12 Th Regional Conference in Mathematical Physics

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Brane Cosmology with String Antisymmetric Field

Closed strings :

$$a^\dagger_\mu \, a^\dagger_\nu |0\rangle \longrightarrow$$

$g_{\mu\nu}$ Graviton

$B_{\mu\nu}$ Kalb-Ramond

φ Dilaton

Open strings

Dirichlet Bound. Cond. $\longrightarrow$ D-Branes

Constant $B_{\mu\nu} \longrightarrow$

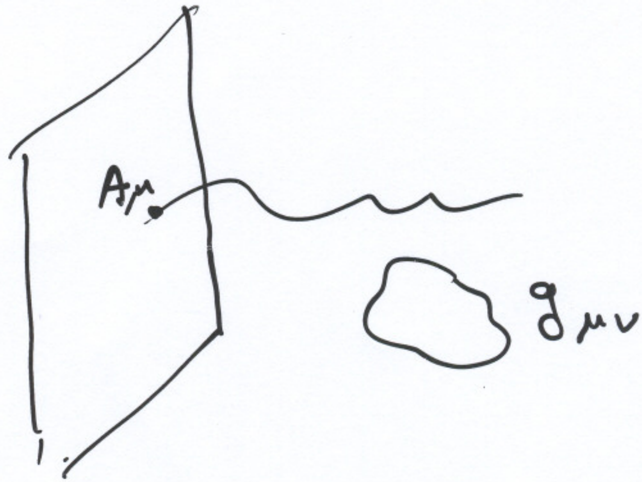
Noncommutative coordinates on Brane

$$[x^i, x^j] \sim B$$

Brane World

1- Gauge Fields on 3-Brane

2- Gravitons, φ, and B_{ab} in the Bulk



Low Energy Theory of string Theory:

1- Yang-Mills

$$\mathcal{L} = F_{\mu\nu} F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

2 - Einstein - Hilbert

$$\mathcal{L} = \sqrt{g}\, R \quad , \quad R = R_{\mu\nu\lambda\rho}\, g^{\mu\lambda} g^{\nu\rho}$$

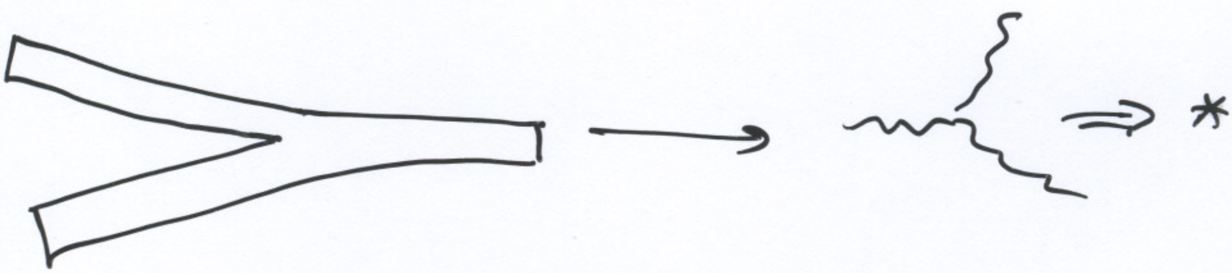
$B \neq 0$

1- Noncommutative Yang–Mills

$$\mathcal{L}_B = F_{\mu\nu} * F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu * A_\nu - A_\nu * A_\mu]$$

$$(f*g)(x) = f(x)\, e^{i \frac{\theta_{\mu\nu}}{2} \overleftarrow{\partial}^\mu \overrightarrow{\partial}^\nu} g(x)$$

$*$ Product coming from scattering of gauge bosons



2 - Graviton Scattering

B covariant constant

$$B_{\mu\nu;\lambda} = 0$$

$$\Longrightarrow \mathcal{L}_B = \sqrt{g+B}\; R_{\mu\nu\lambda\rho} \left(\frac{1}{g+B}\right)^{\mu\lambda} \left(\frac{1}{g+B}\right)^{\nu\rho}$$

$\therefore \quad g_{\mu\nu} \longrightarrow g_{\mu\nu} + B_{\mu\nu}$

Cosmology with $\mathcal{L}_B$

1– $B \neq 0$ on 3–Branes studied $\longrightarrow$

Bounds on B.

Anisotropy in R^4 cosmology.

2– $B \neq 0$ in the Bulk may stabilize Brane Moduli

3– "Toy" Model :

3–brane in 5–brane

$B \neq 0$ in the 5,6 directions

$$\int \mathcal{L} = \int dx^4\, dy^2 \sqrt{-\det(g+B)}\; R_{abcd} \left(\frac{1}{g+B}\right)^{ac} \left(\frac{1}{g+B}\right)^{bd}$$

$$g = \left(\begin{array}{c|c} g^{(4)} & 0 \\ \hline 0 & g^{(2)} \end{array}\right) \quad , \quad B = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \begin{matrix} 0 & B \\ -B & 0 \end{matrix} \end{array}\right)$$

$$ds^2 = -n^2(t,y)\,dt^2 + a^2(t,y)\,d\Sigma_k^3 + g_{AB}\,dy^A dy^B$$

$A, B = 5, 6 \quad , \quad \Sigma_k^3 \quad , \quad k = 0, \mp 1$

Max. Sym. homog. isotrop.

$$B_{AB;C} = 0 \longrightarrow B_{AB} = B_0 \sqrt{g^{(2)}_{AB}}\; \varepsilon_{AB}$$

$$B = B_0 \sqrt{\det g^{(2)}_{AB}}$$

B_0 const.

$$g^{(2)}_{AB} = \begin{pmatrix} b_1^2 & 0 \\ 0 & b_2^2 \end{pmatrix}$$

$$f^2 \equiv 1 + B_0^2$$

Einstein Tensors:

$$E^0_0 = G^0_0(g_{\mu\nu}) + \frac{3n^2}{b_A^2}\left(\frac{\partial_A a}{a}\right)^2 + \frac{3}{f^2}\left(\frac{1}{b_A^2}\frac{\partial_A^2 a}{a} - \frac{1}{n^2}\frac{\dot a}{a}\frac{\dot b_A}{b_A}\right)$$
$$+ \frac{1}{f^2}\left\{ \frac{3}{b_1^2}\frac{\partial_1 a}{a}\left(\frac{\partial_1 b_2}{b_2} - \frac{\partial_1 b_1}{b_1}\right) + \frac{3}{b_2^2}\frac{\partial_2 a}{a}\left(\frac{\partial_2 b_1}{b_1} - \frac{\partial_2 b_2}{b_2}\right) + \frac{1}{b_1 b_2}\left[\partial_1\left(\frac{\partial_1 b_2}{b_1}\right) + \partial_2\left(\frac{\partial_2 b_1}{b_2}\right)\right] - \frac{1}{n^2}\frac{\dot b_1}{b_1}\frac{\dot b_2}{b_2}\right\}$$

$$E^5_5 = -\frac{1}{2}R(g_{\mu\nu}) - \frac{3}{b_A^2}\left(\frac{\partial_A n}{n} + \frac{\partial_A a}{a}\right)\frac{\partial_A a}{a}$$
$$+ \frac{1}{f^2}\left\{ -\frac{1}{b_2^2}\left(\frac{\partial_2^2 n}{n} + 3\frac{\partial_2^2 a}{a}\right) + \frac{1}{n^2}\left[\frac{\ddot b_2}{b_2} - \frac{\dot b_2}{b_2}\left(\frac{\dot n}{n} - 3\frac{\dot a}{a}\right)\right] - \left(\frac{\partial_1 n}{n} + 3\frac{\partial_1 a}{a}\right)\left(\frac{1}{b_1^2}\frac{\partial_1 b_2}{b_2} + \frac{B_0}{b_1 b_2}\frac{\partial_2 b_1}{b_1}\right) + \left(\frac{\partial_2 n}{n} + 3\frac{\partial_2 a}{a}\right)\left(\frac{1}{b_2^2}\frac{\partial_2 b_2}{b_2} - \frac{B_0}{b_1 b_2}\frac{\partial_1 b_2}{b_2}\right)\right\}$$

$$E^6_6 = E^5_5(1 \leftrightarrow 2)$$

$$E_0^{\ 5} = \frac{b_2}{b_1}\left[3\left(\frac{\partial_1 \dot a}{a} - \frac{\dot a}{a}\frac{\partial_1 n}{n} - \frac{\dot b_1}{b_1}\frac{\partial_1 a}{a}\right) + \frac{\partial_1 \dot b_2}{b_2} - \frac{\partial_1 n}{n}\frac{\dot b_2}{b_2} - \frac{\partial_1 b_2}{b_2}\frac{\dot b_1}{b_1}\right]$$
$$+ B_0\left[3\left(\frac{\partial_2 \dot a}{a} - \frac{\dot a}{a}\frac{\partial_2 n}{n} - \frac{\dot b_2}{b_2}\frac{\partial_2 a}{a}\right) + \frac{1}{f^2}\left(\frac{\partial_2 \dot b_1}{b_1} - \frac{\partial_2 n}{n}\frac{\dot b_1}{b_1} - \frac{\partial_2 b_1}{b_1}\frac{\dot b_2}{b_2}\right)\right]$$

$$E_0^{\ 6} = E_0^{\ 5}(1 \leftrightarrow 2)$$

$$E_i^{\ j} = G_i^{\ j}(g_{\mu\nu}) + a^2 \gamma_i^{\ j}\,(*)$$

$$(*) = -\frac{1}{b_A^2}\left(2\frac{\partial_A n}{n} + \frac{\partial_A a}{a}\right)\frac{\partial_A a}{a}$$
$$- \frac{1}{f^2}\left\{ \frac{1}{b_A^2}\left(\frac{\partial_A^2 n}{n} + 2\frac{\partial_A^2 a}{a}\right) - \frac{1}{n^2}\left[\frac{\ddot b_A}{b_A} - \frac{\dot b_A}{b_A}\left(\frac{\dot n}{n} - 2\frac{\dot a}{a}\right)\right] - \frac{1}{n^2}\frac{\dot b_1}{b_1}\frac{\dot b_2}{b_2} + \frac{1}{b_1^2}\left(\frac{\partial_1 b_2}{b_2} - \frac{\partial_1 b_1}{b_1}\right)\left(\frac{\partial_1 n}{n} + 2\frac{\partial_1 a}{a}\right) + \frac{1}{b_2^2}\left(\frac{\partial_2 b_1}{b_1} - \frac{\partial_2 b_2}{b_2}\right)\left(\frac{\partial_2 n}{n} + 2\frac{\partial_2 a}{a}\right) + \frac{1}{b_1 b_2}\left[\partial_1\left(\frac{\partial_1 b_2}{b_1}\right) + \partial_2\left(\frac{\partial_2 b_1}{b_2}\right)\right]\right\}$$

$$R(g_{\mu\nu}) = \frac{6}{n^2}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{\dot{a}}{a}\frac{\dot{n}}{n}\right) + \frac{6}{a^2}k$$

$$G_{00}(g_{\mu\nu}) = 3\frac{\dot{a}^2}{a^2} + 3\frac{n^2}{a^2}k$$

$$G_{ij}(g_{\mu\nu}) = \gamma_{ij}\left[-\frac{2a\ddot{a}}{n^2} - \frac{\dot{a}^2}{n^2} + 2\frac{a\dot{a}\dot{n}}{n^3} - 3k\right]$$

Note dependence on B_0

To get a taste of qualitative consequence of B do a 5-dim reduction :

$$b_2 \longrightarrow 0 \qquad \frac{\partial}{\partial y_2} \longrightarrow 0$$

$$\frac{\kappa^2}{3} T_0^{\,0} = -\frac{1}{n^2}\frac{\dot a}{a}\left(\frac{\dot a}{a} + \frac{1}{f^2}\frac{\dot b}{b}\right) + \frac{1}{b^2}\left[\frac{1}{f^2}\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{1}{f^2}\frac{b'}{b}\right) - \frac{k}{a^2}\right]$$

$$\frac{\kappa^2}{3} T_5^{\,5} = \frac{1}{b^2}\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{1}{n^2}\left[\frac{\ddot a}{a} + \frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{\dot n}{n}\right)\right] - \frac{k}{a^2}$$

$$\frac{\kappa^2}{3} T_0^{\,5} = \frac{n'}{n}\frac{\dot a}{a} + \frac{a'}{a}\frac{\dot b}{b} - \frac{\dot a'}{a}$$

$$\frac{\kappa^2}{3} T_i^{\,j} = \gamma_i^{\,j}\left\{\frac{a^2}{b^2}\left[\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{1}{f^2}\frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + \frac{1}{f^2}\left(2\frac{a''}{a} + \frac{n''}{n}\right)\right]\right.$$
$$\left. + \frac{a^2}{n^2}\left[\frac{\dot a}{a}\left(2\frac{\dot n}{n} - \frac{\dot a}{a}\right) - 2\frac{\ddot a}{a} + \frac{1}{f^2}\frac{\dot b}{b}\left(\frac{\dot n}{n} - 2\frac{\dot a}{a}\right) - \frac{1}{f^2}\frac{\ddot b}{b}\right] - k\right\}$$

where $b \equiv b_1$, $\quad a' \equiv \frac{\partial}{\partial y} a$, etc.

$y \equiv y_1$, $\quad \dot a \equiv \frac{\partial}{\partial t} a$, etc.

A Solution ?

A Remarkable solution of 5-dim with $B=0$ was found in 1999: hep-th/9910219

(P. Binetruy, C. Deffayet, U. Ellwanger, D. Langlois)

define: $$F \equiv \left(\frac{aa'}{b}\right)^2 - \left(\frac{a\dot{a}}{a}\right)^2 - ka^2$$

It was found that:

$$F' = \frac{2a'a^3}{3} G^0_0 \quad , \quad \dot{F} = \frac{2\dot{a}a^3}{3} G^5_5$$

$$G^a_b = \kappa^2 T^a_b \quad , \quad T^a_b = \mathrm{diag}\left(-\rho, P, P, P, P_5\right)$$

For $-\rho = P_5$, $T_5{}^0 = 0$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6}\rho + \frac{\kappa^4}{36}\rho_b^2 + \frac{C}{a^4} - \frac{k}{a^2}$$

$$\left[\frac{a'}{ab}\right] = -\frac{\kappa^2}{3}\rho_b$$ discontinuity across the bran

Again remarkably There is a solution with

$$B \neq 0 :$$

This time for $\quad P_5 = \frac{-2f^2}{f^2+1}\rho \quad , \; f^2 = 1 + B_0^2$

with the same Friedman eq. modified:

$$H = \frac{\kappa^2}{6}\rho + f^2\frac{\kappa^4}{36}\rho_b^2 + \frac{C}{a^4} - \frac{k}{a^2}$$

The way it works:

$$\begin{cases} \frac{1}{f^2}a^2\left(\frac{F}{a^2}\right)' + (a^2)'\left(\frac{F}{a^2}\right) = \frac{2a'a^3}{3}E_0^0 \\ \dot{F} = \frac{2\dot{a}a^3}{3}E_5^5 \end{cases}$$

$$\longrightarrow \quad F = a^4\frac{\kappa^2}{3}T_5^5 = a^4\frac{f^2}{f^2+1}\frac{\kappa^2}{3}T_0^0$$