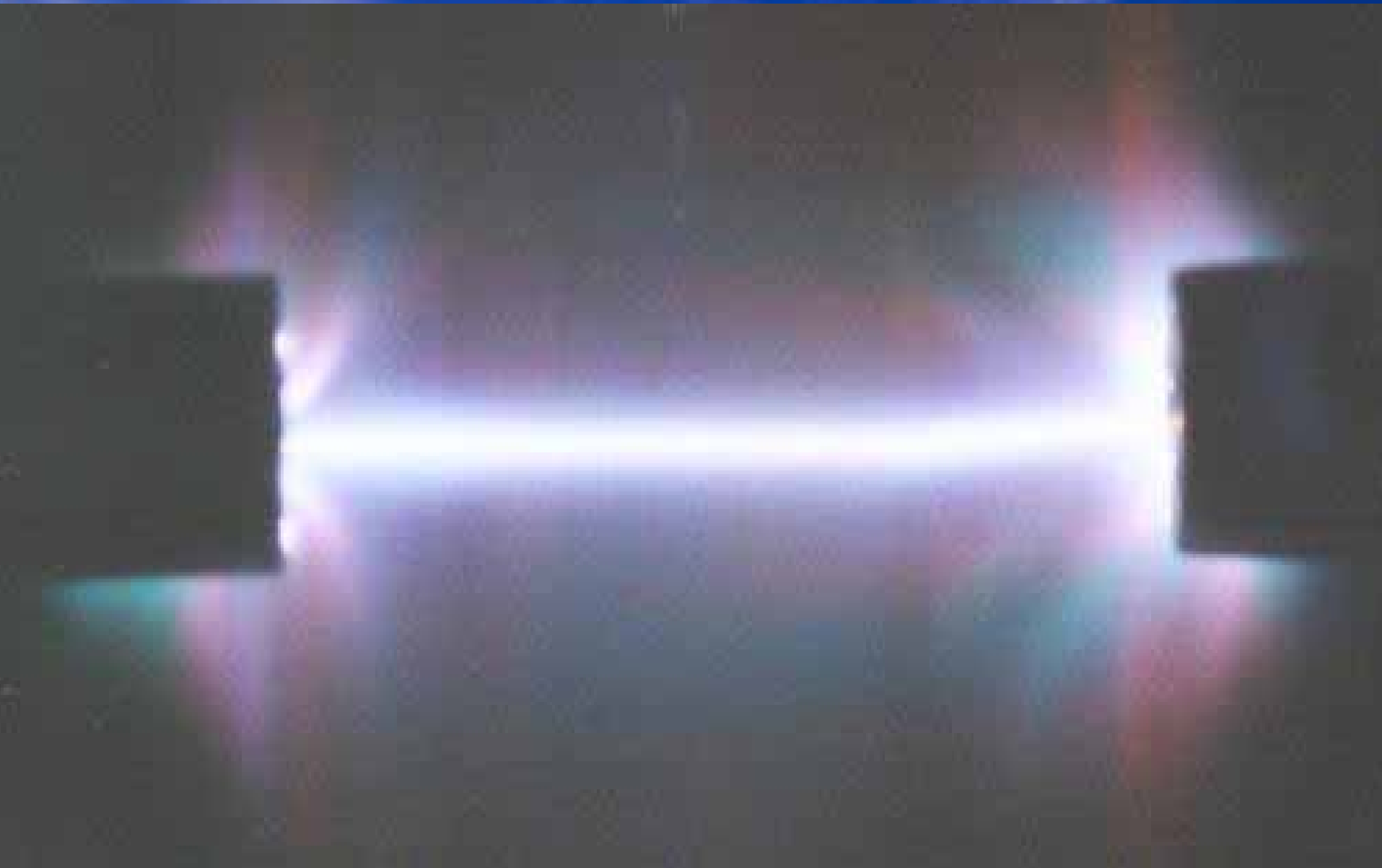
Introduction

What is Z-pinch ?

A plasma column in which current **J** is driven in the axial (z) direction by an electric power source producing an azimuthal (θ) direction magnetic field **B** that tends to confine the plasma by a force (**J** x **B**).

Glass tube

r_0

I_z

Current sheath

r

I_z J_z

$\bar{B}_\theta$

Undisturbed plasma

Length
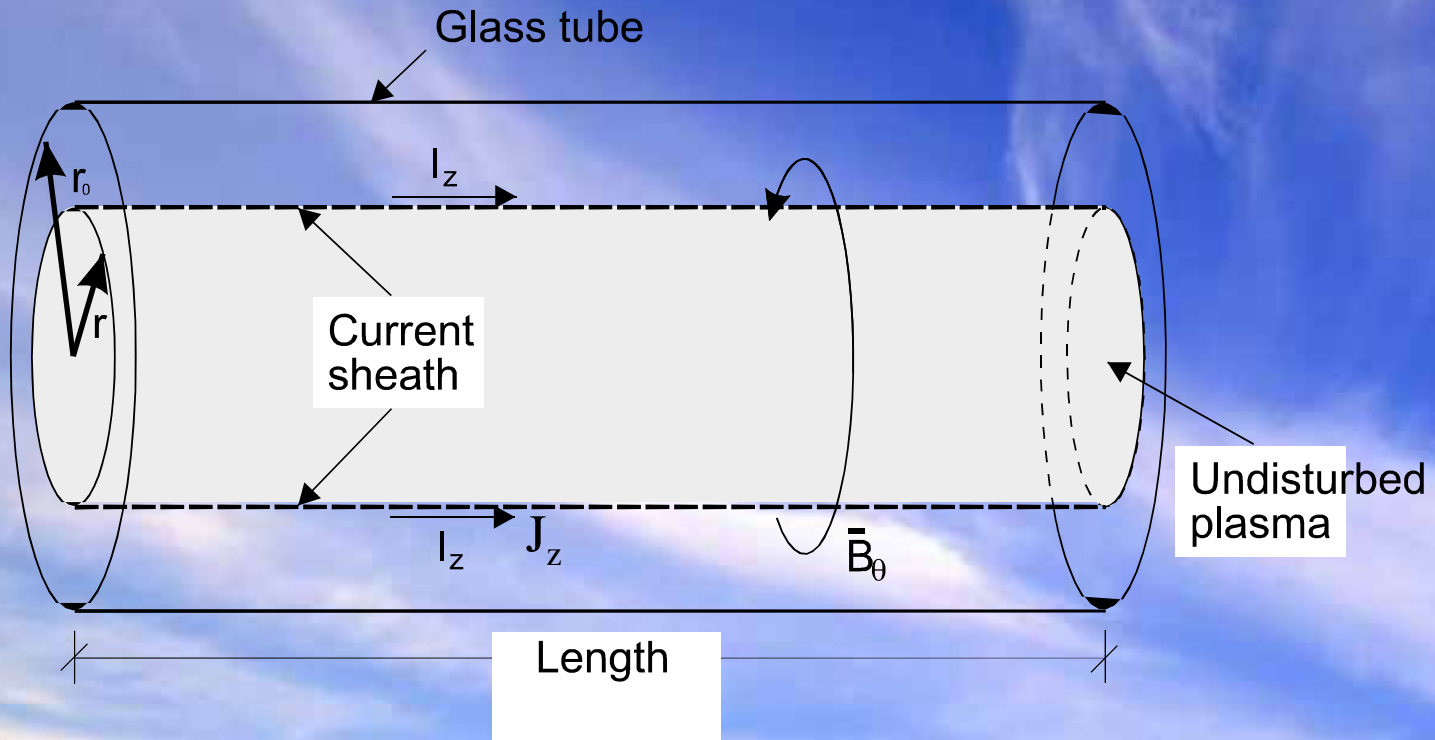
θ -pinch

In θ-pinch device the current flows in the azimuthal direction of cylindrical plasma column. The interaction of the current and axial magnetic field is through ($\mathbf{J}$ x $\mathbf{B}$) force.

Induced Plasma Current

Coil current

Magnetic field from Theta Pinch Coil

Capacitors Bank

Switch

Plasma

Theta Pinch Coil

Vacuum Vessel
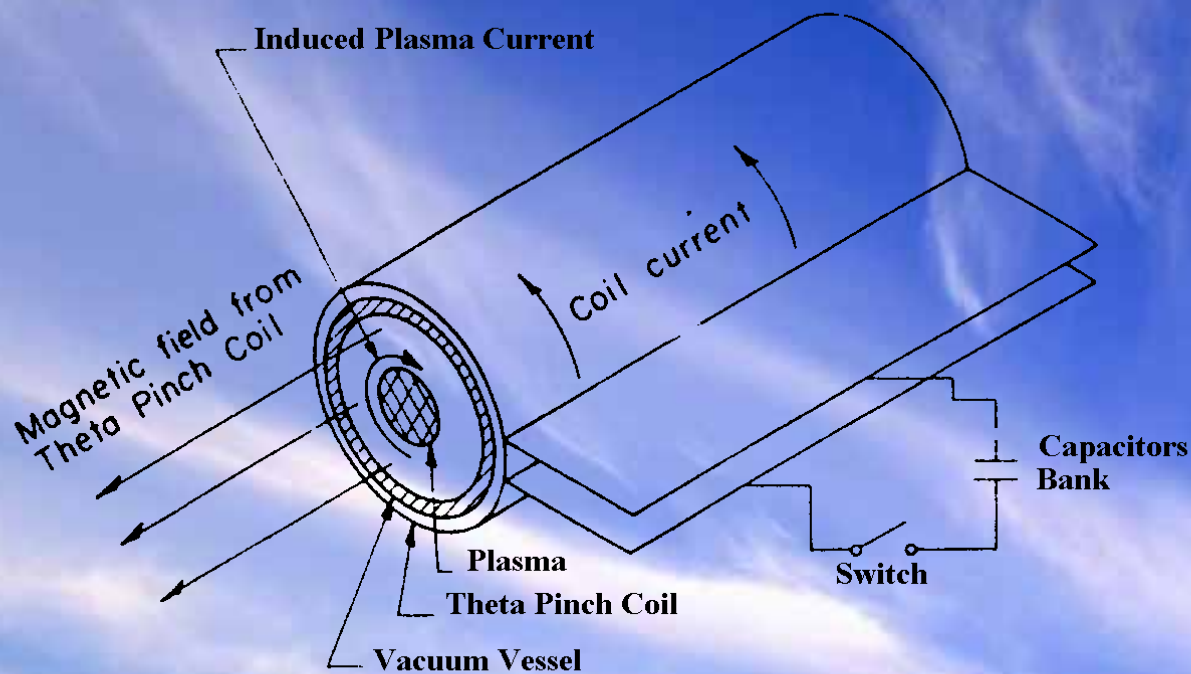
Instabilities in Pinch Devices

Plasma treated by MHD equations admits number of instabilities like **sausage**, **kink** and **Rayleigh-Taylor**.

Sausage Instability

$$B_\theta \sim \frac{1}{r}$$

B<Bo   B >Bo

I   I
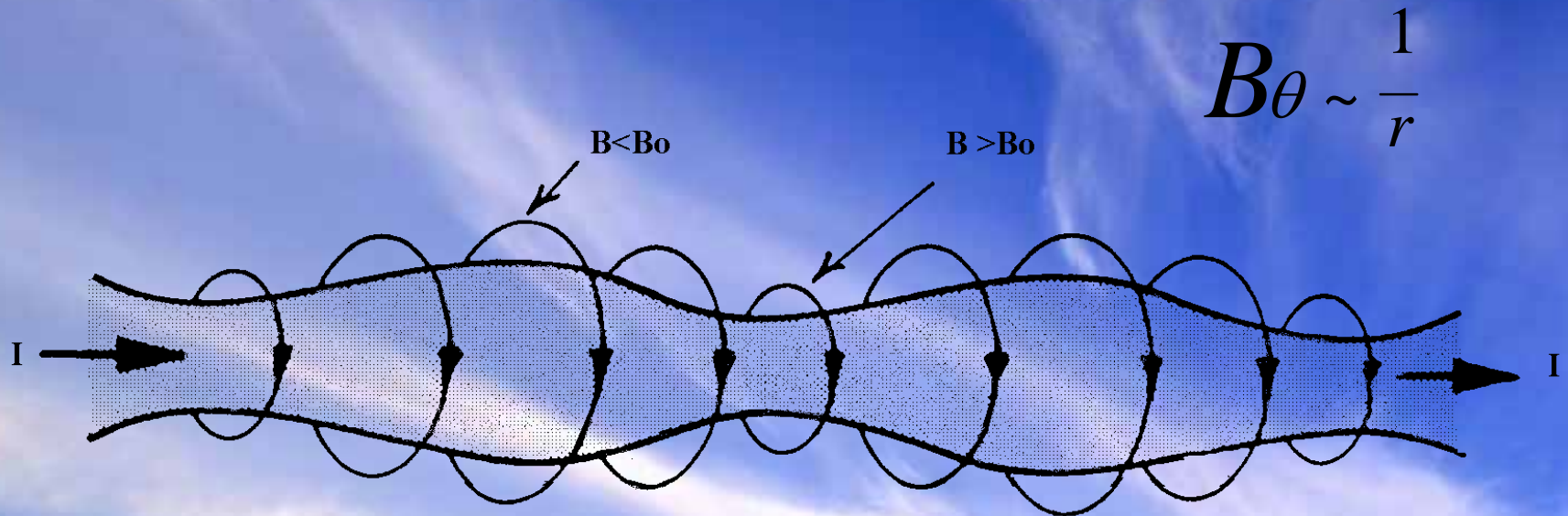
Kink Instability

The kink instability arises due to the formation of bend or kink in the plasma column, although it maintains its uniform circular cross-section. *The density of the magnetic field lines increases on the concave side and decreases on the convex side*. The growth of the instability can be minimized by applying **axial magnetic field**.
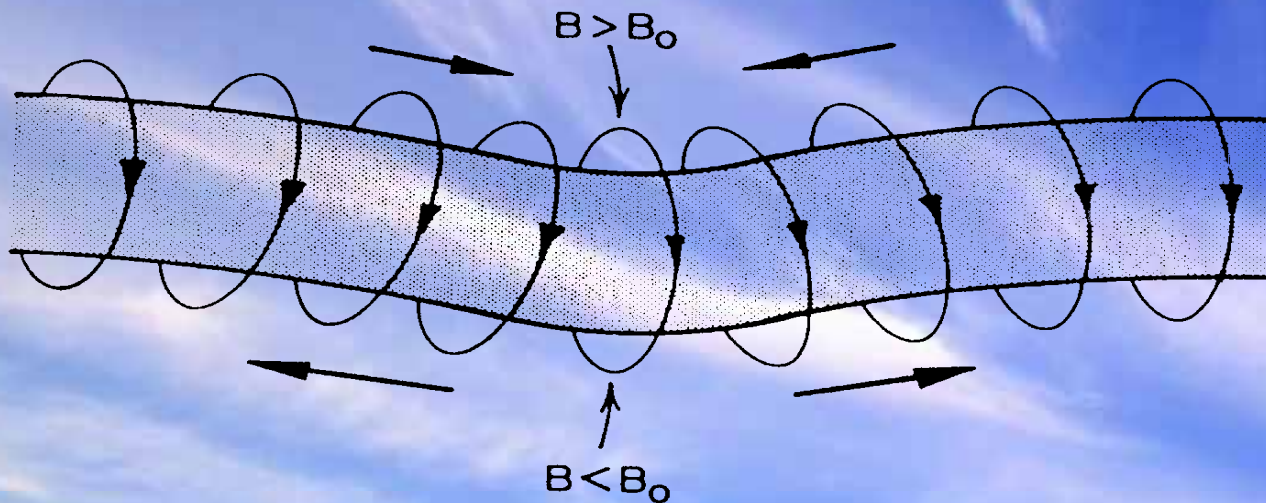
Rayleigh-Taylor Instability

Rayleigh-Taylor instability occurs when a **heavy fluid** is supported by **light fluid** in the presence of gravitational fields. In pinch devices **plasma being heavy fluid** is compress by **mass-less magnetic field**. The instability occurs only if the plasma is **accelerating or decelerating.**

The growth rate of R-T instability is given by

$$\gamma = \sqrt{kg_{eff}}$$

k is the wave number of the perturbation and g_{eff} is the acceleration of the plasma.

Possible Ways to Mitigate R-T in Z-Pinches

R-T instability can be reduced with different stabilization techniques like.

- Spin of the outer shell,

- Thick shell

- Multi-gas-puff liner systems with the inclusion of axial magnetic field B_z.

Major Applications of Z-pinches

- Controlled Thermonuclear Fusion

- Generation of High Magnetic Fields

- Sources of high energy, high intensity charged particles

- X-Ray Lasers etc.

Numerical Model for Z-pinch dynamics

- **Snow plow model**

In this model we assume that

1. Plasma is good conductor; initially the discharge current form a **thin current sheath** on the surface of the plasma column.

2. Plasma ahead of the current sheath is in initial state.

3. The current sheath **sweeps all the mass in its way** as it moves.

Staged Pinch (Z- θ pinch)

The configuration embodies a conventional Z-pinch, imploding onto a co-axial cryogenic fiber (θ -pinch) target plasma. This configuration is also called as Z- θ pinch.

Addresses the problems of **effective compression** and **large current rising time**.

The inductive heating induces θ-pinch discharge on the fiber surface with rise time of few nanosecond and the combined configuration of Z- θ pinch is found stable.

Theoretical model for Z- θ pinch have been proposed by *Rahman et al. (1989),* assuming thin shell of the imploding Plasma.

Glass vessel

J_z

r_{03} r_{02} r_{01}

2a

B_z

B_θ

J_θ

r

J_z

S

c_o
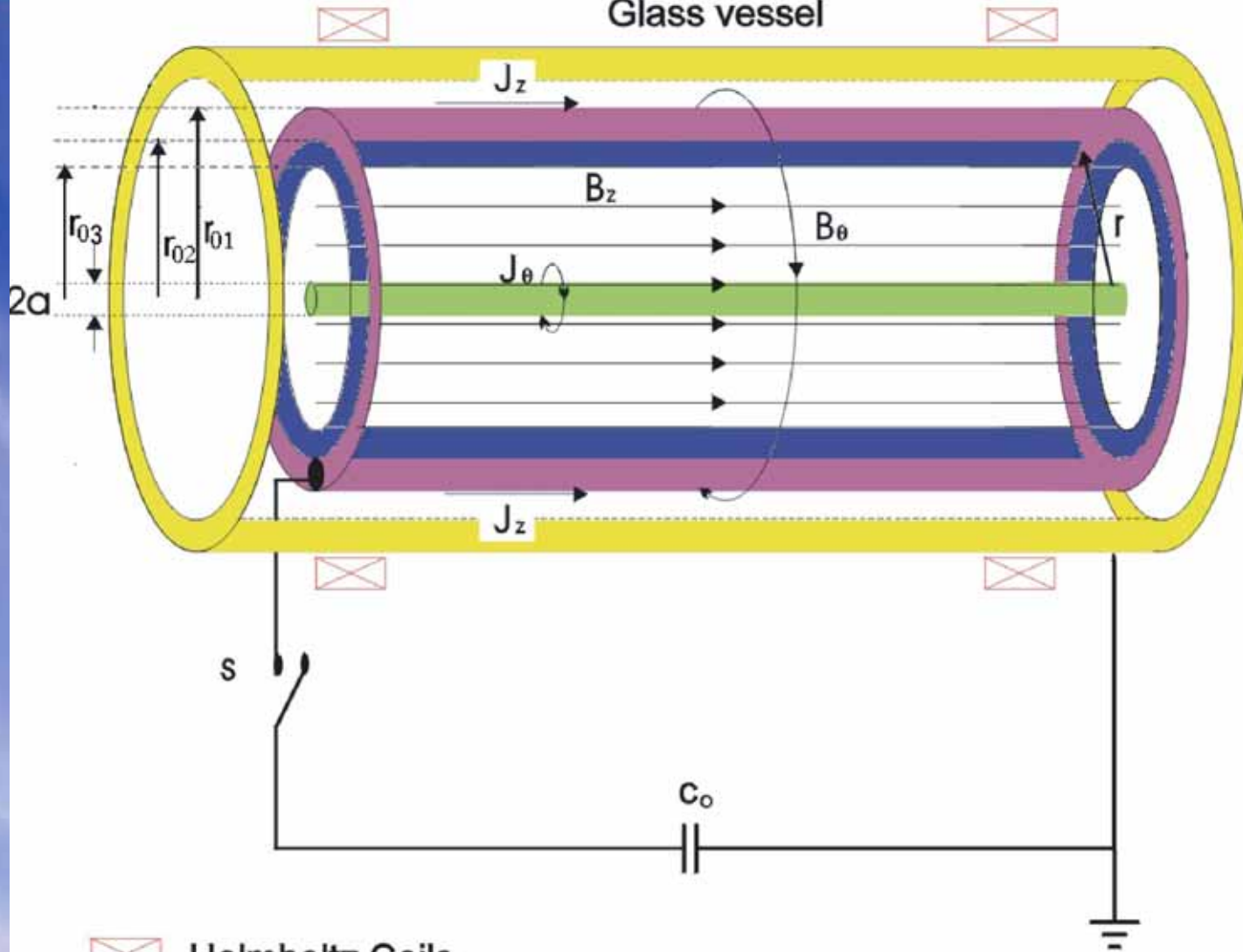
Helmholtz Coils

Gas-Puff (I)

Gas-Puff (II)

D-T Fiber

Two-Gas-Puff Liner System

- ## Outer Z-Pinch Dynamics

The dynamical equation for the outer double-gas-puff Z-pinch plasma based upon modified snow-plow model can be expressed as

$$\frac{d}{dt}\left(M\,\frac{dr}{dt}\right) = -2\pi r\left(P_{mag} - P_{kin}\right)$$

where $M = \pi\left(\rho_0\,r_1^2 - \rho_1\,r^2\right)$ is the mass per unit length accumulated in the current sheath during the implosion.

The total magnetic pressure caused by azimuthal and axial magnetic fields can be written as

$$P_{mag} = \frac{1}{8\pi}\left[\left(\frac{I_z}{5r}\right)^2 + B_0^2\left(1 - \left(\frac{r_0}{r}\right)^4\right)\right]$$

Under adiabatic conditions, the kinetic pressure can be expressed as

$$P_{kin} = P_0 \left(\frac{r}{r_0}\right)^{-2\gamma}$$

The equation of motion for the double gas puff Z-pinch plasma in the dimensionless form can be written as

$$\frac{d}{dt}\left[\left(1+R^2\right)\frac{dR}{dt}\right] = -\frac{a}{R}\left[\sin^2\left(\frac{\gamma}{2t_0}\right) - \frac{b}{R^2}\left(1+R^4\right) - \beta R^{2\eta-1}\right]$$

where $R = (r/r_0)$ is the normalized radius, $a = I_0^2\,/100\,m_0\,r_0^2$ is the measure of external force on the pinch per unit mass, $b = (5\,r_0\,B_0^2\,/I_0^2)^2$, $\beta = 8\pi P_0/B_0^2$ and $\Theta = r_2/r = r_1/r_0$, $\eta = r_2/r = r_1/r_0$.

The above nonlinear differential equation is numerically integrated for I_0 =10 MA, m_0 = 38 μ g/cm, B_0 = 0.02 MG, r_0 = 4 cm and t_0 = 50 nsec. Fig. 1 shows the evolution of the normalized outer z-pinch radius R and the normalized (by I_0) current I_z versus time for various values of α and β. We see that the minimum radius of the imploding outer double gas-puff as a function of α and β. For fixed value of α (= 0.1), finite β, always delays the maximum compression. Small values of β < 0.1 gives higher compression. This indicates that double gas-puff devices can be used for controlled thermonuclear fusion. Whereas, for large β case, the maximum compression occurs at later times. Similarly, large α with fixed β gives fast compression.
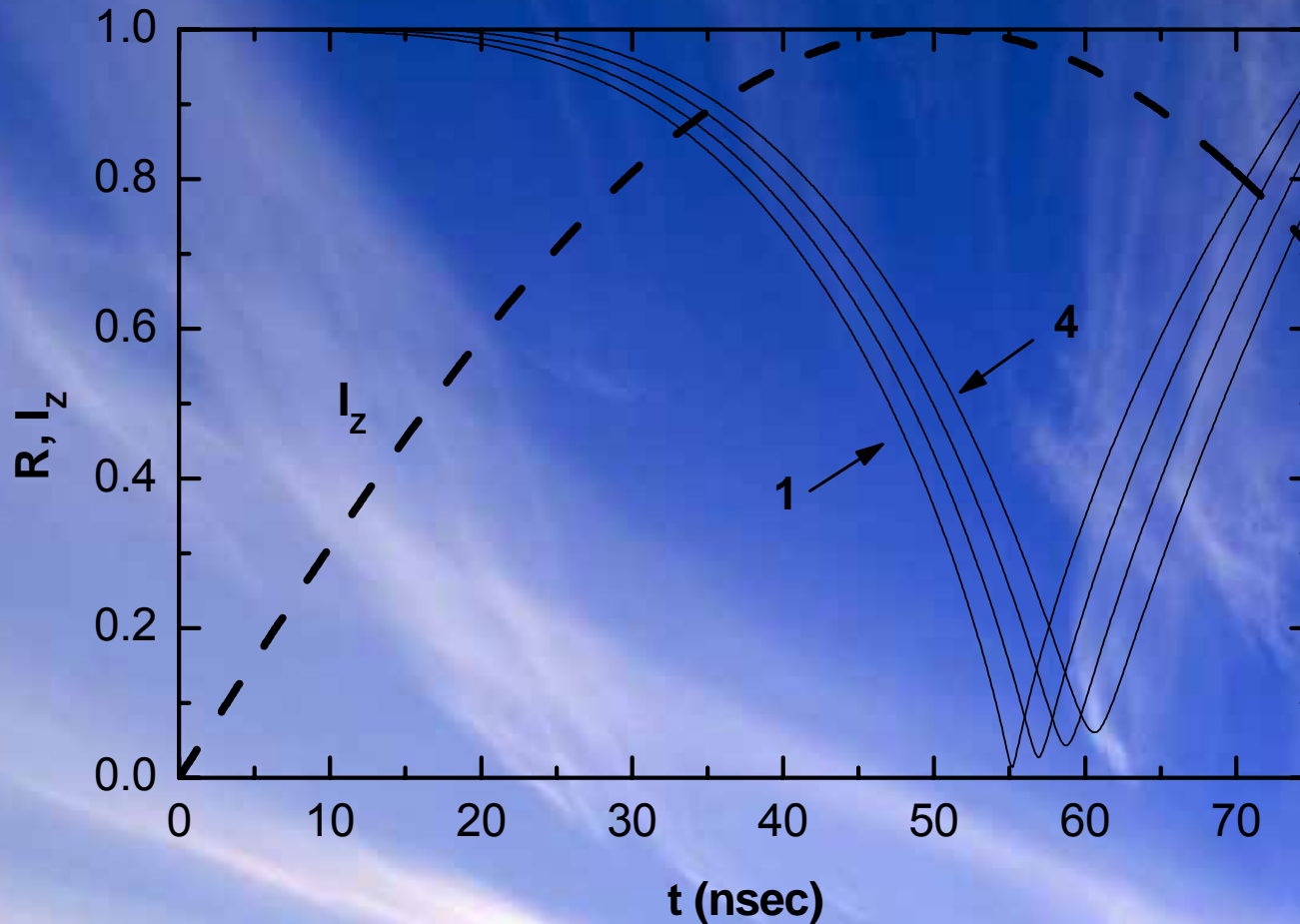
Fig.1: Normalized plots of axial current I_z and outer z-pinch radius R versus time, for different values of β and for $\alpha = 0.1$. Curves 1-4 for $\beta = 0, 0.025, 0.050, 0.075$, respectively.

Dynamics of the Inner θ-pinch Fiber Plasma

Continuity equation

$$\frac{\partial n}{\partial t} + \nabla \bullet (nV) = 0$$

Mass conservation equation

$$mn\left(\frac{\partial}{\partial t} \cdots \right) \cdots P_{mag}$$

$$P_{kin} = (1 + Z)nT \qquad P_{mag} = \frac{B_\theta^2}{8\pi}\left[\frac{1}{R^4} + \left\{ 1 - \left(\frac{a_0}{a} \right)^4 \right\} \right]$$

Z being the charge state of the plasma ions. n is number density.

The energy equation under the adiabatic condition

$$\left(\frac{\partial}{\partial t} \cdots \right) n \cdots P_{ohm} \cdots P_{alpha} \cdots P_{loss}$$

Here, the Ohmic heating term in normalized parameters can be expressed as:

$$P_{ohm} = 1.29 \times 10^{21} \left(\frac{B_0^2}{a_0^2 T_0^{3/2}} \right) \frac{1}{a^2 \, T^{3/2} \, R^4}$$

The energy loss term includes the bremsstrahlung and the cyclotron radiation loss terms:

$$P_{brem} = 3.32 \times 10^{20} \, n_0^2 \left(\frac{\langle T_0 \rangle^{1/2}}{a^4} \right)$$

$$P_{cycl} = 3.88 \times 10^{16} \left(\frac{n_0 B_0^2 T \, T_0}{a^2 \, R^4} \right)$$

The alpha-particle self-heating term can be written as:

$$P_{alpha} = 3.22 \times 10^{26} \frac{n_0^2}{\langle T_0 \rangle^{2/3}} \left(\frac{1}{a^2} - 2 n_\alpha^2 \right)^2 \exp\left(-\frac{19.94}{\langle T_0 \rangle^{1/3}} \right)$$

where n_α can be obtained from the production rate equation

$$\frac{dn_\alpha}{dt} = \frac{9.2 n_0}{\langle T_0 \rangle^{3/2}} \left(\frac{1}{a^2} - 2 n_\alpha^2 \right)^2 \exp\left(-\frac{19.94}{\langle T_0 \rangle^{1/3}} \right) - \frac{2 n_\alpha}{a} \frac{da}{dt}$$

Here P_{Ohm}, P_{brem}, P_{cycl} and P_{alpha} are in units of keV/(nsec-cm³); a_0, B_0 and T_0 are in units of μ m, MG, and k eV, respectively.

The fiber plasma equations in the normalized form can be expressed as

$$\frac{d^2a}{dt^2} = -\frac{200B_0^2}{n_0 a_0^2}\left[\frac{a}{R^4} - 800\left(\frac{n_0 T_0}{B_0^2}\right)\frac{T}{a} - a\left(1 - \frac{1}{a^4}\right)\right]$$

$$\frac{dT}{dt} = -2(\gamma-1)\frac{T}{a}\frac{da}{dt} + \frac{(\gamma-1)\times 10^{22}a^2}{2T_0 n_0}\left[P_{ohm} + P_{alpha} - P_{brem} - P_{cycl}\right]$$

where the fiber radius a and the temperature T are normalized to their initial values a_0 and T_0, n_0 being the initial density of the fiber plasma in the units of 10^{22} cm^{-3}.

Numerical Results and Discussion

Using the typical parameters of the University of California-Irvine experiments, we have numerically integrated the above equations with I_0=10 MA, r =4 cm, t_0 = 50 nsec, M_{o1} = 38 µg/cm, T_0 = 20 eV, B_0 = 20 kG, a_0 = 0.02 cm, n_0 = 10^{22} cm^{-3} for different values of α and for β in the range 0 ≤β ≤0.075. Figure 2(a-d) displays the results for the normalized radius a, density n and temperature T as a function of time during the final stages of collapse for a D-T θ-pinch plasma when ohmic heating, adiabatic heating, α-particle self-heating and radiative losses are included for different values of β with α = 0.1.
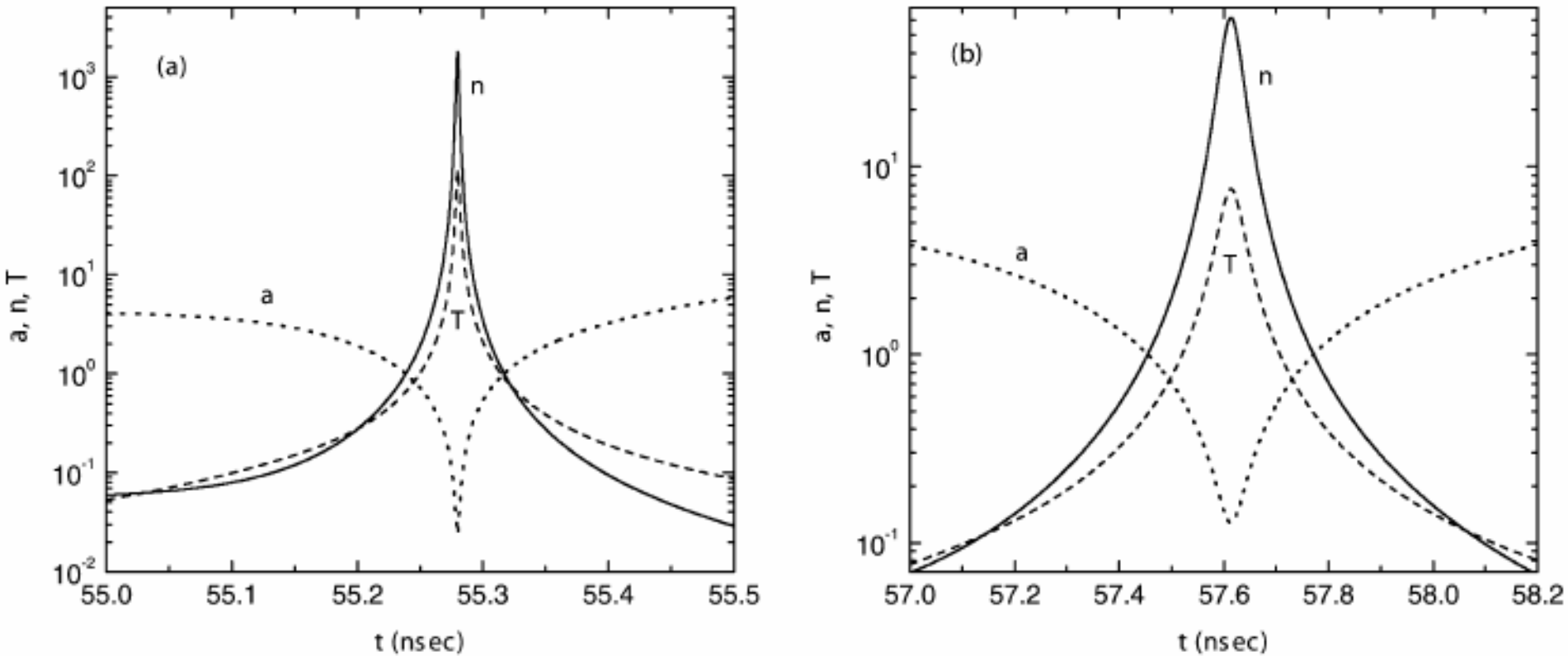
Fig.2 (a, b): Dynamics of the θ-pinch displaying the plots of the normalized fiber radius a, normalized number density n (10^{22} cm^{-3}) and temperature T (k eV) versus time during the last stages of compression with α = 0.1 and for different values of β. Curves 1-4 for β = 0, 0.025, 0.050, 0.075, respectively.
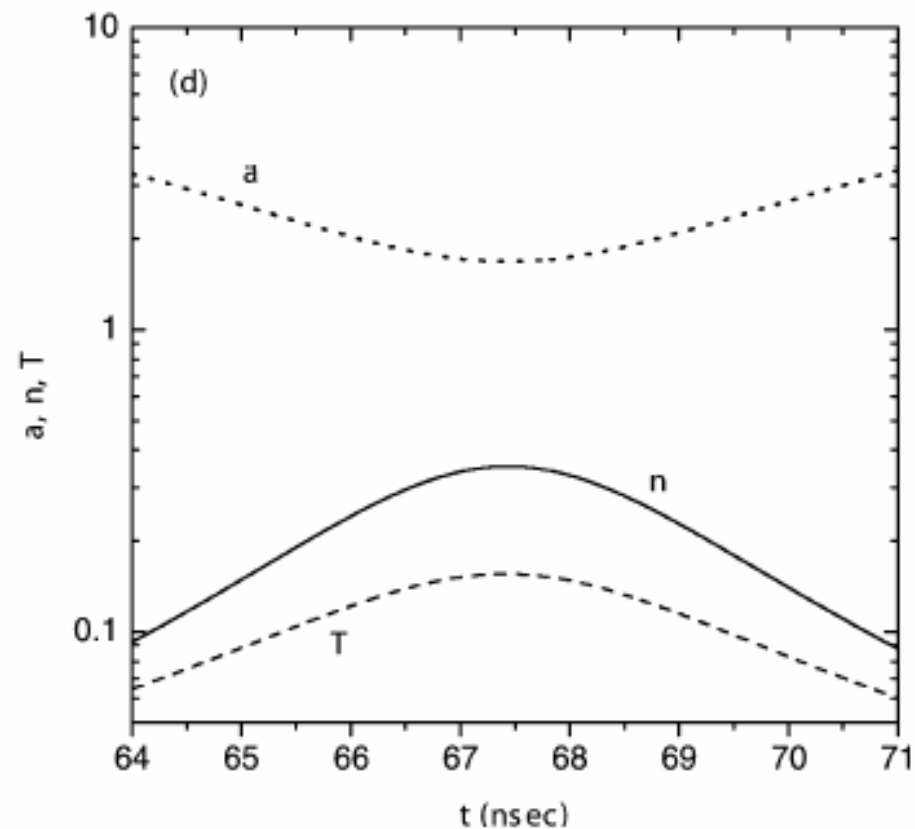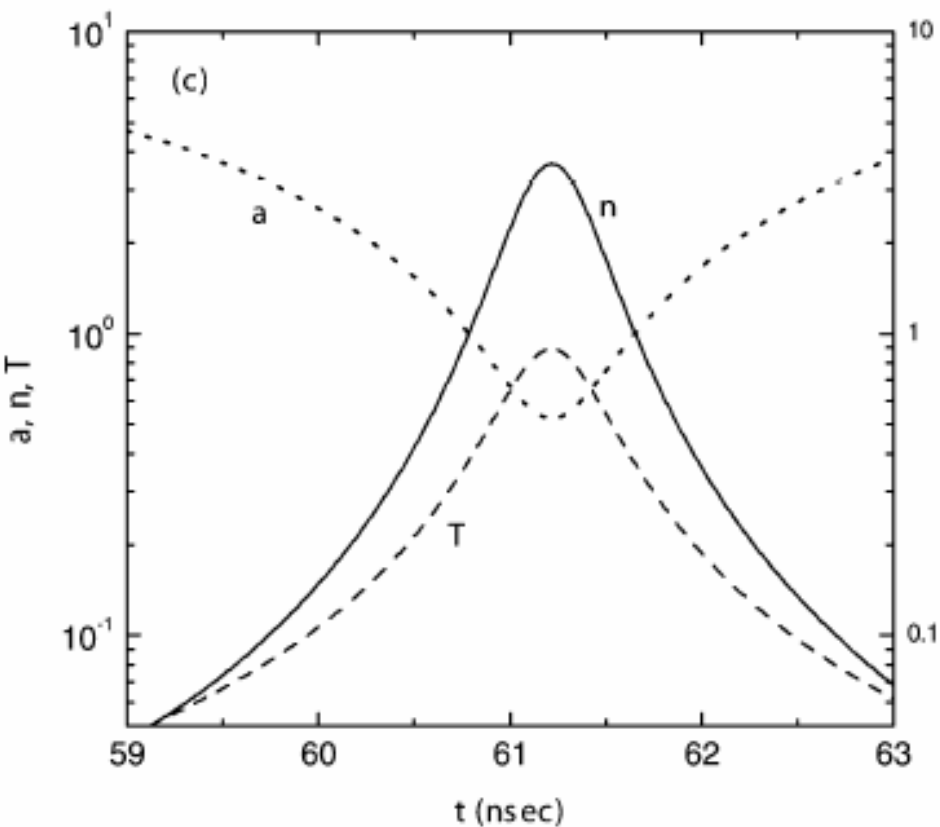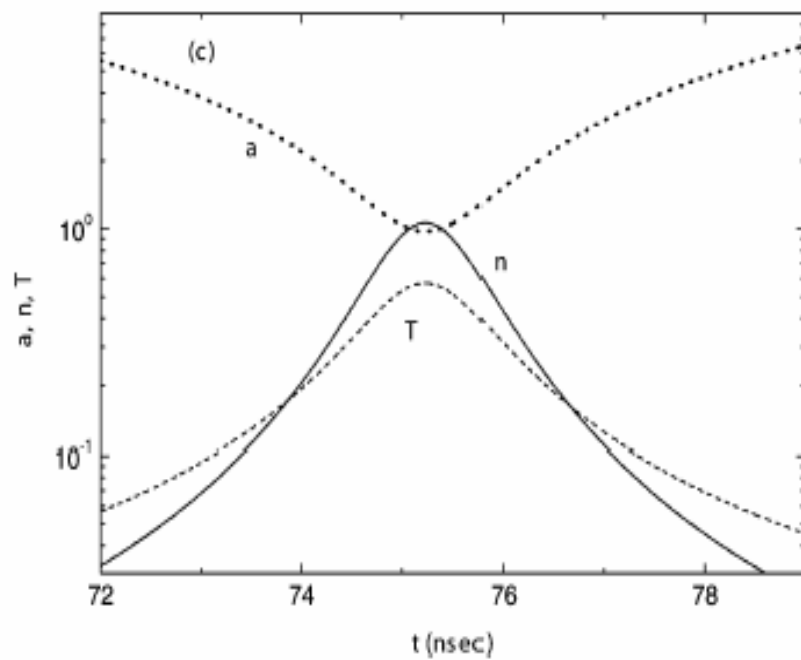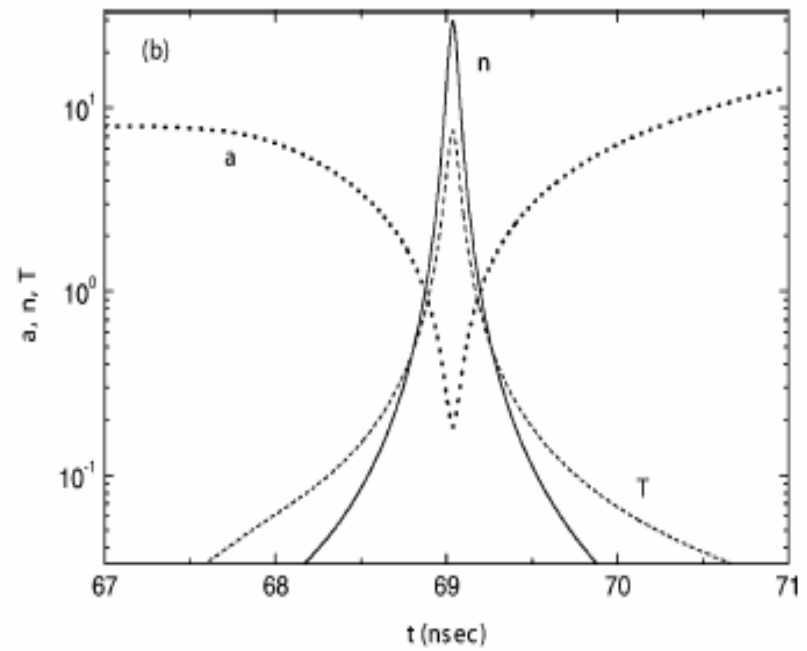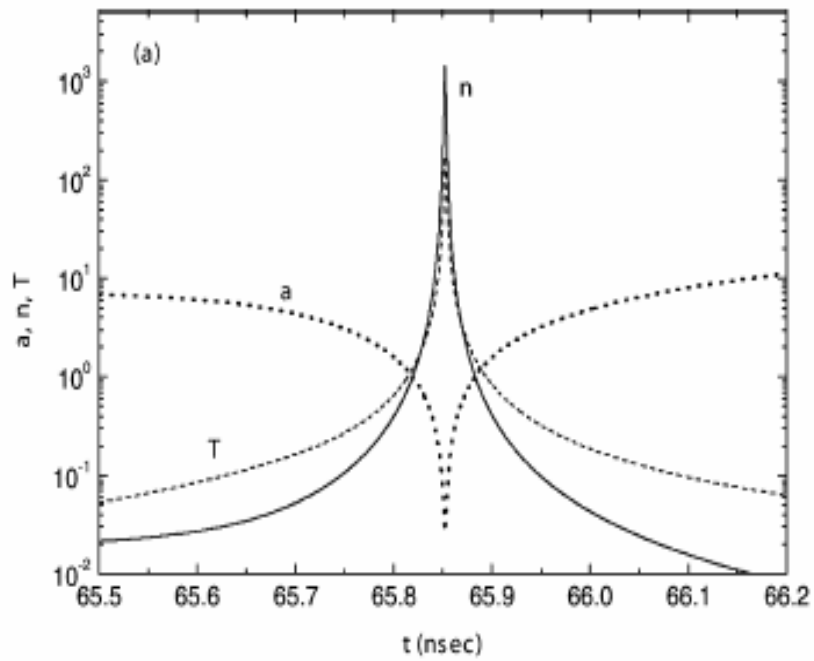
Fig.2 (c, d): Dynamics of the θ-pinch displaying the plots of the normalized fiber radius a, normalized number density n (10^{22} cm⁻³) and temperature T (k eV) versus time during the last stages of compression with α = 0.1 and for different values of β. Curves 1-4 for β = 0, 0.025, 0.050, 0.075, respectively.

For α = 0.5

It is evident from the graphs that without the kinetic pressure term ($\beta=0$), one can obtain high density ($n \sim 10^{25}$ cm^{-3}) and high temperature ($T \sim 100$ keV) plasma with $\alpha = 0.1$. However, for any finite-β value with fixed $\alpha = 0.1$, seems to reduce the maximum compression, and leads to lower temperatures and densities.

For example, for $\beta = 0.025$, the maximum density (Fig. 2(b)) at peak compression is about $100\ n_0$ and the temperature is about 10 keV. On the other hand, for $\beta = 0.075$, the maximum density (Fig.2(d)) at peak compression is about $0.3\ n_0$ with a temperature of the order of 0.05 keV.

On the other hand, if we choose a relatively large value of $\alpha = 0.5$, then the maximum density and temperature at peak compression drastically reduces (see e.g., Fig. 3(a)-(c)) for any finite β.

From these results one may conclude that the double gas-puff staged pinch *can be used for controlled thermonuclear fusion with small β and high α values*. Our numerical results demonstrates that large α (for a large density ratio of the test to the driver gas at the interface position) gives fast compression while high-β gives slow compression. The finite-β effect also delays the timing of the maximum compression. Thus for optimum choice of α and β parameters, the double gas-puff staged pinch can be used as a more feasible approach to achieve fusion conditions with enhanced stability.

Questions

Thanks !

References:

1) Physica Scripta, vol. **72**, 399 - 403 (2005).
2) Modern Physics Letters B, vol. **19**, 1095 - 1105 (2005).