Gravitational Collapse with Negative Energy Field

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I - Introduction

The mathematical framework for our cosmological model has been discussed by Hoyle, Burbidge and Narlikar (1995; HBN hereafter), and we outline briefly its salient features.

To begin with, it is a theory that is derived from an action principle based on Mach's Principle, and assumes that the inertia of matter owes its origin to other matter in the universe.

This leads to a theoretical framework wider than general relativity as it includes terms relating to inertia and creation of matter.

II – The Equations

Thus the equations of general relativity are replaced in the theory by

$$R_{ik} - \frac{1}{2} g_{ik} R + \lambda g_{ik} = 8\,\pi G$$

$$[T_{ik} - f(C_i C_k - \frac{1}{4} g_{ik} C^l C_l)] \qquad (1)$$

With the coupling constant f defined as

$$f = \frac{2}{3\tau^2} \qquad (2)$$

[We have taken the speed of light c = 1.]

Here $\tau = \hbar / mp$ is the characteristic life time of a Plank particle with mass $mp = \sqrt{3\hbar / 8\pi G}$. The gradient of C with respect to spacetime coordinates x^i (i = 0, 1,2,3) is denoted by C_i

II – The Equations

Although the above equation defines *f* in terms of the fundamental constants it is convenient to keep its identity on the right hand side of Einstein's equations since there we can compare the *C*-field energy tensor directly with the matter tensor.

Note that because of positive *f*, the *C*-field has *negative* kinetic energy. Also, the constant λ is *negative* in this theory.

III – Creation Condition

The question now arises of why astrophysical observation suggests that the creation of matter occurs in some places but not in others. For creation to occur at the points A_0, B_0, ….. it is necessary classically that the action should not change (i.e. it should remain stationary) with respect to small changes in the spacetime positions of these points, which can be shown to require

$$C_i\,(A_0)\,C^i\,(A_0) = C_i\,(B_0)\,C^i\,(B_0) = m^2{}_P\,. \qquad (3)$$

III – Creation Condition

This is in general not the case: in general the magnitude of $C_i(X)C^i(X)$ is much less that $m^2{}_P$. However, as one approaches closer and closer to the surface of a massive compact body $C_i(X)C^i(X)$ is increased by a general relativistic time dilatation factor, whereas m_P stays fixed.

This suggests that we should look for regions of strong gravitational field such as those near collapsed massive objects. In general relativistic astrophysics such objects are none other than black holes, formed from gravitational collapse.

IV – Collapse and Bounce

Theorems by Penrose, Hawking and others (see Hawking and Ellis 1973) have shown that provided certain positive energy conditions are met, a compact object undergoes gravitational collapse to a spacetime singularity. Such objects become black holes before the singularity is reached. However, in the present case, the negative energy of the *C*-field intervenes in such a way as to violate the above energy conditions. What happens to such a collapsing object containing *C*-field apart from ordinary matter?

IV – Collapse and Bounce

We conjecture that such an object does not become a black hole. Instead, the collapse of the object is halted and the object bounces back, thanks to the effect of the *C*-field. We will refer to such an object as a compact massive object (CMO) or a near-black hole (NBH). In the following section we discuss the problem of gravitational collapse of a dust ball with and without the *C*-field to illustrate this difference.

V – Dust Ball

Consider how the classical Oppenheimer-Snyder problem of gravitational collapse is changed under the influence of the negative energy C-field. First we describe the classical problem. We write the spacetime metric inside a collapsing dust ball in comoving coordinates (t, r, θ, φ) as

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - \alpha r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right] \qquad (4)$$

where (t, r, θ, φ) are constant for a typical dust particle and t is its proper time. Let the dust ball be limited by $r \leq r_b$.

V – Dust Ball

In the original Oppenheimer-Snyder problem we may describe the onset of collapse at $t = 0$ with $a(0) = 1$ and $\dot{a}(0) = 0$. The starting density ρ_0 is related to the constant α by

$$\alpha = \frac{8\pi G\rho_0}{3} \tag{5}$$

The field equations (1) *without* the *C*-field then tell us that the equation of collapse is given by

$$\dot{a}^2 = \alpha\left(\frac{1-a}{a}\right) \tag{6}$$

and the spacetime singularity is attained $a(t) \longrightarrow 0$ as $t \longrightarrow t_S$, where

$$t_S = \frac{\pi}{2\sqrt{\alpha}} \tag{7}$$

V – Dust Ball

Note that we have ignored the λ - term as it turns out to have a negligible effect on objects of size small compared to the characteristic size of the universe.

The collapsing ball enters the event horizon at a time $t = t_{\mathrm{H}}$ when

$$r_{\mathrm{b}} a(t_H) = 2GM, \tag{8}$$

where the gravitational mass of the dust ball is given by

$$M = \frac{4\pi}{3} r^3_b \rho_0 \quad = \quad \frac{\alpha\, r^3_b}{2G} \tag{9}$$

This is the stage when the ball becomes a black hole.

V – Dust Ball

When we introduce an ambient C -field into this problem, it gets modified as follows. In the homogeneous situation under discussion, C is a function of t only. Let, as before $a(0) = 1\dot{a}(0) = 0$ and let $\dot{C}$ at $t = 0$, be given by β. Then it can be easily seen that the equation (6) is modified to

$$\dot{a}^2 = \alpha\left(\frac{1 - a}{a}\right) - \gamma\left(\frac{1 - a}{a^2}\right) \tag{10}$$

where $\gamma = 2\pi G f \beta^2 > 0$. Also the earlier relation (5) is modified to

$$\alpha = \frac{8\,\pi G \rho_0}{3} - \gamma \tag{11}$$

V – Dust Ball

It is immediately clear that in these modified circumstances $a(t)$ cannot reach zero, the spacetime singularity is averted and the ball bounces at a minimum value $a_{\min} > 0$, of the function $a(t)$.

Writing $\lambda = \gamma / \alpha$**,** we see that the second zero of $\dot{a}(t)$ occurs at $a_{\min} = \lambda$. Thus even for an initially weak C-field, we get a bounce at a finite value of $a(t)$.

VI – Black Hole

But what about the development of a black hole? The gravitational mass of the black hole at any epoch t is estimated by its energy content, i.e., by,

$$M = \frac{4\pi}{3} r^3_b\, a^3(t)\, \{\rho - \frac{4}{3} f \dot{C}^2\}$$

$$= \frac{\alpha\, r^3_b}{2G}\, (\lambda + \lambda - \frac{\lambda}{a}). \qquad (12)$$

VI – Black Hole

Thus the gravitational mass of the dust ball *decreases* as it contracts and consequently its effective Schwarzschild radius decreases. This happens because of the reservoir of negative energy whose intensity rises faster than that of dust density. Such a result is markedly different from that for a collapsing object with positive energy fields only. From (12) we have the ratio

$$\mathrm{F} \equiv \frac{2GM(t)}{r_b a(t)} = \alpha\, r_b^2 \left\{ \frac{1+\lambda}{a} - \frac{\lambda}{a^2} \right\} \qquad (13)$$

Hence

$$\frac{\mathrm{dF}}{\mathrm{d}a} = \frac{\alpha\, r_b^2}{a^2} \left\{ \frac{2\lambda}{a} - (1+\lambda) \right\} \qquad (14)$$

VI – Black Hole

Hence,

$$\frac{dF}{da} = \frac{\alpha\, r^2_b}{a^2} \left\{ \frac{2\lambda}{a} - (1 + \lambda) \right\} \qquad (14)$$

We anticipate that $\lambda << 1$, i.e., the ambient *C*-field energy density is much less than the initial density of the collapsing ball. Thus *F* increases as a decreases and it reaches its maximum value at $a \cong 2\lambda$

This value is attainable, being larger than a_{min}.

Denoting this with F_{max}, we get

$$F_{max} \cong \frac{\alpha\, r^2_b}{4\lambda} \qquad (15)$$

VI – Black Hole

In general $\alpha\, r^2_b \ll 1$ for most astrophysical objects.

For the Sun, $\alpha\, r^2_b \cong 4 \times 10^{-8}$, while for a white dwarf it is ~ 4×10^{-6}. We assume that λ, although small compared to unity, exceeds such values, thus making $F_{max} < 1$.

In such circumstances black holes do not form.

We consider scenarios in which the object soon after bounce picks up high outward velocity. From (10) we see that maximum outward velocity is attained at $a = 2\lambda$ and it is given by

$$\dot{a}^2_{max} \approx \frac{\alpha}{2\lambda} \qquad (16)$$

VII – A White Hole

As $\lambda \ll 1$ we expect $\dot{a}_{max}$ to attain high values. Likewise the *C*-field gradient ($\dot{C}$ in this case) will attain high values in such cases.

Thus, such objects after bouncing at a_{min} will expand and as $a(t)$ increases the strength of the *C*-field falls while for small $a(t)$ $\dot{a}$ increases rapidly as per equation (10). This expansion therefore resembles an explosion. Further, the high local value of the *C*-field gradient will trigger off creation of Planck particles.

VII – A White Hole

It is worth stressing here that even in classical general relativity, the external observer never lives long enough to observe the collapsing object enter the horizon. Thus all claims to have observed black holes in X-ray sources or galactic nuclei really establish the existence of compact massive objects, and as such they are consistent with the NBH concept.

VIII – Conclusion

The introduction of a negative energy field

(1) permits creation of matter near dense regions,

(2) reverses gravitational collapse at finite density,

and

(3) can lead to bounce that develops into an explosive situation.