RECENT APPLICATIONS OF THE WEYL ANOMALY

ISLAMABAD
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33 YEARS OF THE WEYL ANOMALY

1973 2 SALAM PROTEGES (CAPPER & DUFF) ON FIRST POSTDOCS IN TRIESTE:

CONFORMAL INVARIANCE UNDER WEYL RESCALING OF THE METRIC TENSOR

$$g_{\mu\nu}(x) \rightarrow \Omega^2(x)\, g_{\mu\nu}(x)$$

NO LONGER SURVIVE IN THE QUANTUM THEORY $\Rightarrow$ DESER, DUFF & ISHAM

$$g^{\mu\nu}\langle T_{\mu\nu}\rangle = c\,F - a\,G + d\,\Box R \equiv \frac{\mathcal{A}}{2}$$

$$F = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

$$G = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

CFT COUPLED TO GRAVITY

$$a = \frac{1}{720(4\pi)^2}\left[2N_0 + 11N_{1/2} + 124N_1\right]$$

$$c = \frac{3}{2}\alpha = \frac{1}{120(4\pi)^2}\left[N_0 + 3N_{1/2} + 12N_1\right]$$

N_s = NUMBER OF FIELDS WITH SPIN s

a AND c ARE SCHEME-INDEPENDENT BUT α IS NOT (e.g $12N_1$ DIM REG
$-18N_1$ ZETA FN)

CAN ALWAYS ADJUST BY ADDING $\sqrt{g}R^2$

INTERESTING SPECIAL CASE N=4 Y.M.

$U(N)$: $(N_1, N_{1/2}, N_0) = (N^2, 4N^2, 6N^2)$

$$a = c = N^2/4(4\pi)^2$$

$\mathcal{N} = 4$ U(N) YANG-MILLS

INTERESTING SPECIAL CASE OF CFT

$$(N_1, N_{1/2}, N_0) = (N^2, 4N^2, 6N^2)$$

$$a = c = \frac{N^2}{64\pi^2}$$

$$\mathcal{A} = \frac{N^2}{32\pi^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)$$

TWENTY YEARS OF THE WEYL ANOMALY

DEDICATED TO ABDUS SALAM

TRIESTE 93

BLACK HOLE PHYSICS

COSMOLOGY

STRING THEORY

STATISTICAL MECHANICS

RECENT APPLICATIONS

1998 THE "HOLOGRAPHIC" WEYL ANOMALY IN ADS/CFT CORRESPONDENCE

2000 WEYL ANOMALY AND CORRECTIONS TO NEWTON'S LAW :
COMPLENTARITY OF THE MALDACENA AND RANDALL-SUNDRUM PICTURES

2000 REVIVAL OF (STAROBINSKY) WEYL ANOMALY DRIVEN INFLATION

2002 WEYL ANOMALY AND THE GRAVITON MASS IN ADS :
COMPLEMENTARITY OF THE MALDACENA AND KARCH-RANDALL PICTURES

2005 INFRA-RED DIAGNOSTIC

HOLOGRAPHIC WEYL ANOMALY

ADS/CFT MALDACENA

GUBSER, KLEBANOV, POLYKOV

WITTEN

GRAVITY ON $(d+1)$ DIM MANIFOLD M

$\equiv$ CFT ON d DIM MANIFOLD ∂M

$$Z_{grav}[\phi_{(0)}] = \int_{\phi_0} d\phi \exp(-S[\phi])$$

ϕ on M $\Rightarrow$ ϕ_0 on ∂M CORRESPOND TO O

$$Z_{CFT}[\phi_{(0)}] = \left\langle \exp \int_{\partial M} d^d x \, O \, \phi_{(0)} \right\rangle$$

$$\boxed{Z_{grav} = Z_{CFT}}$$

$$S = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{\det G_{MN}} \left(R + 2\Lambda \right)$$

DIVERGES WHEN EVALUATED AT SOLUTIONS G_{MN} OF

$$R_{MN} - \frac{1}{2} g_{MN} R = \Lambda G_{MN}$$

COORDINATE CHOICE

$$G_{MN}\, dx^M dx^N = \frac{L^2_{d+1}}{4} \rho^{-2} d\rho^2 + \rho^{-1} g_{\mu\nu} dx^\mu dx^\nu$$

$$g(x,\rho) = g_{(0)}(x) + \rho\, g_{(1)}(x) + \cdots \rho^{d/2} g_{(d)} + \rho^{d/2} \ln\rho\, h_{(d)} + \cdots$$

$$S[g_0] = \frac{1}{16\pi G} \int d^d x \sqrt{g_{(0)}} \Big[\epsilon^{-d/2} a_{(0)} + \epsilon^{-d/2+1} a_{(2)} + \cdots + \epsilon^{-1} a_{(d-2)} - \ln\epsilon\, a_{(d)} + \text{FINITE} \Big]$$

WEYL ANOMALY

DIVERGENCES CANCELLED BY COUNTERTERMS. BREAKS D=5 GENERAL COVARIANCE $\Rightarrow$ D=4 ANOMALY

HENNINGSON & SKENDERIS

$$\delta g_{(0)} = 2\delta\sigma \, g_{(0)}$$

$$\delta S = \int_{\partial M} d^d x \sqrt{g_0}\, \mathcal{A}\, \delta\sigma$$

$$\mathcal{A} = \frac{1}{16\pi G_{d+1}}\left(-2\, a_{(d)}\right)$$

EXAMPLE D=4

$$\mathcal{A} = \frac{L_5^2}{64\pi G_5}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)$$

$AdS_5 \times S^5$ TYPE IIB

GEOMETRY OF N COINCIDENT

3-BRANES

$$N^2 = \frac{\pi L_5^3}{2 G_5}$$

MALDACENA: DUAL TO $\mathcal{N}=4$ $U(N)$ YANG-M

$\Rightarrow$

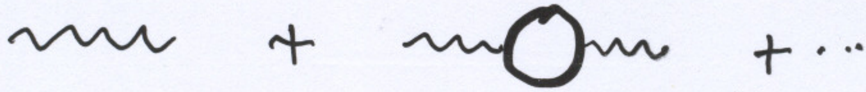
$$A = \frac{N^2}{32\pi^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)$$

SAME AS 1-LOOP CFT ANOMALY

(NON-RENORMALIZATION THEOREM)

1 LOOP CORRECTIONS TO NEWTON'S LAW

PhD WITH SALAM.



$$V(r) = \frac{G_4 M}{r}\left(1 + \frac{\alpha G_4}{r^2} + \cdots\right)$$ MJD 1972

$$\alpha = \frac{1}{45\pi}\left(12 n_1 + 3 n_{1/2} + n_0\right) = \frac{8(4\pi)^2}{3\pi} c$$

(SAME COEFFICIENT AS IN WEYL ANOMALY)

MALDACENA N=4 CFT

$$(n_1, n_{1/2}, n_0) = (N^2, 4N^2, 6N^2)$$

$$\hbar N^2 = \pi L_5{}^3 / 2G_5 \qquad G_4 = 2G_5 / L_5$$

$$V(r) = \frac{G_4 M}{r}\left(1 + \frac{2L_5{}^2}{3r^2} + \cdots\right)$$

SAME AS RANDALL-SUNDRUM USING

D=5 CLASSICAL CALCULATION!

SAME α APPEARS IN GRAVITON MASS!

ANOMALY-DRIVEN INFLATION

FOR LARGE NUMBER OF MATTER FIELDS, CAN NEGLECT GRAVITON LOOPS

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_4 \langle T_{\mu\nu} \rangle$$

MATTER EFFECTIVELY CFT IF $m^2 \ll R$
BUT ANOMALY

$$\mathcal{A} = g^{\mu\nu} \langle T_{\mu\nu} \rangle \neq 0$$

MEANS FOR MAXIMALLY SYMMETRIC SPACES

$$\langle T_{\mu\nu} \rangle = \frac{1}{4} g_{\mu\nu} \mathcal{A}$$

EFFECTIVE COSMOLOGICAL CONSTANT
$\Rightarrow$ (UNSTABLE) DE SITTER INFLATION!
STAROBINSKY 1980

REVIVAL

VILENKIN 1985
HAWKING HERTOG & REALL 2000

① LOOKED AT $\mathcal{N}=4$ $U(N)$ YANG MILLS IN LARGE N LIMIT $\Rightarrow$ AdS/CFT

② UNIVERSE ENTERS DE SITTER PHASE TROUGH TUNELLING FROM "NOTHING" VIA COSMOLOGICAL INSTANTON

③ GET AGREEMENT WITH CMB DATA AT EXPENSE OF FINE-TUNING COEFFICIENT OF

$$\sqrt{g}R^2$$

COUNTERTERM (WITHOUT CONSTRAINING N)

GRAVITON MASS IN AdS_4

KARCH-RANDALL : AdS_4 BRANE IN AdS_5 BULK $\Rightarrow$

GRAVITON MASS

$$M^2 = \frac{3L_5^2}{2L_4^2}$$

L_5 = RADIUS OF AdS_5
L_4 = RADIUS OF AdS_4

AdS/CFT INTERPRETATION

EXTRA 3 DEGREES OF FREEDOM COME FROM MASSIVE SPIN 1 BOUND STATE

$M^2 = \frac{96\pi G}{L_4} c$ c WEYL ANOMALY !

$= 3N^2G/2\pi L_4^4$ AGREES WITH K-R RESULT!