12th Regional Conference on Mathematical Physics

Islamabad, 31st March 2006

On Gauss-Bonnet Gravity

(hep-th/0509126)

Naresh Dadhich

IUCAA, Pune

(nkd@iucaa.ernet.in)

Plan of the Talk

- Universal nature of Gravity
- Classical Motivation for Higher Dimension and Gauss-Bonnet-Lovelock Gravity
- Simple examples
  - Dynamics of dust collapse
  - Does GB anticipate QG effects?
  - Is it a Bridge between Classical & Quantum Gravity?

Gravity is Universal

Dynamics } Self-determined
Dimensions

- Everything about it is self-determined
- We can’t prescribe anything
- Let us follow / probe Universality

Dynamics

- Universal Linkage: *massive, $m = 0$*
- Can be described by Spacetime curvature: R_{abcd}
- Bianchi identity

$$R_{ab(cd;e)} = 0$$

▼ *contraction*

$$G^{ab}{}_{;b} = 0$$

$$G_{ab} = R_{ab} - \tfrac{1}{2} R g_{ab}$$

$$R_{ab} = R^{c}{}_{acb}\ ,\ R = g^{ab} R_{ab}$$

- Integration of Bianchi identity

$$G_{ab} = \kappa T_{ab} - \lambda\, g_{ab} \quad , \quad T^{ab}{}_{;b} = 0$$

- On the left: Second order differential operator on g_{ab}
- On the right: Source/charge: Universal energy-momentum of matter, conserved
- Λg_{ab}: new constant relative to " ; " derivative
- Einstein Equation

Dynamics of Einstein Gravity

- Fully determined by Spacetime Curvature

Two constants: κ & λ

- κ : as for any force determined by experiment : Strength = - $8\pi G/c^2$
- λ : New, why?

- Matter/Energy → Gravity ≡ R_{abcd}
- Matter = 0 → Spacetime homogeneous & isotropic

Does it imply flat $R_{abcd} = 0$?

NO!

- Homogeneity & Isotropy

=► Constant curvature

$$R_{abcd} = \kappa\,(g_{ac}\,g_{bd} - g_{ad}\,g_{bc})$$

=► *dS / AdS*

Matter (Newtonian Gravity) free limit

is *not* flat

but *dS/AdS*

➢ Why Λ ?

- In absence of matter, Spacetime is homogeneous & isotropic, but not necessarily flat.
- Background Spacetime is not fixed but dynamic.
- Derivative is “ ; ”
- Spacetime is not at zero potential.

Dynamic Spacetime means

- It bends / curves $\longrightarrow$ micro-structure
- It can't be at zero potential
- λ has thus to be determined by micro-structure of Spacetime $\longrightarrow$ Quantum Gravity/Spacetime
- It is a new constant of Nature
- New Physics required!!!

Higher Dimension

- Einstein Equation

$$G_{ab} = kT_{ab} - \Lambda\, g_{ab}\,, \quad D \geq 2$$

  - D = 2,3 not big enough for gravity dynamics
  - So we come to 4-D

- Is it big enough to accommodate gravity fully?

- How do we know it doesn't propagate in extra D?

➢ Gravity ≡ Curvature

How do we keep curvature confined in 4-D ?

- Does higher D get curved or Not ?
- Higher D embedding 4-D is flat, $R_{abcd} = 0$?
- For an arbitrary 4-D curved 10-D required

- General 4-D Gravity can penetrate down to 10-D

<u>Extra dimension Required…</u>

Gravity is self-interactive

- Self interaction means iterations, first, second, …

$$\nabla^2\phi \neq 0 = -\tfrac{1}{2}(\nabla\phi)^2$$

- Einstein equation contains first iteration

$$(\partial^2, (\partial)^2)\, g_{ab} = 0$$

☞ Can't stop at first…

- Second iteration: Basic Field Entity : R_{abcd}

$$R^2_{abcd} \rightarrow \text{also squares } (\partial^2)^2$$

Self-interaction

- Quasilinear Equation
  - Highest order of derivative, ∂^2 to be linear
- Gauss-Bonnet combination

$$R^2 - 4\, R_{ab} R^{ab} + R_{abcd} R^{abcd}$$

ensures quasi-linearity, $(\partial^2)^2$ cancel out

- Gauss-Bonnet is non-trivial in $D \geq 5$
- Thus we have to go to 5-D to physically realize second iteration of self-interaction

Self-interaction

- D = 2 , 3: Not big enough for free Gravity

- D = 4: Not big enough for self-interaction

- If matter is confined to 3-space/brane,
  (this is for matter dynamics to decide)
  gravity does propagate in extra 5th D

Gauss-Bonnet Gravity

- In 5-D there is no Matter

$$G_{AB} = \alpha\, H_{AB}\, (R^2)$$

- Spacetime is homogeneous & isotropic
- It should be Einstein space

$$G_{AB} \propto g_{AB} \propto H_{AB}$$

$$\Rightarrow R_{abcd} = \kappa\, (g_{ac}\, g_{bd} - g_{ad}\, g_{bc})$$

$$\Rightarrow \text{Constant curvature}$$

- Gravitational field – negative energy density
- Hence 5-D Bulk is *AdS*

Motivation for Brane-World Gravity

Where does this iteration chain end?

- If matter lives on 3-brane, it ends in 5-D *AdS* Bulk
  - *AdS* Bulk has Weyl = 0, No free gravity to go any further
  - It is embeddable in 6-D flat Spacetime
- Iteration naturally stops at the second

It is purely classical motivation for Brane-World like model.

Gauss-Bonnet Gravity

- Theorem: A 4-D curved Spacetime is isometrically embedded in 5-D Einstein space.

(Campbell, Romero, Tavakol, Dahia)

- If 5-D Einstein space satisfies E-GB equation

$$G_{AB} = \alpha H_{AB}(R^2)$$

It is *AdS* / *dS*

- Spacetime of constant curvature is a solution of the Equation

5-D GB Gravity

- Two distinct situations arise:

(a) Einstein-GB Gravity in 5-D without reference to Brane

- $\alpha \to 0$ Einstein limit exists
- Qualitatively similar to GR with GB correction

(b) Gravity leaks from the Brane into Bulk

- No matter in Bulk
- GB term in Bulk sourced by gravity leaked from Brane
- No $\alpha \to 0$ limit exists

- Mass point in E-GB depicts this

Mass point in E-GB

- Boulware-Deser solution

$$ds^2 = f\,dt^2 - f^{-1}\,dr^2 - r^2\,d\Omega_3{}^2$$

$$f = 1 - (r^2 / 2\alpha)\,[\,-1 \pm \{1 + 4\alpha\,(m^2/r^4 + \Lambda)\}^{1/2}\,]$$

- Two Solutions $\oplus$ & $\ominus$
  - (a) $\oplus$ Bulk solution – BS
  - (b) $\ominus$ Brane-Bulk solution – BBS

As argued earlier, they represent two different physical situations.

Solutions Continued

$$f \xrightarrow[r \to \infty]{} 1 - m^2/r^2 - \Lambda r^2 \qquad \text{BS } S\text{-}dS$$

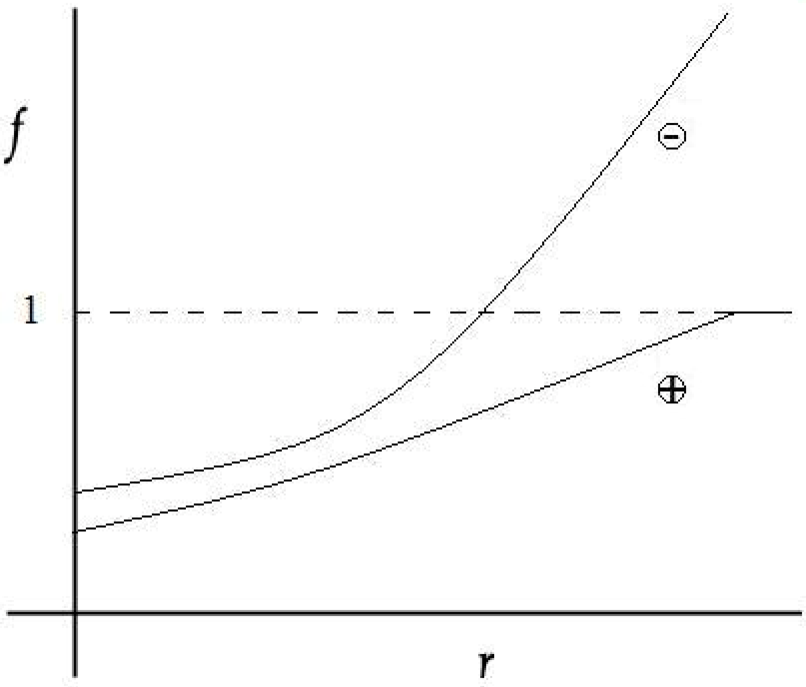
$$1 + m^2/r^2 + (1/\alpha + \Lambda)\, r^2 \qquad \text{BBS } AS\text{-}AdS$$

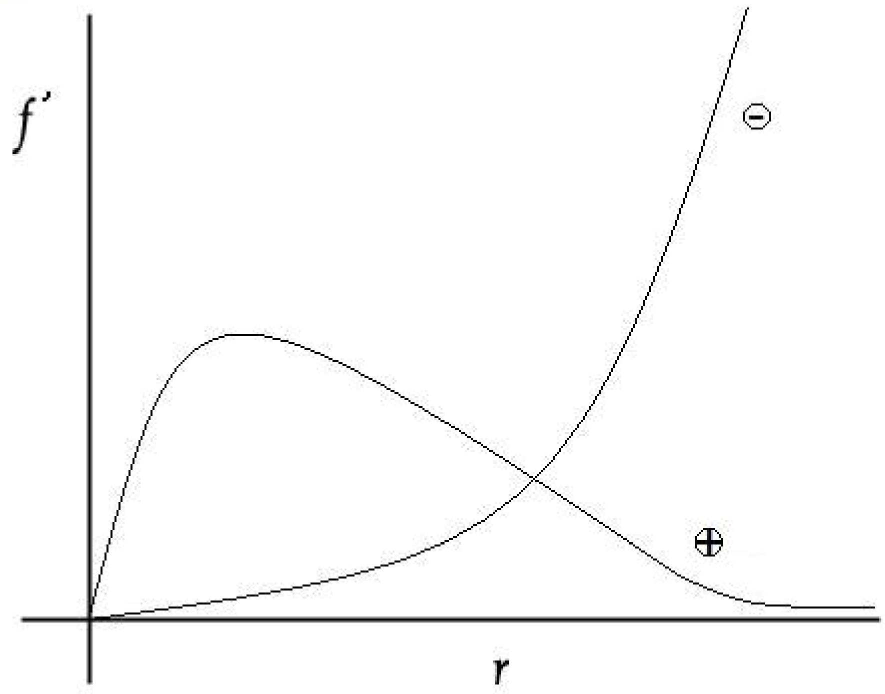
$$\xrightarrow[r \to 0]{} 1 - (\pm\, m/\sqrt{\alpha}) + r^2/2\alpha \qquad \text{BS / BBS}$$

No $\alpha \to 0$ limit for small r

➢ f & f ’ Plots

D = 5





➢ Singularity and Horizon

- Horizon: $r_h = \sqrt{(m^2 - \alpha)}$ BS
  - BH: $m^2 > \alpha$, NS: $m^2 < \alpha$

  α has no effect at large r

- No Horizon, naked singularity: BBS

  α is always non-ignorable

- Singularity is Weak, Metric non-singular

  $R_{abcd} \sim \rho \sim 1/r^2$ But $\int \rho dv \rightarrow 0$

- Timelike / Null (Maeda & Torii)

➢ Gravitational Force

- For large r
  - $f' \sim -m^2/r^3$, $m^2/r^3 - r/\alpha$ *BS/BBS (S/AS-AdS)*
- For small r
  - $f' \sim -r/2\alpha$ *Always AdS*
- Change in radial dependence $1/r^3 \sim r$
  Similar to LQC, ρ : $1/a^3 \sim a^n$, $n > 0$

Does **GB** *anticipate* **QG** *effects?*

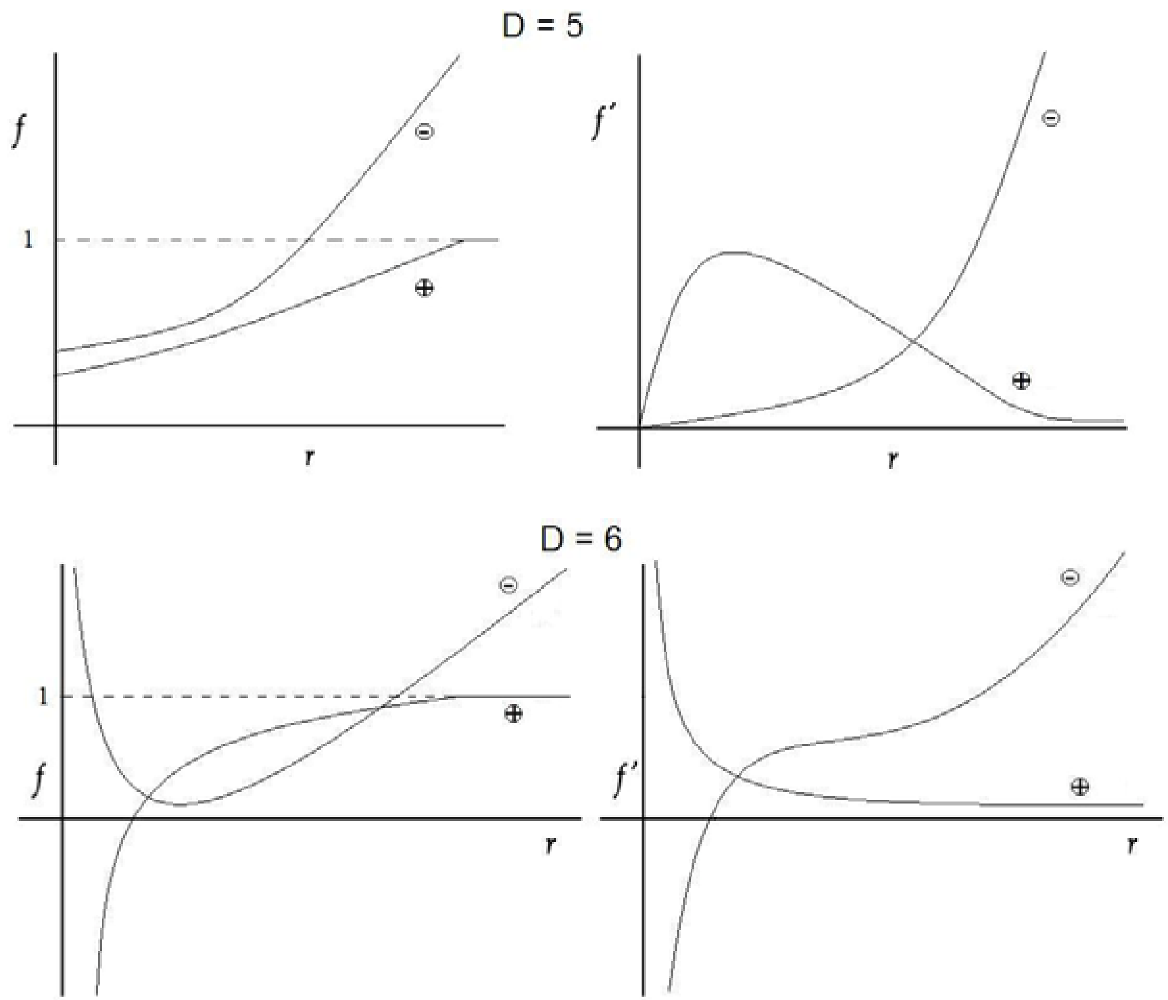
Homogeneous Dust Collapse / Cosmology

m^2 : attractive / repulsive

α : attractive

In 5-D, α dominates

In 6-D, m^2 dominates



In 6-D BBS

- There is No Singularity
- Dust sphere can be
  - Oscillating / Bouncing
  - Stable static
- Friedmann open universe
  - No Big Bang
  - Oscillating Universe

GB Effects

- Smoothens / Weakens Singularity
- Force changes power law for small and large *r*
- In 6-D there is no singularity
  - Force turns repulsive ⟶ Bounce
  - Oscillating sphere / universe
- All this resonates well with the LQC results

(Ashtekar, Bojowald, Parampreet, Goswami, Joshi, Tavakol, Maartens, Lidsey, …)

Lovelock Action

- $L = L1\ (R) + L2\ (R^2) + L3\ (R^3) + \ldots$

  Mass point Spacetime:

  - L1 Unique One solution
  - L2 Unique Two Solutions
  - L3 Unique One solution (?)

Hunches / Conjectures

- Is GB a bridge?
  - Classical ←→ QG
  - String ←→ LQG
  - GB ←→ LQC (Parampreet, Copeland, Lee, Lidsey, Mizuno)
- Intermediatory semi-Classical-Quantum limit.
- Desirable features like
  - Weakened / Smoothed Singularity
  - Bouncing / Oscillating Universe

  Could be brought down to 4-D by Dilaton Coupling ?

Hunches / Conjectures

- Gravity can’t fully remain confined to 4-D
- Physical Spacetime in the large is of constant curvature:
  - dS/AdS
  - Not Flat Minkowski
  - Must have micro-structure
- Classical Limit to LQG may be dS/AdS ?

Thank You!