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Top Quark Physics

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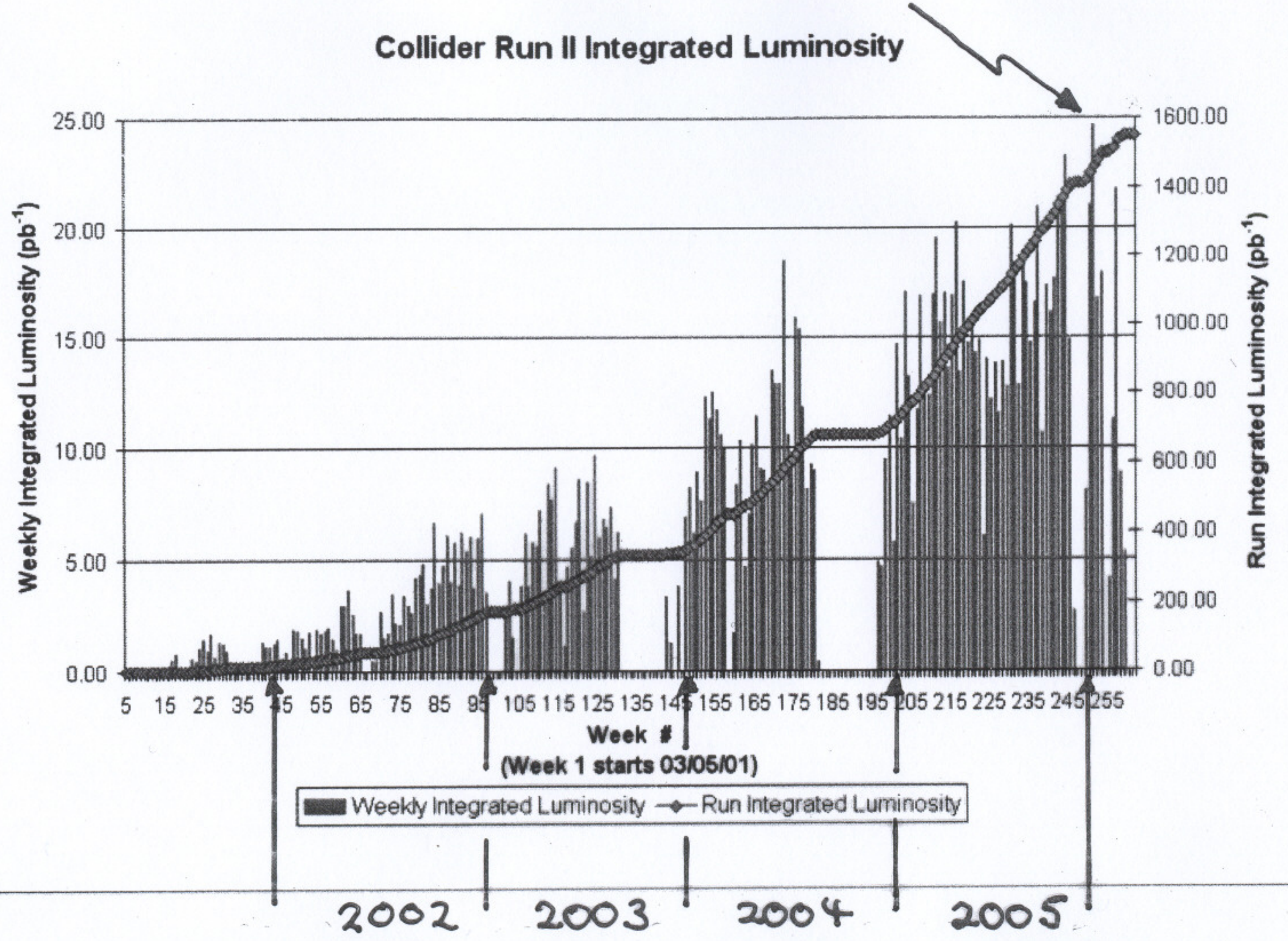
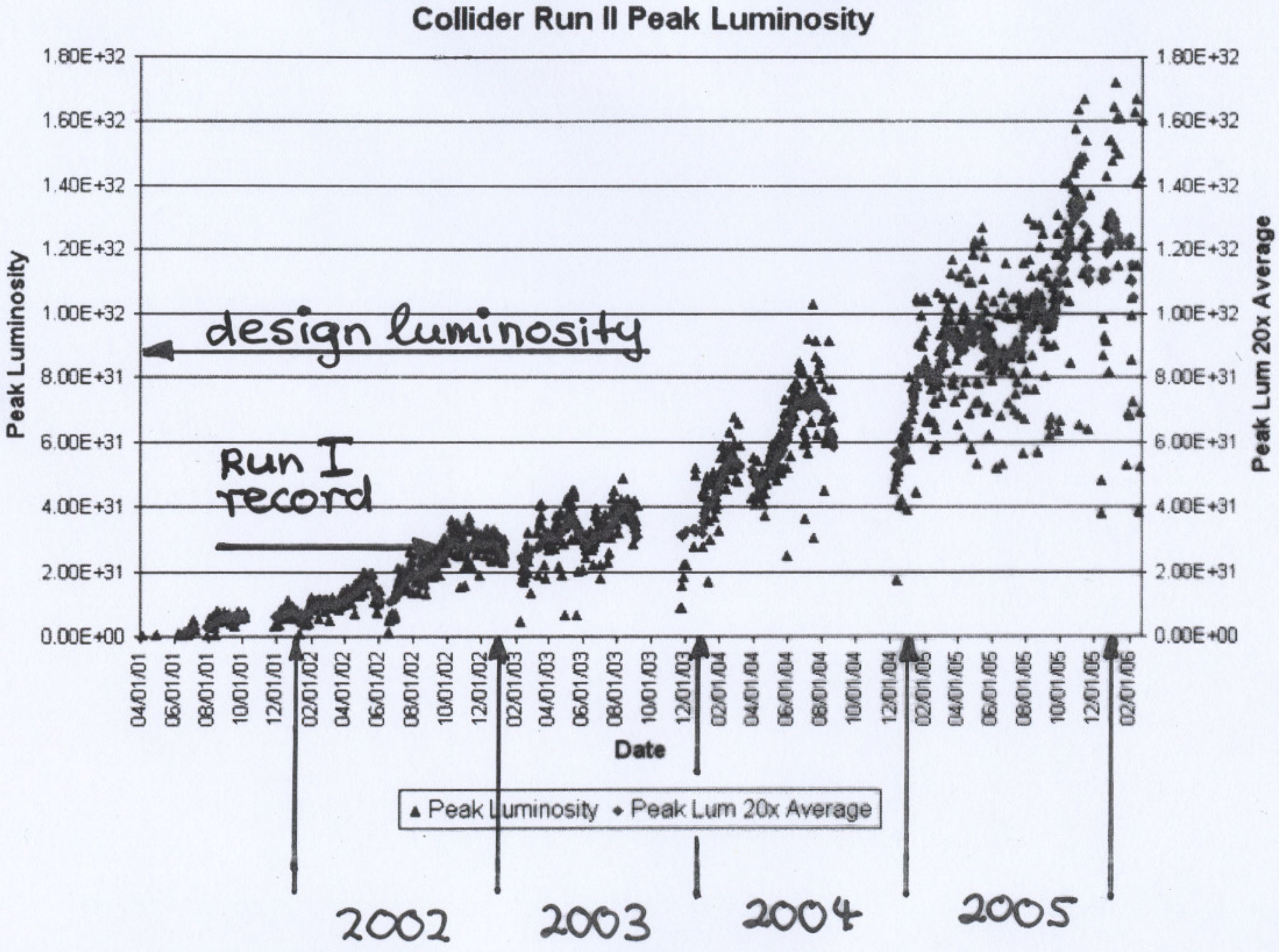
If Tevatron performs
as here can reach 1.3 fb^{-1}/y





design luminosity

Run I record

2002

2003

2004

2005

TOP QUARK YIELD

PAST

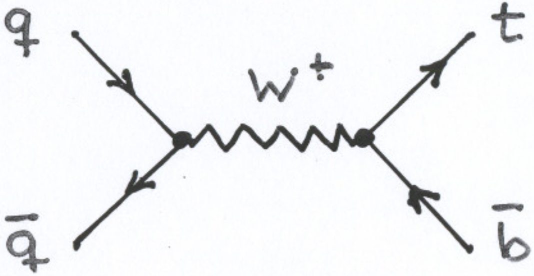
- Top quark discovered by CDF and D0 (Tevatron) collaborations in 1995
- Run I Tevatron $\sqrt{s} = 1.8$ TeV
  approx 500 $t\bar{t}$ pairs at each detector (CDF, D0)

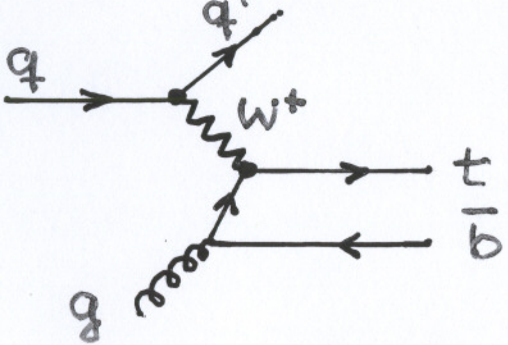
PRESENT

- Run II Tevatron $\sqrt{s} = 2.0$ TeV (started 2001)
- $\sigma(t\bar{t}) \simeq 6.8$ pb
  For integrated luminosity of 1 fb^{-1} around 7000 $t\bar{t}$ pairs expected
- $\sigma(t) \simeq 2.5$ pb (not yet seen)

↑
approx. 40% of $t\bar{t}$

- singly produced top quark expected to be almost <u>100%</u> polarized (weak production!)





- top quark mass

current world average $\underline{m_t = 172.7 \pm 2.9 \text{ GeV}}$

expected uncertainty at end of run ($4-8 fb^{-1}$):

$$\underline{\delta m_t \simeq 1.3 \text{ GeV}}$$

FUTURE

A. LHC to start in 2007

13% $q\bar{q} \rightarrow t\bar{t}$
87% $gg \rightarrow t\bar{t}$

- $\sigma(t\bar{t}) \simeq 800$ pb
  $\sigma(t) \simeq 300$ pb
- cross section 100 - fold increased
  luminosity 10 - fold increased (low luminosity run)

  ↷ $\simeq 10^7$ $t\bar{t}$ - pairs per year
  (one $t\bar{t}$ - pair every 4 s)
- later : high luminosity run
  - first years : low luminosity run $10\ fb^{-1}/y$
  - later : high luminosity run $100\ fb^{-1}/y$

B. International Linear Collider (ILC)

possibly starting in 2015

- $(1-4) \times 10^5$ $t\bar{t}$-pairs/y

  depending on $\sqrt{s}$

  $\sqrt{s} = 350, 500, 800$ GeV

- high degree of polarization of top through tuning of beam polarizat

TOTAL TOP QUARK WIDTH

$$\Gamma_t = \underset{\text{1.54 GeV}}{\Gamma_t^{\text{Born}}} \Big(1 \underset{\substack{\text{QCD (one-loop)} \\ \text{NLO}}}{- 8.5\%} \underset{\substack{\text{electroweak (one-loop)} \\ \text{NLO}}}{- 1.54\%}$$

$$\underset{\substack{\text{QCD (two-loop)} \\ \text{NNLO}}}{- 2.12\%} \underset{m_b \neq 0}{- 0.27\%} \underset{\text{finite width of W}}{- 1.55\%} \Big)$$

$$= \Gamma_t^{\text{Born}} \cdot 0.89$$

$$= 1.37 \text{ GeV}$$

Top quark width not measurable at hadron colliders ($\tau_t = 4.8 \cdot 10^{-25} s$). Must wait for ILC to do measurements in $t\bar{t}$ threshold region.

$$\tau_B \simeq 10^{-12} s$$

MISCELLANEOUS REMARKS

- Top quark decays almost 100% to $t \to b + W^+$

  Unitarity of KM-matrix: $|V_{bt}|^2 + \underbrace{|V_{bc}|^2}_{(0.04)^2} + \underbrace{|V_{bu}|^2}_{(0.004)^2} = 1$

  D0 (2006): $V_{tb} > 0.78$ @ 95% C.L.

- Standard Model (SM) coupling $t \to b + W^+$ is left-chiral ($\gamma^\mu(1-\gamma_5)$). The W^+ and b-quark will be in a polarized state.

- Top quark retains its polarization from birth till it decays. It decays before it can hadronize:

$$\tau_{top} \simeq \frac{1}{1.37 \text{ GeV}} \ll \frac{1}{\Lambda_{QCD}}$$

↳ hadronization scale $\simeq$ 300 MeV

- Polarized top quark decay $t(\uparrow) \to b + W^+ (\to \ell \nu_e)$ described by 3 unpolarized and 5 polarized structure functions

$t\bar{t}$ PRODUCTION CROSS SECTION

- NLO QCD (one-loop) (1988)

  including spin-spin correlation effects (2001)

- Next-to-Next-to-Leading Log NNLL (2003)

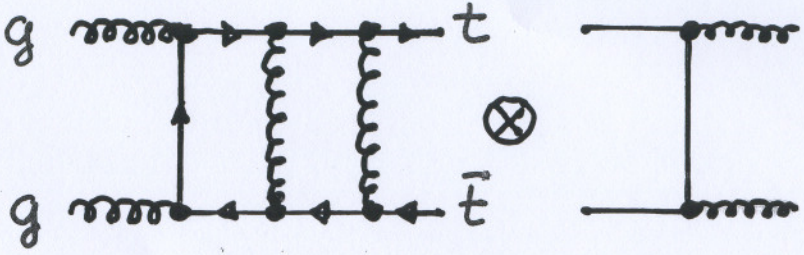
- NNLO QCD (two-loop)

  Challenge of the future

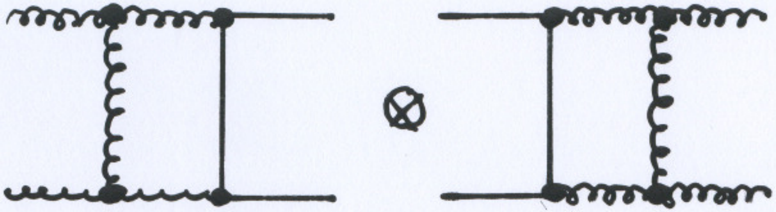
  Will reduce theoretical errors due to renormalization and factorization scale dependence

  Will require the effort of many theorists

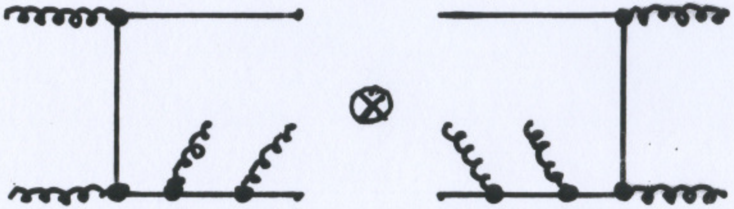
GENERIC NNLO QCD DIAGRAMS $\mathcal{O}(\alpha_s^4)$



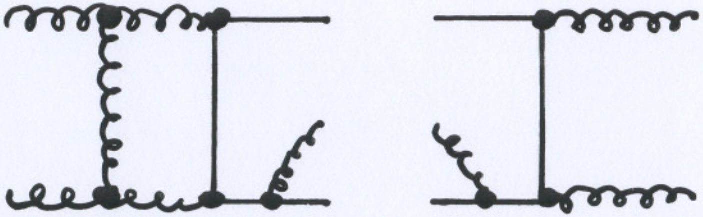
two-loop-by-Born



loop-by-loop



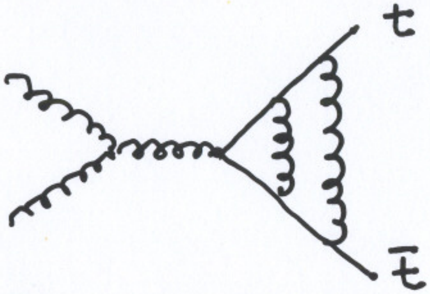
tree-by-tree



one-loop-by-tree

A FEW PARTIAL RESULTS EXIST:

1. Two-loop contribution



(Aachen, Freiburg, Zurich, UCLA, Bologna (2005))

results expressed in terms of <u>1-dimensional harmonic polylogarithms</u> of maximum weight four

2. loop-by-loop contributions (Mainz 2005-2006) including spin

results expressed in terms of <u>multiple polylogarithms</u> of maximum depth and weight four.

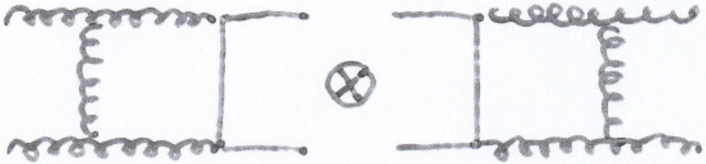
Partial results give a glimpse of the function structure that appears at NNLO

* 1) is a two mass scale problem (s, m_t)
versus 2) (s, t, m_t)

$O(\alpha_s^4)$ LOOP-BY-LOOP CONTRIBUTION

Z. Merebashvili, M. Rogal, JGK
(2005-2006)

$$\Big(\underbrace{\frac{a_{-2}}{\varepsilon^2}+\frac{a_{-1}}{\varepsilon}+a_0}_{\text{known}}+\underbrace{a_1\varepsilon+a_2\varepsilon^2}_{\text{new}}+\ldots\Big)\Big(\frac{a_{-2}}{\varepsilon^2}+\frac{a_{-1}}{\varepsilon}+a_0+a_1\varepsilon+a_2\varepsilon^2+\ldots\Big)^*$$



$O(\varepsilon^2)$ terms come from

i) spin algebra

$$g_{\mu\nu}g^{\mu\nu}=n=4-2\varepsilon$$

ii) Passarino-Veltman decomposition of tensor integrals

$$g_{\mu\nu}g^{\mu\nu}=n=4-2\varepsilon$$

iii) scalar master one-loop integrals

MULTIPLE POLYLOGARITHMS (Goncharov 1998)

$$Li_{m_k,\cdots,m_1}(x_k,\dots x_1) = \int_0^{x_1x_2\cdots x_k} \left(\frac{dt}{t}\circ\right)^{m_1-1} \frac{dt}{x_2x_3\cdots x_k - t}\circ$$

$$\left(\frac{dt}{t}\circ\right)^{m_2-1} \frac{dt}{x_3\cdots x_k - t}\circ\cdots\circ\left(\frac{dt}{t}\circ\right)^{m_k-1}\frac{dt}{1-t}$$

where the iterated integrals are defined by

$$\int_0^{\lambda} \frac{dt}{a_n-t}\circ\cdots\cdots\frac{dt}{a_1-t} = \int_0^{\lambda}\frac{dt_n}{a_n-t_n}\int_0^{t_n}\frac{dt_{n-1}}{a_{n-1}-t_{n-1}}\times\cdots\times\int_0^{t_2}\frac{dt_1}{a_1-t_1}$$

weight : $w = m_1 + m_2 + \cdots m_k$

depth : k

m_i and k positive integers

MULTIPLE POLYLOGARITHMS (Goncharov 1998)

$$\mathrm{Li}_{m_k,\dots,m_1}(x_k,\dots x_1) = \int_0^{x_1 x_2 \dots x_k} \left(\frac{dt}{t}\circ\right)^{m_1-1} \frac{dt}{x_2 x_3 \cdots x_k - t} \circ$$

$$\left(\frac{dt}{t}\circ\right)^{m_2-1} \frac{dt}{x_3 \cdots x_k - t} \circ \dots \circ \left(\frac{dt}{t}\circ\right)^{m_k-1} \frac{dt}{1-t}$$

where the iterated integrals are defined by

$$\int_0^\lambda \frac{dt}{a_n - t} \circ \dots \frac{dt}{a_1 - t} = \int_0^\lambda \frac{dt_n}{a_n - t_n} \int_0^{t_n} \frac{dt_{n-1}}{a_{n-1} - t_{n-1}} \times \dots \times \int_0^{t_2} \frac{dt_1}{a_1 - t_1}$$

weight : $w = m_1 + m_2 + \cdots m_k$

depth : k

m_i and k positive integers

SPECIAL CASES OF MULTIPLE POLYLOGARITHMS

- classical polylogarithms

$$\mathrm{Li}_n(z) = \int_0^z \frac{\mathrm{Li}_{n-1}(x)}{x}\,dx \quad n \geq 2 \quad ; \quad \mathrm{Li}_1(z) = -\ln(1-z)$$

multiple polylogarithms of weight n and depth 1

- Nielsen's generalized polylogarithms

- one-dimensional harmonic polylogarithms

  (Remiddi , Vermaseren 2000)

- two-dimensional harmonic polylogaritms

  (Gehrmann , Remiddi 2001)

a very efficient program for the numerical evaluation of multiple polylogarithms has been written by Weinzierl & Vollinga, (Mainz) based on GiNaC .

UNPOLARIZED TOP QUARK DECAYS

$$d\Gamma \sim \underset{\uparrow \text{ hadron tensor}}{H^{\mu\nu}} \; \underset{\uparrow \text{ lepton tensor}}{L_{\mu\nu}}$$

Hadron tensor described by three invariant structure functions

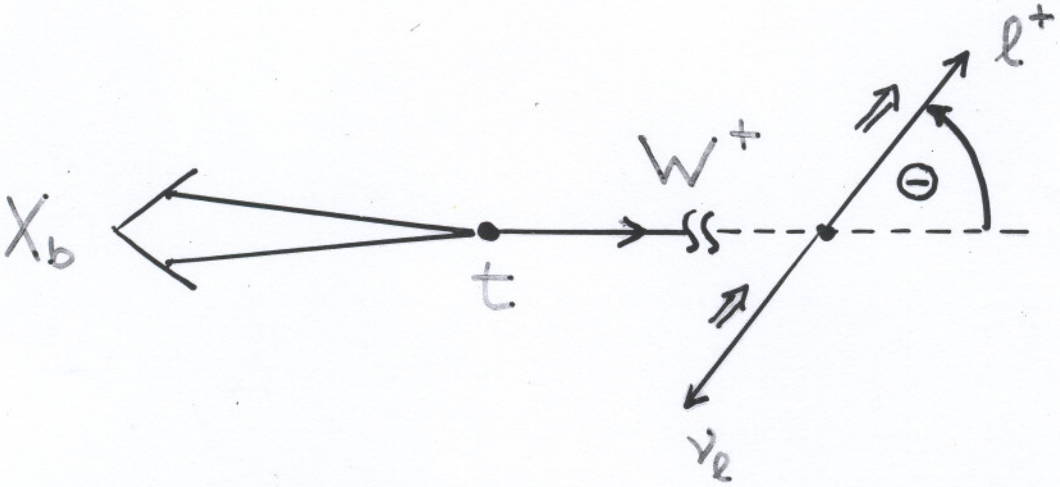
$$\boxed{H^{\mu\nu} = -g^{\mu\nu} H_1 + p_t^{\mu} p_t^{\nu} H_2 - i \varepsilon^{\mu\nu\rho\sigma} p_{t\rho} q_\sigma H_3}$$

or, by three helicity structure functions

$$\begin{aligned} H_+ &= H_1 + |\vec{q}| m_t H_3 \\ H_- &= H_1 - |\vec{q}| m_t H_3 \\ H_L &= H_1 + |\vec{q}|^2 \frac{m_t^2}{m_W^2} H_2 \end{aligned} \qquad \overset{\text{Born}}{\boxed{\begin{aligned} &= 0 \qquad x^2 = \frac{m_W^2}{m_t^2} \\ &= 2 m_t^2 (1-x^2) \\ &= m_t^2 \frac{1}{x^2} (1-x^2) \end{aligned}}}$$

The aim is to measure the three structure functions or moments of them.

Unpolarized top decay



$$\boxed{\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\Theta} = \; & \mathcal{F}_+ \; \tfrac{3}{8}(1+\cos\Theta)^2 \quad \text{harder} \\ & + \mathcal{F}_- \; \tfrac{3}{8}(1-\cos\Theta)^2 \quad \text{softer} \\ & + \mathcal{F}_L \; \tfrac{3}{4}\sin^2\Theta\end{aligned}}$$

$$\mathcal{F}_+ + \mathcal{F}_- + \mathcal{F}_L = 1 \qquad \mathcal{F}_+ = H_+/\Gamma$$

etc.

Measurement errors

	$0.1\,fb^{-1}$	$1\,fb^{-1}$	$10\,fb^{-1}$	$100\,fb^{-1}$	$\mathcal{O}(\alpha_s)$
$\delta\mathcal{F}_L$	50%	6.5%	2.1%	0.7%	-1.1%
$\delta\mathcal{F}_+$	15%	2.6%	0.8%	0.3%	0.1%

RUN I RUN II LHC

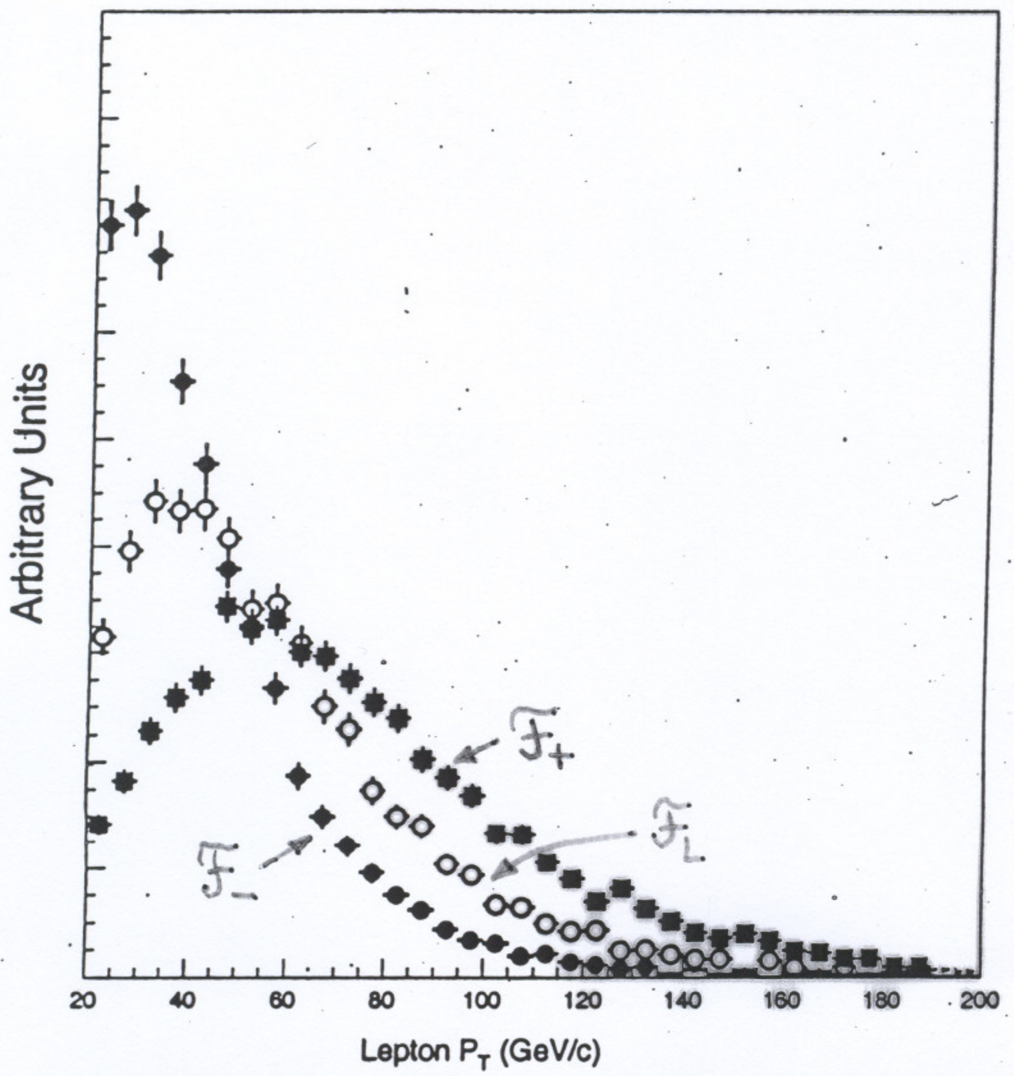
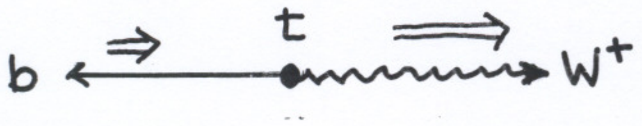


FIG. 1. Lepton P_T distributions for the three W helicities. The solid circles are from negative helicity W^+ and positive helicity W^-, the open circles are from longitudinal W^+ and W^-, and the closed squares are from positive helicity W^+ and negative helicity W^-. All three distributions are normalized to the same area.

ALLOWED AND FORBIDDEN HELICITIES OF W^+ ($m_b=0$)

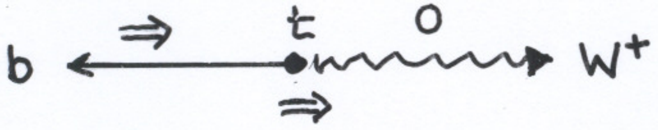
BORN TERM

$\mathcal{F}_+$:



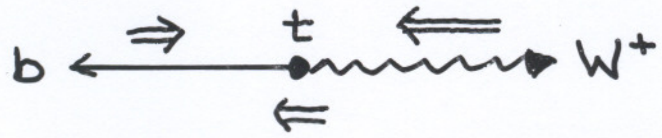
forbidden 0

$\mathcal{F}_0$:



allowed $= \frac{1}{1+2x^2}$

$\mathcal{F}_-$:



allowed $= \frac{2x^2}{1+2x^2}$

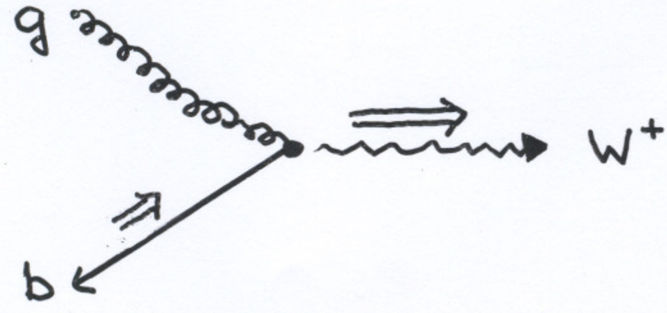
$x = \frac{m_W}{m_t}$

$\mathcal{F}_+ = 0$

$\mathcal{F}_0 = 0.703$

$\mathcal{F}_- = 0.297$

$O(\alpha_s)$



allowed

Experimental RESULTS ON $\mathcal{F}_+ , \mathcal{F}_- , \mathcal{F}_0$

$$\mathcal{F}_+ = \frac{\Gamma_+}{\Gamma} \qquad \mathcal{F}_- = \frac{\Gamma_-}{\Gamma} \qquad \mathcal{F}_0 = \frac{\Gamma_L}{\Gamma}$$

- CDF at Tevatron II (2006)

$$\mathcal{F}_0 = 0.74 {}^{+0.22}_{-0.34}$$

$$\mathcal{F}_+ = 0.00 {}^{+0.20}_{-0.19}$$

based on integrated luminosity of 200 pb^{-1}

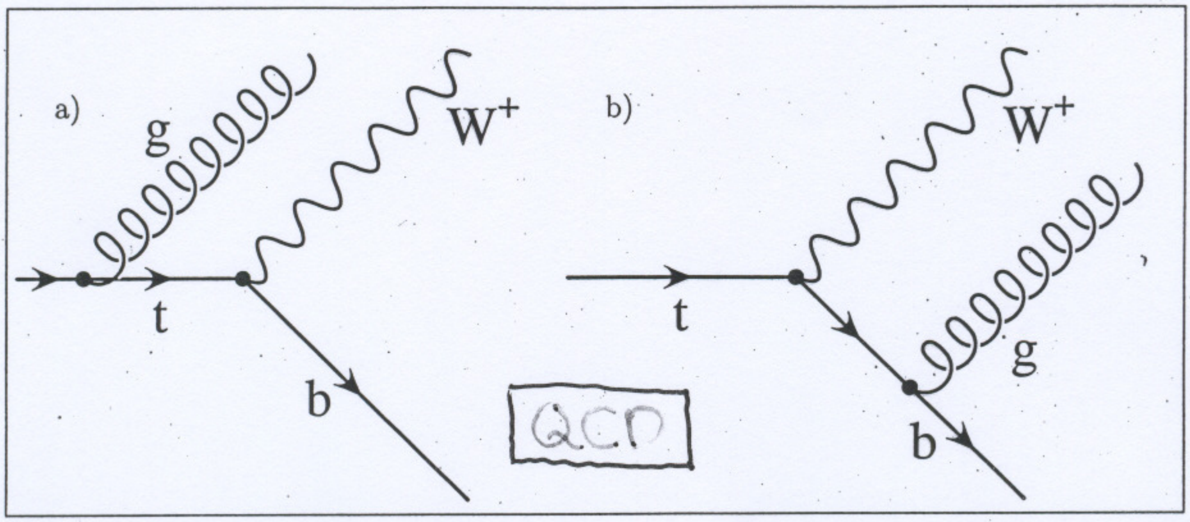
- D0 at Tevatron II (2006)

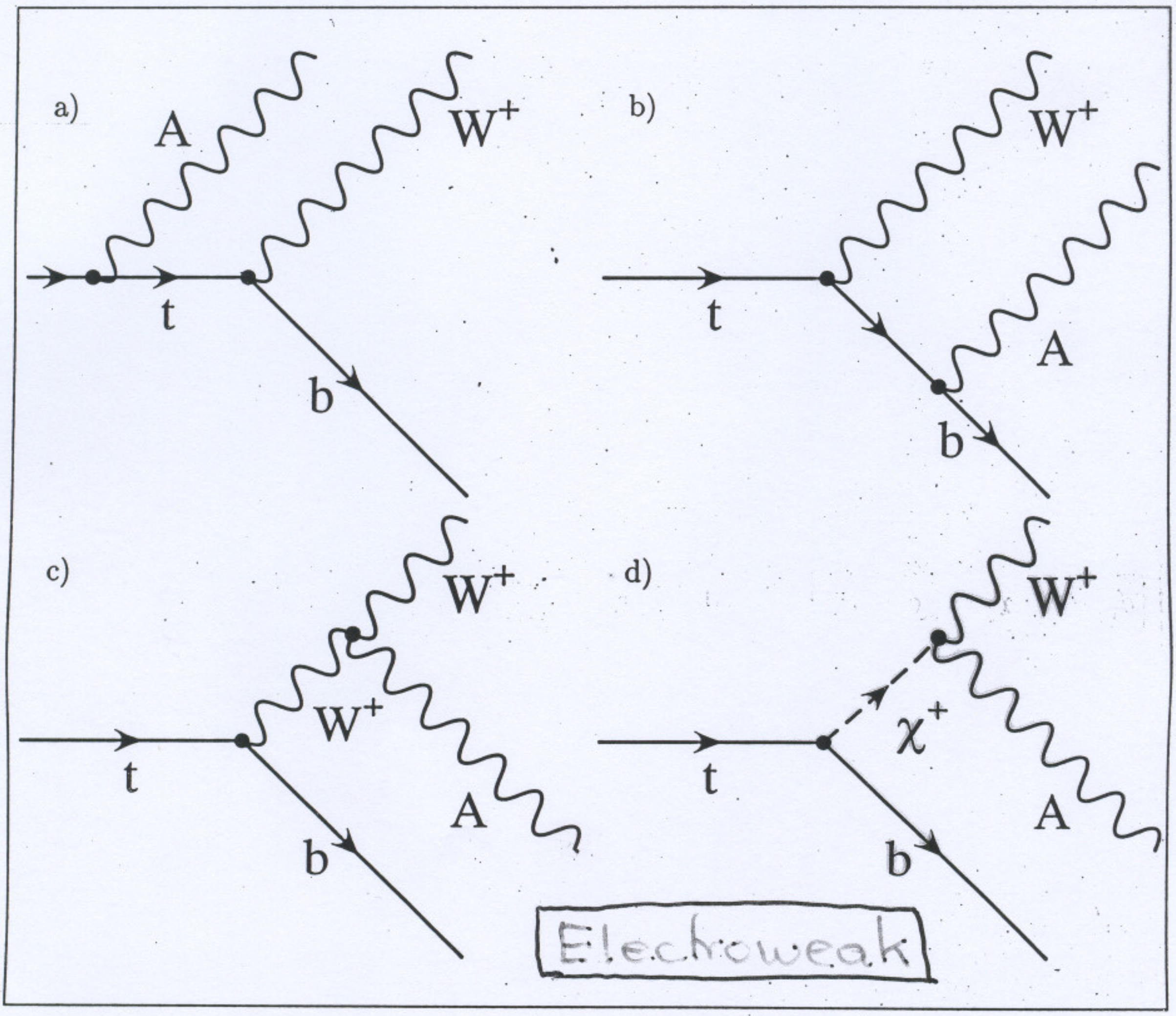
$$\mathcal{F}_+ = 0.00 \pm 0.13 \,(\text{stat.}) \pm 0.07 \,(\text{syst.})$$

based on 230 pb^{-1}

Measurements compatible with SM results

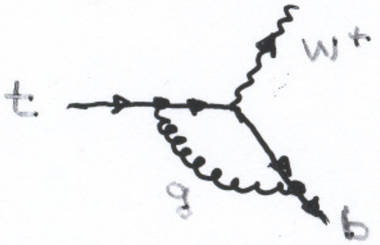
Tree-graph contributions



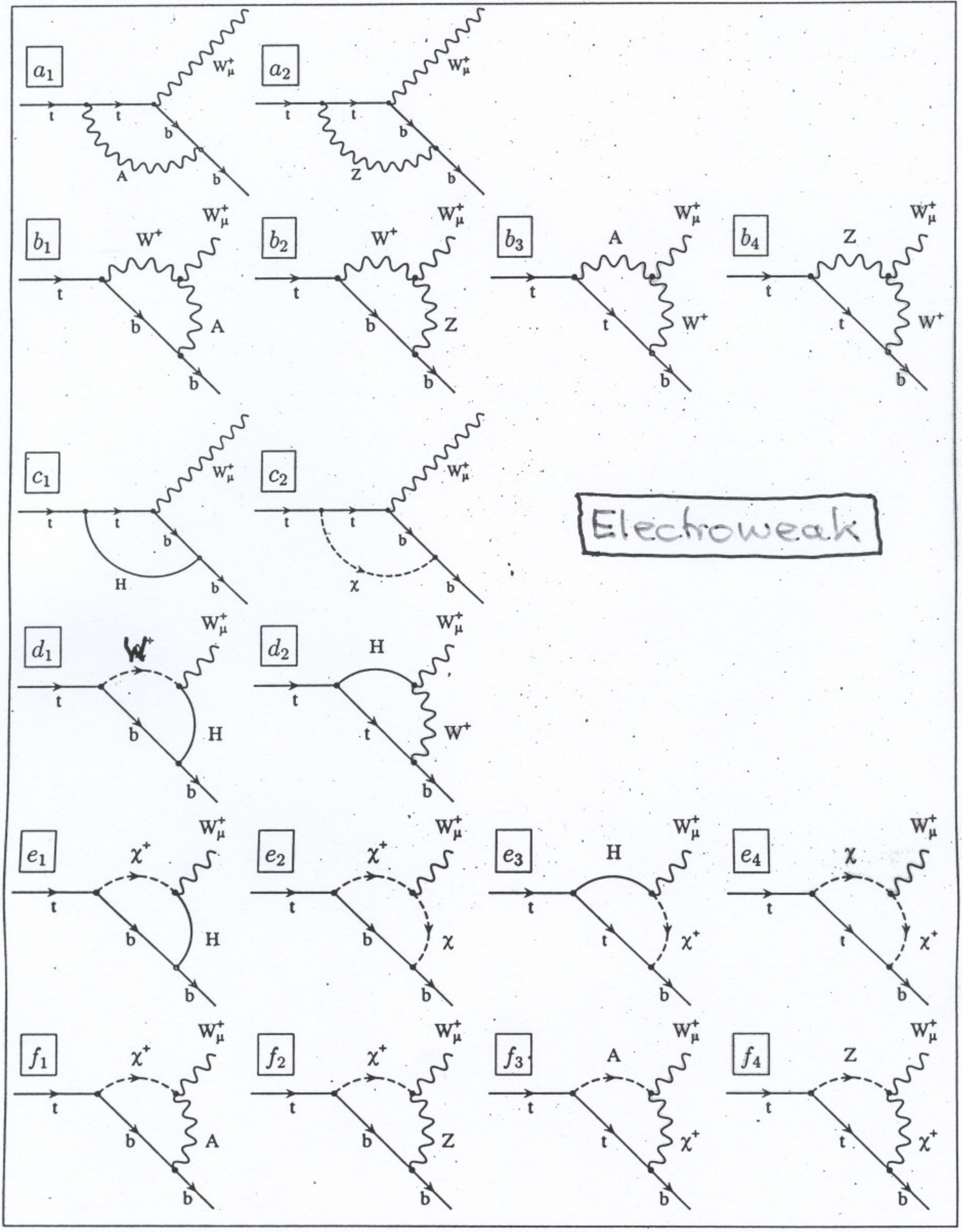


Feynman-'t Hooft gauge

One-loop contributions (vertex)



QCD



$\chi^\pm$: charged Goldstone boson
χ_3 : neutral Goldstone boson
H : physical Higgs

Feynman-'t Hooft gauge

SAMPLE RESULTS

QCD (one-loop) + Electroweak (one-loop)

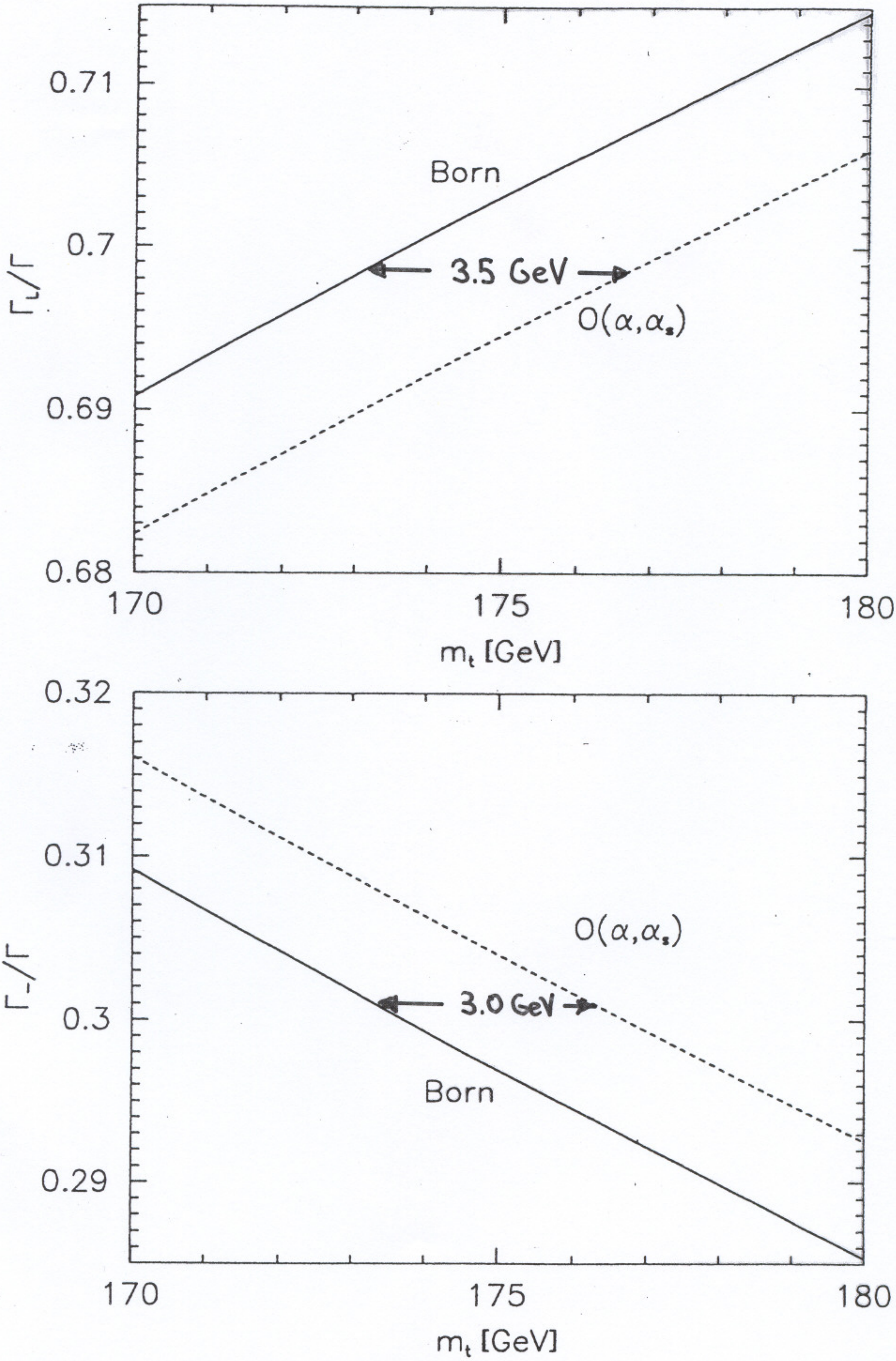
$$\mathcal{F}_+ = 0.00136$$

- can be pretty sure that if experimentally $\mathcal{F}_+ \cong 1\%$ this must be due to new physics and <u>not</u> radiative corrections
- right-handed current

$$\bar{u}_b \gamma^\mu (1-\gamma_5) u_t \to \bar{u}_b \left[\gamma^\mu (1-\gamma_5) + \underline{\delta_R \gamma^\mu (1+\gamma_5)} \right] u_t$$

  - $\mathcal{F}_+ = 1\%$ corresponds to $\delta_R = \begin{matrix} 0.18 \\ -0.15 \end{matrix}$
  - $\delta_R = 0.03$ leads to $\mathcal{F}_+ = 0.00122$

$\uparrow$ indirect model dependent bound $\delta_R \leq 0.03$ from rare B-decays ($b \to s + \gamma$)



1% rel. error on $\frac{\Gamma_L}{\Gamma}$: $\Delta m_t = 3$ Ge

1% rel. error on $\frac{\Gamma_-}{\Gamma}$: $\Delta m_t = 1.2$ GeV

FIG. 2. Top mass dependences of the ratios Γ_L/Γ and Γ_-/Γ. Full line: Born term. Dashed line: Corrections including (QCD), electroweak (EW), finite-width (FW) and ($m_b \neq 0$) Born term corrections.

POLARIZED TOP QUARK DECAYS

$$t(\uparrow) \to X_b + W^+(\to \ell^+ \nu_\ell)$$

- singly produced top quarks are almost 100 % polarized
- $\sigma_t \simeq \frac{1}{2} \sigma_{t\bar{t}}$ in hadronic production
- top retains its polarization since it decays before it can hadronize

<u>Angular decay distribution</u> :

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta_P \, d\cos\theta \, d\phi} = \frac{1}{4\pi} \Big\{ & \frac{3}{8} \big(\Gamma_{\underline{U}} + P_t \cos\theta_P \, \Gamma_{\underline{\underline{UP}}}\big)(1+\cos^2\theta) \\ & + \frac{3}{4} \big(\Gamma_{\underline{L}} + P_t \cos\theta_P \, \Gamma_{\underline{\underline{LP}}}\big) \sin^2\theta \\ & + \frac{3}{4} \big(\Gamma_{\underline{F}} + P_t \cos\theta_P \, \Gamma_{\underline{\underline{FP}}}\big) \cos\theta \\ & + \frac{3}{2\sqrt{2}} \Gamma_{\underline{\underline{IP}}} \, P_t \sin\theta_P \sin 2\theta \cos\phi \\ & + \frac{3}{\sqrt{2}} \Gamma_{\underline{\underline{AP}}} \, P_t \sin\theta_P \sin\theta \cos\phi \Big\}\end{aligned}$$

$P_t = |\vec{P}_t|$: polarization of top

previously
$$\Gamma_+ = \frac{1}{2}(\Gamma_U + \Gamma_F)$$
$$\Gamma_- = \frac{1}{2}(\Gamma_U - \Gamma_F)$$

- three unpolarized + five polarized structure functions

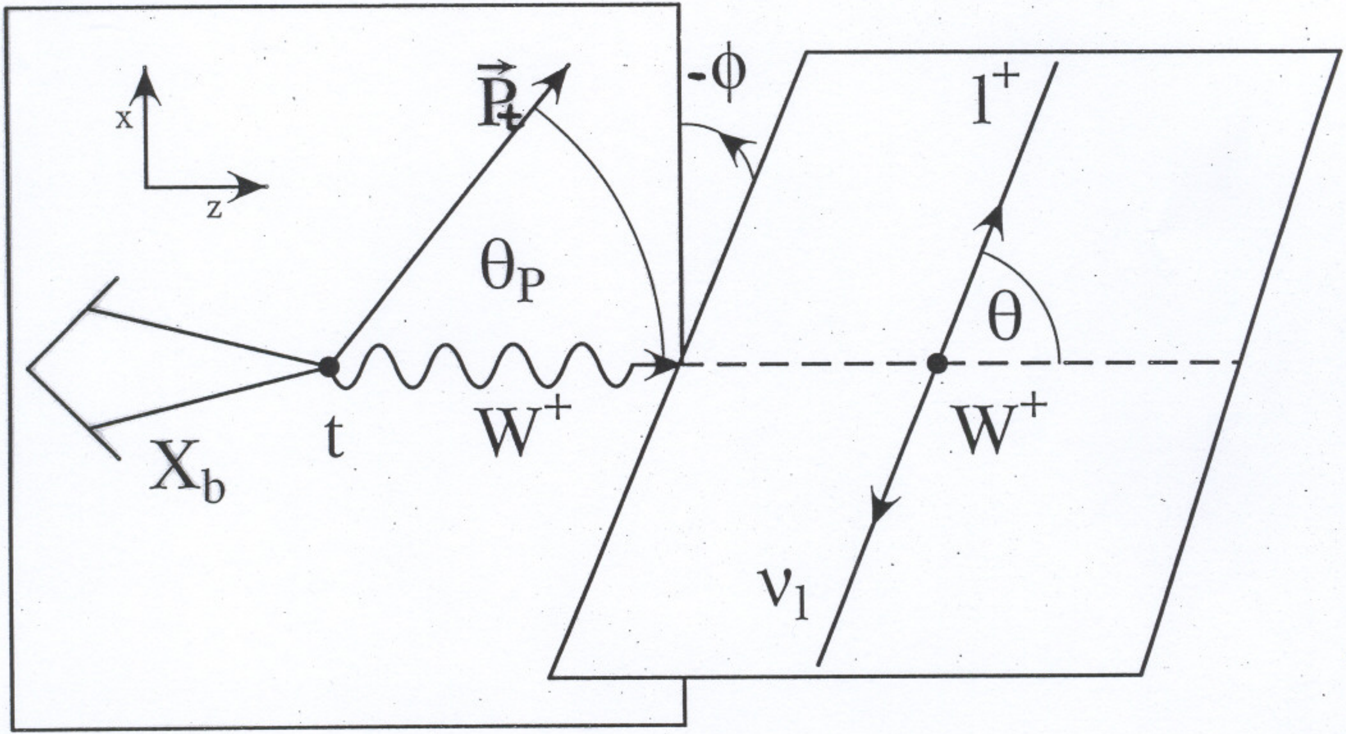
Eight invariant structure functions defined through covariant expansion ($m_\ell = 0$) (no imaginary part of one-loop contribution)

$$
\begin{aligned}
H^{\mu\nu}(q^2, q_0) = & -g^{\mu\nu}\underline{H_1} + p_t^{\mu} p_t^{\nu}\underline{H_2} \\
& - i\varepsilon^{\mu\nu\rho\sigma} p_{t\rho} q_\sigma \underline{H_3} \\
& - q\cdot s_t \left(-g^{\mu\nu}\underline{\underline{G_1}} + p_t^{\mu} p_t^{\nu}\underline{\underline{G_2}} \right. \\
& \qquad \left. - i\varepsilon^{\mu\nu\rho\sigma} p_{t\rho} q_\sigma \underline{\underline{G_3}} \right) \\
& + (s_t^{\mu} p_t^{\nu} + s_t^{\nu} p_t^{\mu})\underline{\underline{G_6}} \\
& + i\varepsilon^{\mu\nu\rho\sigma} p_{t\rho} s^t_\sigma \underline{\underline{G_8}} \\
& (+ i\varepsilon^{\mu\nu\rho\sigma} q_\rho s^t_\sigma \underline{\underline{G_9}})^*
\end{aligned}
$$

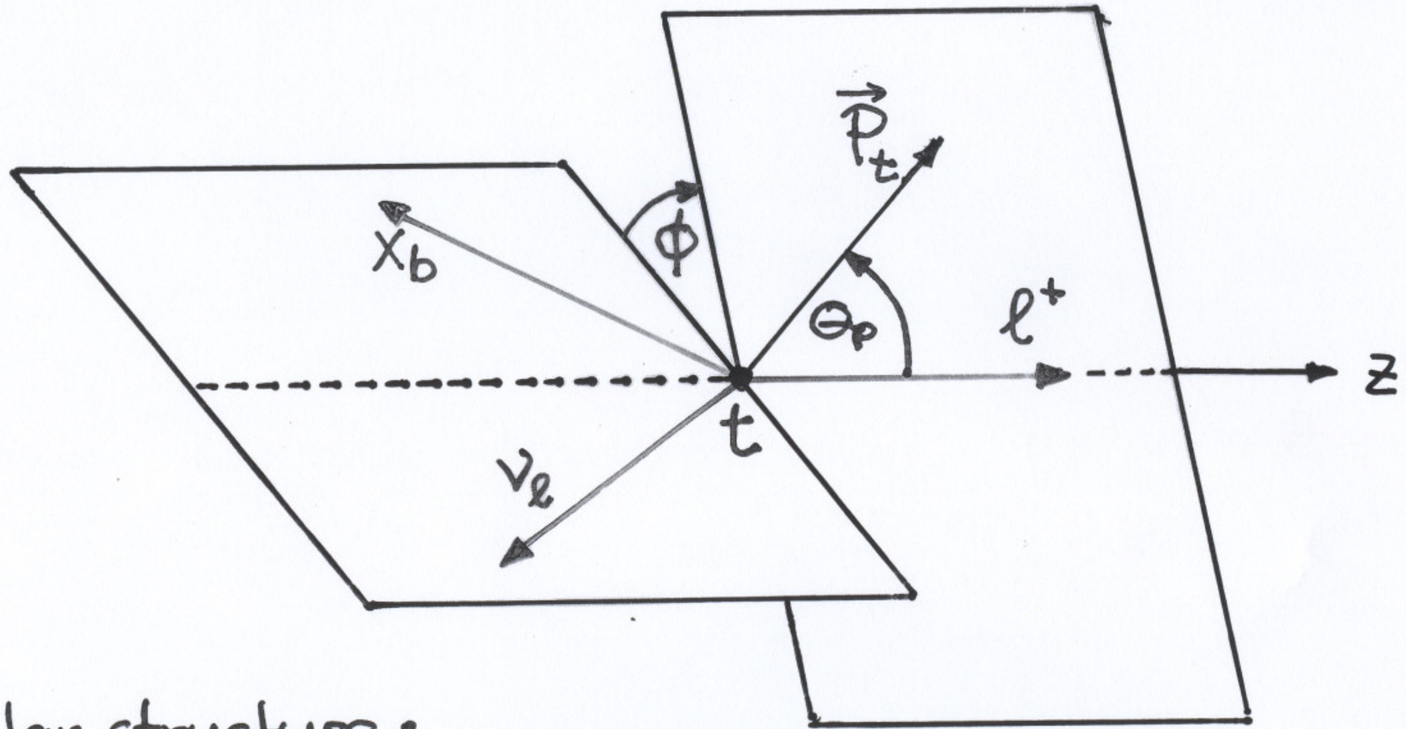
Eight helicity structure functions are linearly related through

$$
\begin{aligned}
\underline{H_U} &= 2H_1 \\
m_W^2\,\underline{H_L} &= m_W^2 H_1 + |\vec{q}|^2 m_t^2 H_2 \\
\underline{H_F} &= 2|\vec{q}|\, m_t H_3 \\
\underline{\underline{H_U^P}} &= 2|\vec{q}|\, G_1 \\
m_W^2\,\underline{\underline{H_L^P}} &= |\vec{q}|\,(m_W^2 G_1 + |\vec{q}|^2 m_t^2 G_2 - 2q_0 m_t G_6) \\
\underline{\underline{H_F^P}} &= 2|\vec{q}|^2 m_t G_3 - 2m_t G_8 - 2q_0 G_9 \\
\underline{\underline{H_I^P}} &= \frac{1}{\sqrt{2}}\frac{m_t}{m_W}|\vec{q}|\, G_6 \\
\underline{\underline{H_A^P}} &= -\frac{1}{\sqrt{2}}\frac{m_t q_0}{m_W} G_8 - \frac{1}{\sqrt{2}} m_W G_9
\end{aligned}
$$

* not linearly independent (Schouten identity) kept for technical reasons


x
z
$\vec{P}_t$
$-\phi$
l^+
θ_P
θ
X_b
t
W^+
W^+
ν_l

REST FRAME DECAY OF POLARIZED TOP QUARK



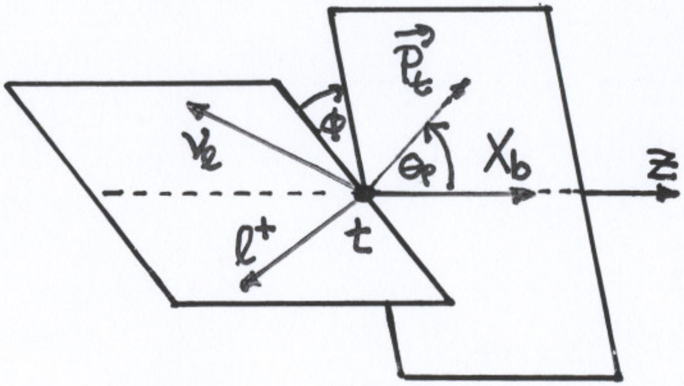
Angular structure:

$$\frac{d\Gamma}{dx\, d\cos\theta\, d\phi} = \frac{1}{4\pi}\left(\frac{d\Gamma_A}{dx} + P\left(\frac{d\Gamma_B}{dx}\cos\theta_p + \frac{d\Gamma_C}{dx}\sin\theta_p\cos\phi\right)\right)$$

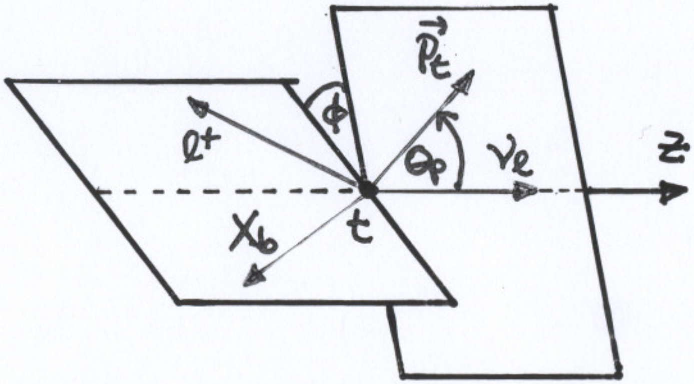
$$\boxed{\text{curiosity: } \frac{d\Gamma_C(\text{Born})}{dx} = 0}$$

$$x = \frac{2E_\ell}{m_t}$$

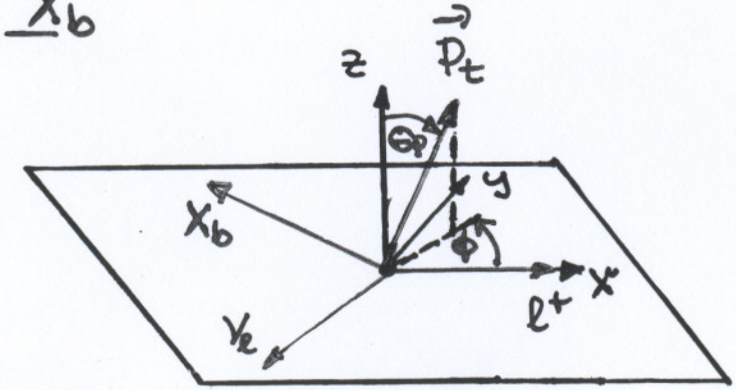
OTHER MEASUREMENTS POSSIBLE



z along X_b



z along ν_ℓ

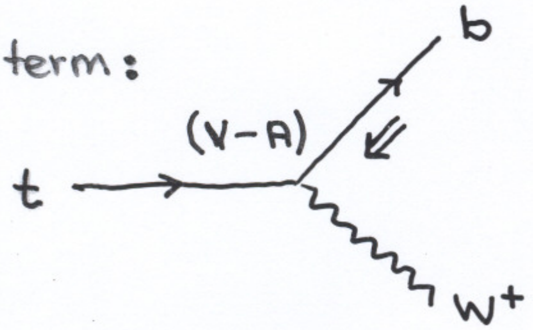


z perpendicular to decay plane

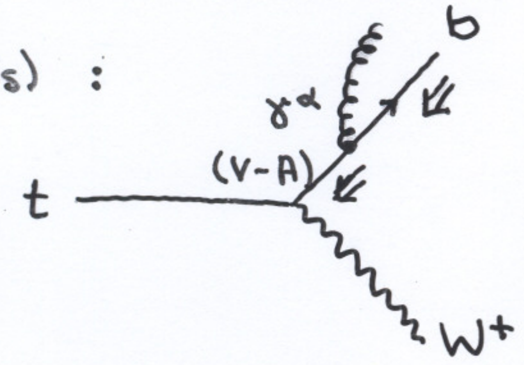
BOTTOM QUARK POLARIZATION ($m_b \to 0$)

- Naively one expects $P_b = -1$ as $m_b \to 0$

Born term:

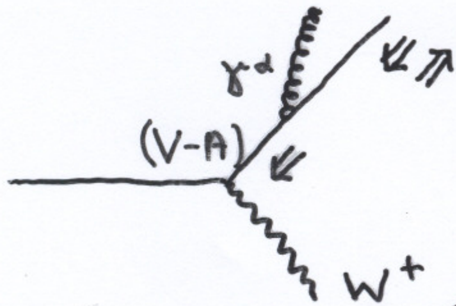


$\mathcal{O}(\alpha_s)$:



γ^α does not flip helicity of massless bottom quark

- In reality there is a helicity flip contribution as $m_b \to 0$ (Lee - Nauenberg 1964)



helicity flip factor

Helicity flip is a m_b/m_b effect

collinear singularity

and survives $m_b \to 0$ limit

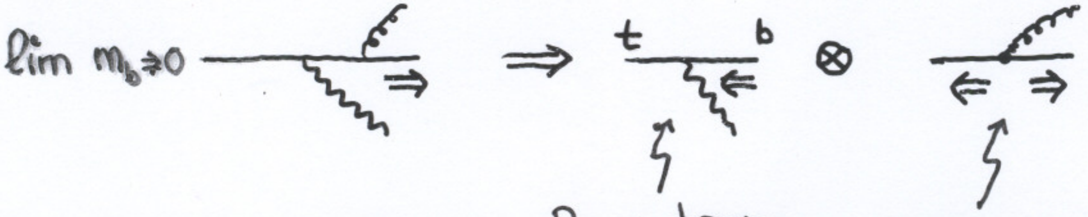
- called anomalous helicity flip contribution since it is not there in a $m_b = 0$ theory.
  Also related to the (VVA)-triangle anomaly in the approach of Dolgov, Zakharov

- Bottom quark polarization for $m_b \to 0$

$$P_b^{\ell} = \frac{\Gamma_{\lambda_b = +\frac{1}{2}} - \Gamma_{\lambda_b = -\frac{1}{2}}}{\Gamma_{\lambda_b = +\frac{1}{2}} + \Gamma_{\lambda_b = -\frac{1}{2}}}$$

$$\to \frac{-1 - \frac{\alpha_s}{2\pi}\left(\frac{21-4\pi^2}{4}\right)}{\underset{\underset{\text{Born term}}{\uparrow}}{1} + \frac{\alpha_s}{2\pi}\left(\frac{25-4\pi^2}{4}\right)} = -1 + \underset{\underset{\text{1.7\% effect}}{\uparrow}}{\frac{\alpha_s}{2\pi}}$$

- Helicity flip contribution can be calculated in the equivalent particle approach of Falk & Sehgal (1994) :



Born term

described by universal splitting function

$$D_{hf} = \frac{\alpha_s}{2\pi} z$$

z : fractional energy of gluon

Helicity flip contribution :

$$\Gamma_{hf} = \int_0^1 dz\, \Gamma^{Born} \frac{\alpha_s}{2\pi} z$$

$$= \Gamma^{Born} \frac{1}{2} \frac{\alpha_s}{2\pi}$$

Polarization of bottom quark :

$$P_b = -1 + \frac{\alpha_s}{2\pi}$$

↳ 1.7% effect

CONCLUSIONS

We are looking forward to a wealth of data on top quark production and decay in the near future.

Thanks to my collaborators :

H. S. Do, M. Fischer, S. Groote, A. Kadeer, D. Kubistin, W. S. Huo, M. Mauser, Z. Merebashvili, M. Rogal