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Complex Transformations

and the

Cosmological Constant Problem

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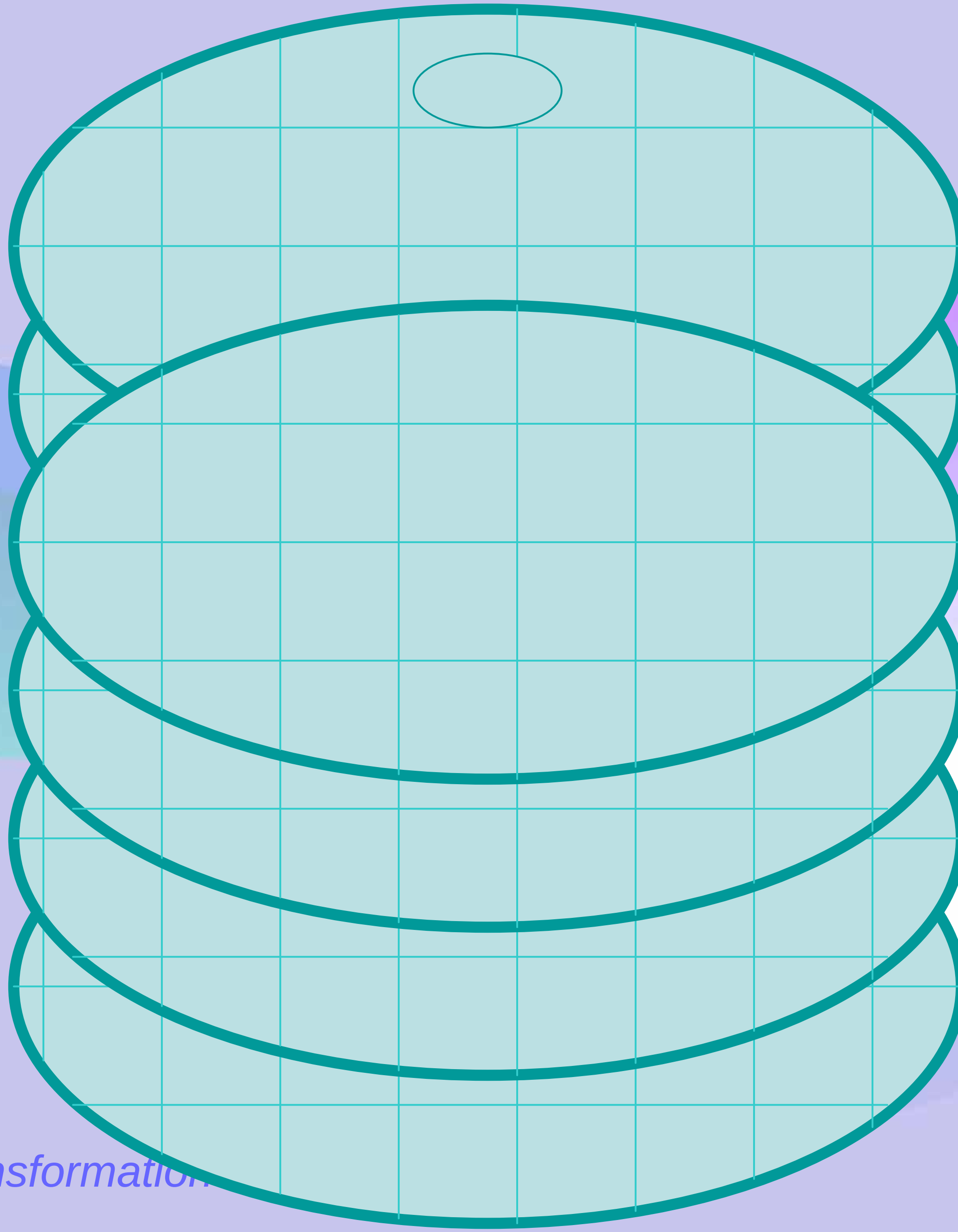
The Cosmological Constant Problem

$$L = \sqrt{g}\,(\Lambda + \frac{1}{8\pi G} R)$$

stretchability
very small

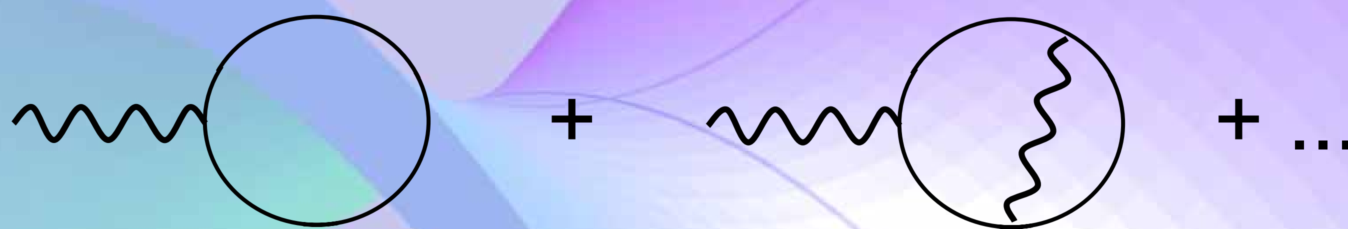
stiffness
very large



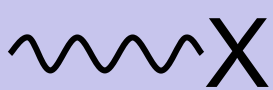
During inflation, one can try to follow the effects of

quantum corrections

on the cosmological cnstant term



but the subtraction term is crucial:



$$\Lambda \rightarrow \Lambda + \delta\Lambda$$

Does Λ^{eff} tend to zero during inflation ???

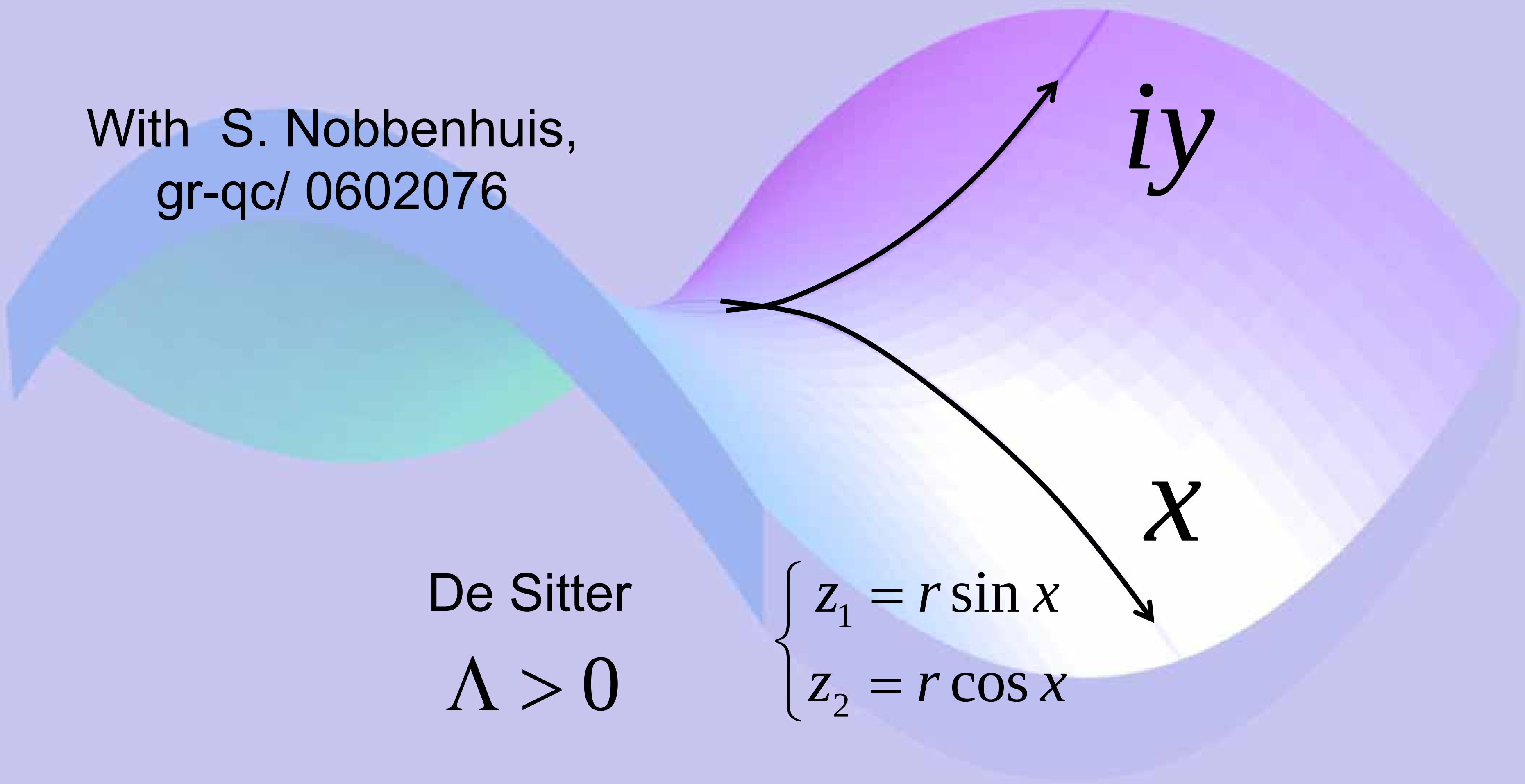
Various theories claim such an effect, but the physical forces for that remain obscure.
Can thist happen ?

Anti-De Sitter

$$\Lambda < 0$$

$$\begin{cases} z_1 = r \sinh y \\ z_2 = r \cosh y \end{cases}$$

With S. Nobbenhuis,
gr-qc/ 0602076



De Sitter

$$\Lambda > 0$$

$$\begin{cases} z_1 = r \sin x \\ z_2 = r \cos x \end{cases}$$

1. Classical scalar field:

$$\phi(\vec{x},t) \rightarrow \phi(\vec{y},\tau)\ , \qquad \vec{x} = i\vec{y}, \quad t = i\tau$$

$$T_{00} = \tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\vec{\partial}\phi)^2 + V(\phi)\ ; \qquad \pi = \partial_0 \phi(x)$$

$$x^\mu = i y^\mu$$

$$T_{00}{}^{y} = -T_{00} = \tfrac{1}{2}\pi_y{}^2 + \tfrac{1}{2}(\vec{\partial}_y\phi)^2 - V(\phi)\ ; \qquad \pi_y = i\pi(iy)$$

Gravity: $$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = -8\pi G\, T_{\mu\nu}$$

$$-R_{\mu\nu}{}^{y} + \tfrac{1}{2} g_{\mu\nu} R\, R_{\mu\nu}{}^{y} - \tfrac{1}{2} g_{\mu\nu} R^y + \Lambda\, g_{\mu\nu}(iy) = -8\pi G\, T_{\mu\nu}{}^{y}\, T_{\mu\nu}{}^{y}$$

2. Non-relativistic quantum particle:

$$H = \frac{p^2}{2m} + V(x)\ ; \quad p = -i\frac{\partial}{\partial x}\ ; \qquad V(x) = -V(ix)\ ; \quad V(x) = x^2\, V_0(x^4)$$

Example: $V(x) = x^2\, e^{-\lambda x^4}$

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle$$

$$x = iy \ , \qquad p = -ip_y \ , \qquad p_y = -i\,\partial/\partial y$$

$$[p,\, x] = -i$$
$$[p_y, y] = -i$$

$$H_y = -H = \frac{p_y^2}{2m} + V(y)$$

$$H_y\left|\psi\right\rangle = -E\left|\psi\right\rangle$$

There can exist only *one* state, $|0\rangle$, obeying *boundary conditions* both at $|x| \to \infty$ and $|y| \to \infty$

$$E_0 = 0$$

Note: change of hermiticity properties

$$x = x^\dagger \;\rightarrow\; y = -y^\dagger \;\rightarrow\; y = y^\dagger$$

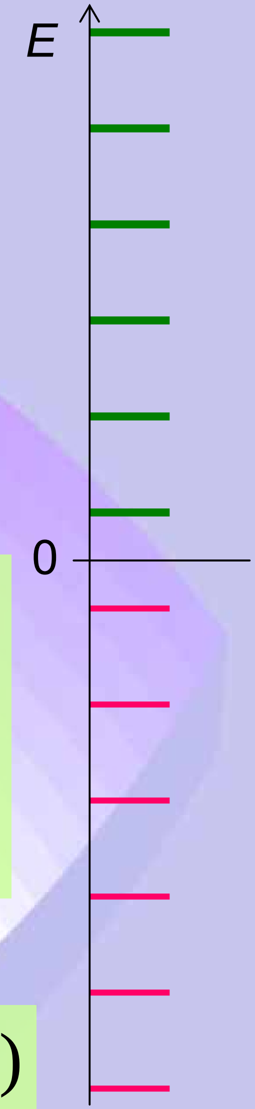
There is a condition that must be obeyed: *translation invariance*:

$x \rightarrow x + a$, a real or complex

$E = 0$ only if $V = 0$

3. Harmonic oscillator:

$$H = \frac{p^2}{2m} + \tfrac{1}{2} m\omega^2 x^2$$



$$a = \sqrt{\frac{m\omega}{2}}\left(x + \frac{ip}{m\omega}\right) \; ; \quad a^\dagger = \sqrt{\frac{m\omega}{2}}\left(x - \frac{ip}{m\omega}\right)$$

$$H = \omega(a^\dagger a + \tfrac{1}{2})$$

$$a_y = \sqrt{\frac{m\omega}{2}}\left(y + \frac{ip_y}{m\omega}\right) \; ; \quad a_y^\wedge = \sqrt{\frac{m\omega}{2}}\left(y - \frac{ip_y}{m\omega}\right) = -a_y{}^\dagger$$

$$a_y = -ia^\dagger \; , \quad a_y^\wedge = -ia \; , \qquad H = -\omega(a_y^\wedge a_y + \tfrac{1}{2})$$

Change hermiticity but not the algebra:

$$a_y{}^\dagger := +a_y^\wedge \; ; \quad H \to -H_y = -\omega(n + \tfrac{1}{2})$$

We get all negative energy states, but the vacuum is *not* invariant !

4. Second quantization:

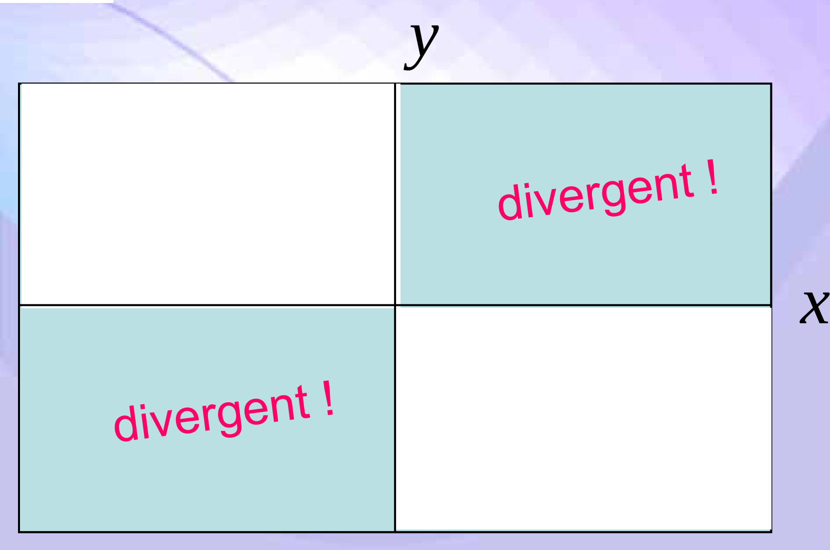
$$[\pi(\vec{x},t),\ \phi(\vec{x}\,',t)] = -i\delta^3(\vec{x}-\vec{x}\,')$$

$$[\pi(i\vec{y},t),\ \phi(i\vec{y}\,',t)] = -i\delta^3(i\vec{y}-i\vec{y}\,')$$

$$\delta(x) \equiv \sqrt{\frac{-i\mu}{\pi}}\, e^{i\mu x^2}$$

$$\delta(iy) = \sqrt{\frac{-i\mu}{\pi}}\, e^{-i\mu y^2} \rightarrow -i\delta(y)$$

Definition of delta function:



or $\delta(iy) \ = +i\delta(y)$

$$[\pi(\vec{x},t),\ \phi(\vec{x}\,',t)] = -i\delta^3(\vec{x}-\vec{x}\,')$$

$$[\pi(i\vec{y},t),\ \phi(i\vec{y}\,',t)] = -\delta^3(\vec{y}-\vec{y}\,')$$

(Now, just 1 space-dimension)

$$\phi(\vec{x},\ t) = \int \frac{dp}{\sqrt{2\pi\cdot 2p^0}}\left(a(p)\,e^{i(px)} + a^\dagger(p)\,e^{-i(px)}\right)$$

$$\pi(\vec{x},t) = \int dp\sqrt{\frac{2p^0}{2\pi}}\left(-ia(p)\,e^{i(px)} + ia^\dagger(p)\,e^{-i(px)}\right)$$

$$(px) = px - p^0 t$$

$$p^0 = \sqrt{p^2+m^2}$$

$$\phi(i\vec{y},\ i\tau) = \int \frac{dq}{\sqrt{2\pi\cdot 2q^0}}\left(a_y(q)\,e^{i(qy)} + a_y^\wedge(q)\,e^{-i(qy)}\right)$$

$$\pi(i\vec{y},\ i\tau) = \int dq\sqrt{\frac{q^0}{2\pi}}\left(-ia_y(q)\,e^{i(qy)} + ia_y^\wedge(q)\,e^{-i(qy)}\right)$$

$$(q\,y) = q\,y - q^0\tau$$

$$q^0 = \sqrt{q^2-m^2}$$

x -space hermiticity conditions:

$$a_y{}^\dagger = a_y\ ;\quad a_y^{\wedge\dagger} = a_y^\wedge \quad \rightarrow \quad a_y^\wedge = a_y{}^\dagger$$

Then, the Hamiltonian becomes:

$$H = \int dy \left(\tfrac{1}{2}\pi(iy)^2 - \tfrac{1}{2}(\partial_y \phi)^2 + \tfrac{1}{2} m^2 \phi(iy)^2 \right) =$$

$$-i\int dq\, q^0 \left(a_y^\wedge(q)\, a_y(q) + \tfrac{1}{2} \right) = -i\int dq\, q^0 \left(n + \tfrac{1}{2} \right)$$

Express the new a_y , $a_y^\wedge$ in terms of a , $a^\dagger$

$$a(p) = \iint \frac{dx\, dq}{2\pi\sqrt{4p^0 q^0}}$$

$$\left\{ (p^0 - iq^0) a_y(q) e^{(q-ip)x} + (p^0 + iq^0) a_y^\wedge(q) e^{(-q-ip)x} \right\}$$

" = 0 by contour integration "

$$q = \pm ip \quad \rightarrow \quad p^0 = -iq^0$$

In that case: $a(p) = i^{1/2} a_y(q)\ , \quad q = ip\ , \quad q^0 = ip^0$

and then ***the vacuum is invariant*** !

The vacuum energy is forced to vanish because of the $x \leftrightarrow ix$ symmetry

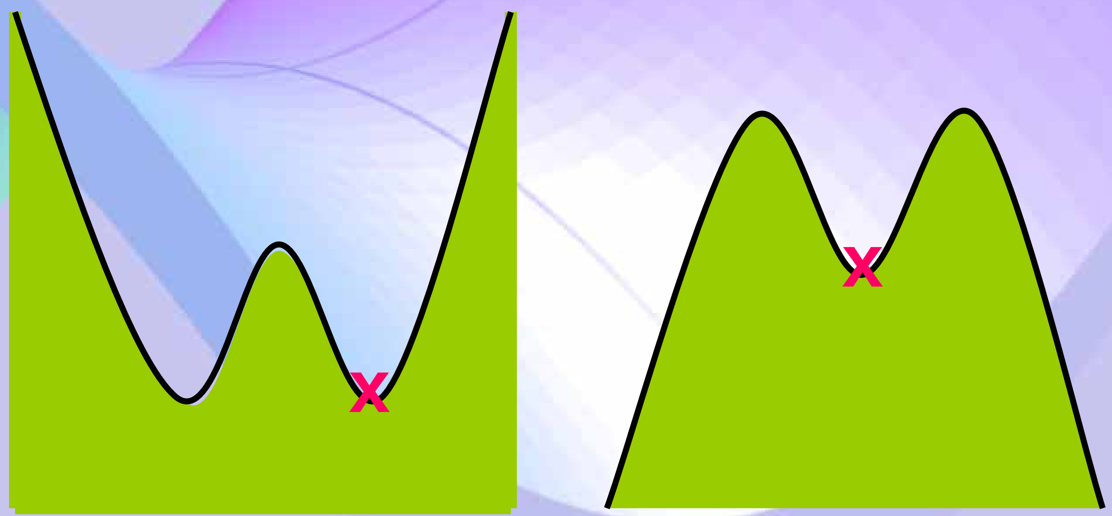
5. Difficulties

Particle masses and interactions appear to violate the symmetry:

$$V(\phi) \leftrightarrow -V(\phi)$$

Higgs potential:

Perhaps there is a Higgs *pair* ?



The pure Maxwell case can be done, but gauge fields ??

$$D_\mu = \partial_\mu + ig\, A_\mu^a T^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + ig\, f^{abc}\, A_\mu^b\, A_\nu^c$$

$$\partial_\mu \to i\,\partial_\mu$$

$$A_\mu^a \to iA_\mu^a$$

$$f^{abc} \to i\, f^{abc}$$

The image of the gauge group then becomes non-compact ...

The $x \leftrightarrow ix$ symmetry might work, but only if, at some scale of physics, Yang-Mills fields become emergent ...

questions: relation with supersymmetry ... ?
interactions ...
renormalization group ....

The END