QUANTUM GRAVITY AND HAWKING RADIATION

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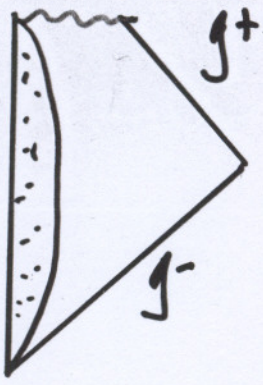
Plan :

1. Standard picture of Black Hole Evaporation

2. A new picture proposed by Ashtekar-Bojowald

3. A toy model of 1+1 black holes

Classical Collapse



What happens in Quantum Theory?

No theory of QG. So look at Quantum matter on classical geomet

Hawking: Black hole radiates at

$$kT_H = \left(\frac{m_{Planck}}{M}\right) m_{Planck} c^2 \sim \frac{1}{M}$$

$\Rightarrow$ radiation $\rightarrow$ mass loss $\rightarrow$ higher temp

$\downarrow$

more radtn.

Note: Evaporation is VERY SLOW.

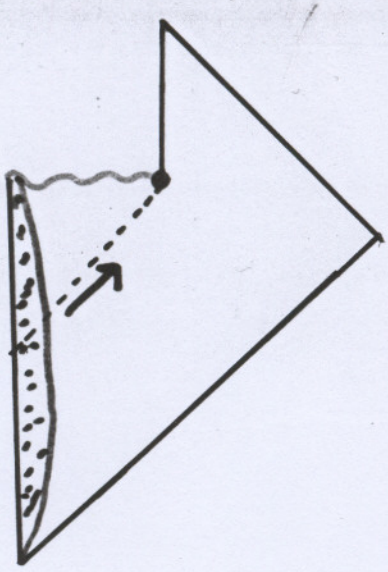
- From mass loss by "σ_{TH}" radtn, pble to estimate time scale for evaporation ~ $\left(\frac{1}{M}\frac{dM}{dt}\right)^{-1}$.

- Given a classical BH one can perturb it and estimate time it takes to settle down.

$$\frac{T_{\text{settling down}}}{T_{\text{evaporation}}} \sim \frac{m_p^2}{M^2} \ll 1$$

⇒ Model evaporation as a quasistatic process

i.e. as a 1 parameter family of BHs of decreasing mass.



Final State (5)

≈ (mp) + Thermal Radtn

= pure if (mp) has enough correltns with radtn

↳ ? ruled out by low enrgy physics arguments

Loophole: BH Lifetime V. LARGE → small contn can accumulate & give different pict

AA - MB:

- Key ingredient: In std picture,

Final state = {radtn + (mp)} + BOUNDARY OF SPTIME

AA-MB expect classical sing is resolv by QG & quantum evoltn is define thru classically singlr region and "beyond" to "other side of singlrty".

• Consequence: Event Horizon not relevant structure

→ Classical geom near ∞ not reliable

small changes near ∞ can remove event horizon (HAJICEK) because EH is a GLOBAL concept ⇒ Need a quasilocal defn of black hole ⇒ Replace EH by "Dynamical Horizon" (AA et al)

≈ smooth, 3d hypersurf foliated by marginally trapped 2 surfaces

Turns out if DH is

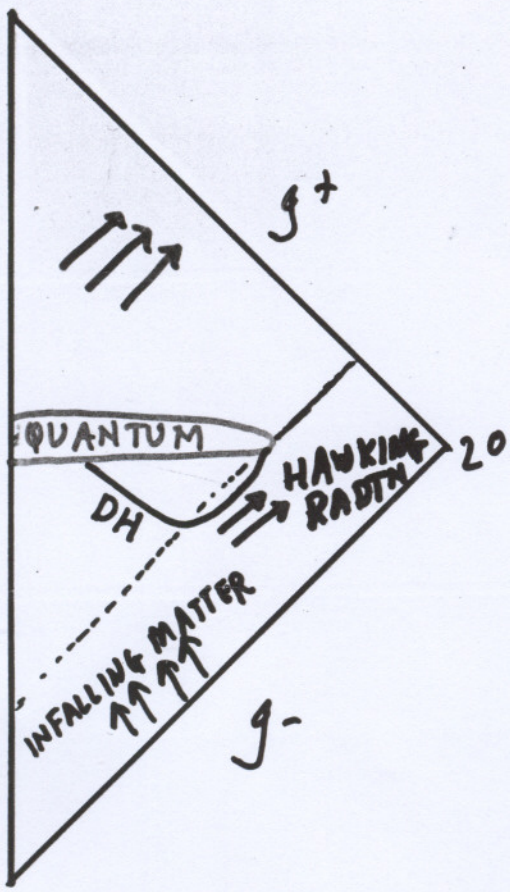
• spike → its spatial cross-sections have increasing area (like Apparent Horizon)

• null → no change in area

If a time like world tube →

area decreases

Sing resoltn + dynamical horizon:



- Sing resolved, no bdry, significant fraction of ADM mass evolves thru singlrty
- What evaporates is AREA of dynamical horizon
- Missing info is in correltns recovered on "other side of singlrty"

CGHS model :

$$S_{CGHS} = \underbrace{S(^2g_{ab}, \phi)}_{\text{action with "cosmological const" } K \sim \frac{1}{L}} - \frac{1}{2}\int d^2x \sqrt{-g}\, g^{ab} \nabla_a f \nabla_b f$$

Solution :

- 2 dim $\Rightarrow$ g_{ab} cfmlly flat :

$$ds^2 = \underbrace{-(dt)^2 + (dx)^2}_{\Omega} = \underbrace{dx^+ dx^-}_{\Omega} \qquad x^\pm = x \pm t$$

- 'f' is cfmlly coupled $\Rightarrow \left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}\right) f = 0$

$$f = f_{(+)}(x^+) + f_{(-)}(x^-)$$

- $\Omega = 1$ + double integrals of T_{ab}

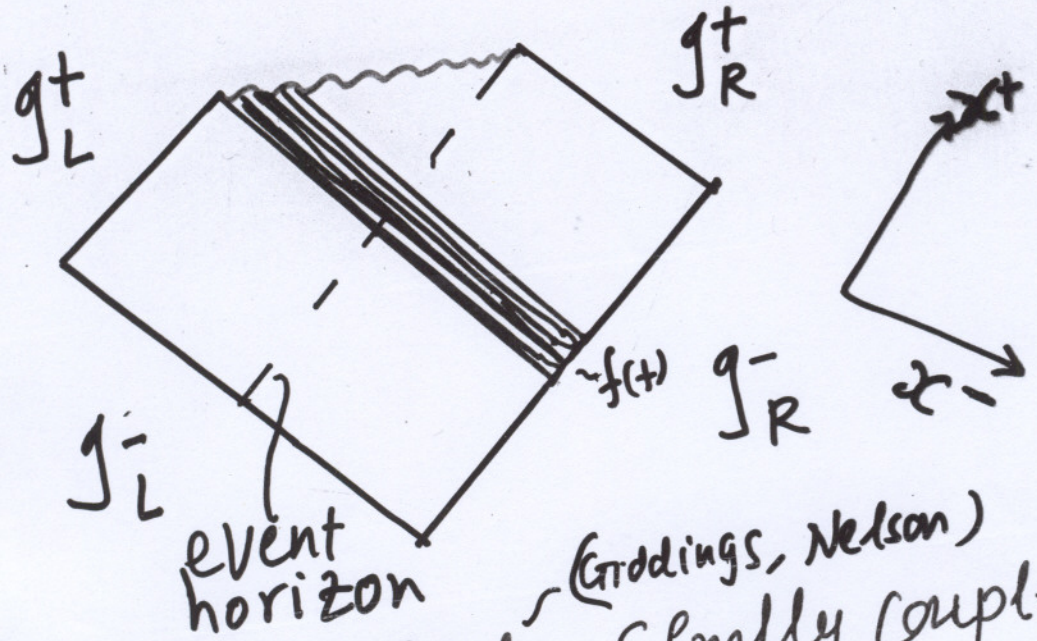
(T_{ab} : matter stress energy)

$$T_{ab} = \partial_a f \partial_b f - \frac{1}{2} g_{ab} (\partial^c f \partial_c f)$$

- ϕ also determined by f.

$f = 0 \Rightarrow \Omega = 1 \Rightarrow$ FLAT SPTIME

$f_{(-)} = 0$: CGHS black hole



Hawking effect: (Giddings, Nelson)
on black hole, focus on (-) modes. Then
- modes for freely falling observers @ $\mathcal{I}^-_L$ $\neq$ for $\mathcal{I}^+_R$
- Part of data on $\mathcal{I}^-_L$ falls into singularity

$\Rightarrow |0(-)\rangle$ on $\mathcal{I}^-_L$ = mix'd state on $\mathcal{I}^+_R$ with late time thermal behaviour

$T_H = \kappa/2\pi$

Quantum Theory: 'f' satisfies K.G. eqn. on $ds^2 = dx^+ dx^-$ FLAT spacetime. ⇒ Can quantize $\hat{f} = \hat{f}_{(+)} + \hat{f}_{(-)}$

$$\hat{f}_{(+)} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk}{\sqrt{2k}} \left[\hat{a}_{(+)}(k) e^{\mp ikx^\pm} + h.c. \right]$$

- Can define INVERSE conformal factor operator $\hat{\Omega}(x^+, x^-)$

  $\hat{\Omega} = 1 +$ double integrals of $: \partial_a f \partial_b f$

- "Quantum geometry" defined by contravariant metric oprtr

$$\hat{g}^{ab} = \hat{\Omega} \left(\frac{\partial}{\partial x^+}\right)^a \left(\frac{\partial}{\partial x^-}\right)^b)$$

Consequences of this quantization:

1. Recovery of classical geometry:
Classical black hole was for $f(+)=0$, $f(-)$

Quantum states $\in \mathcal{H}_+ \otimes \mathcal{H}_-$

$\Rightarrow$ Choose $|\Psi\rangle = \underbrace{|f(+)\rangle}_{\text{coherent state}} \otimes |0(-)\rangle$

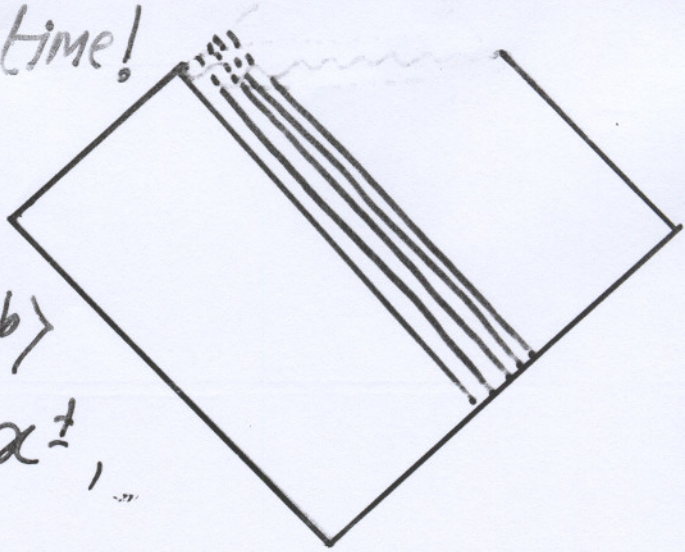
Then classical geometry recovered from quantum coherent state thru exp values i.e.

$$\langle \hat{\Omega} \rangle = \Omega_{classical} \quad \langle \hat{g}_{ab} \rangle = g^{ab}_{classical}$$

But also get:

Quantum extn of sptime!

$\hat{f}$ lives on entire $x^\pm$ plane; $\langle \hat{\Omega}(x^+, x^-) \rangle$, $\langle \hat{g}_{ab} \rangle$ computble for all $x^\pm$,



even "beyond" singlrty where

$$\langle \hat{\Omega} \rangle < 0$$

Not only exp values but oprtr eqns, oprtrs ($\hat{g}^{ab}$, $\hat{f}$ etc) well defined "beyond" sing $\Rightarrow$ Singlrty is resolved.

2. Fluctnations of geometry:

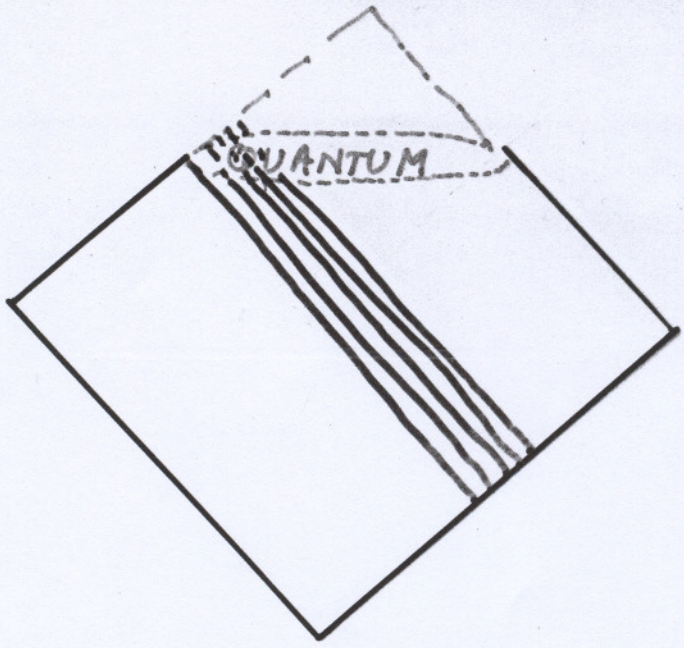
→ Smear $\hat{\Omega}$ along non-null curve in $ds_0^2 = dx^+ dx^-$, $\hat{\Omega}(\alpha) = \int \alpha(x)\, \hat{\Omega}(x)\, dx$

'x' is coordinate on curve

For large class of $\alpha(x)$ of compact support, $\hat{\Omega}(\alpha)$ is a well defined oprtr.

$\Rightarrow \Delta\hat{\Omega}(\alpha)$ can be computed.

→ Seems that fluctuations $\Delta\hat{\Omega}(\alpha)$ are very large for curves near classical singlrty. If so, then classical geometry (= exp values) is not reliable



3. Hawking Radtn ? : Recall that non perturbtve quantum state underlying the geometry-dilaton-matter system is $|\psi\rangle = |f(t)\rangle \otimes |0-\rangle$. $|0-\rangle$ restricted to ρ_R^t = mix'd state, $T_H = k/2\pi$. Info ein $|0-\rangle$ regained thru correltns with oprtrs on "other side" of sing.

1.-3. above = hints of AB paradigm.

But still open issues....

Open issues:

- If picture correct, need to know how detailed back reaction, $O(\hbar)$ corrections to classical eqns emerge from nonpert oprtr eqns.

- Need a good way of deciding when fluctuations are "Large" e.g. Are fluctuations of geometry large at ℓ_L?

However basic features of AB do seem to be exhibited despite above.

Note: Viewpoint NOT new - Mikovič, Kuchař, Romano ...

Summary:

- Stdrd evaprtn involves quasistatic approxtn and hence has BDRY to sptime @ sing lrty.
- Loophole: small corrections may accumulate, alternative picture pble.
- AB proposal: singlrty resolved by QG, no longer bdry of sptime, info recovered on "other side".
- CGHS model can be quantized. Quantzn seems to exhibit feauture of AB paradigm. key open issues remain, work in progress.