Update on De Sitter Solution in N=4 gauged Supergravity

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Motivation:

de Sitter space-time:

maximally symmetric solution of Einstein's eqns with +ve cosmological const (Λ)

D-dim dS space, as hyperboloid in $R^{1,D}$

$$x^{0\,2} - \sum_{i=1}^{D} x^{i\,2} = - H^{-2}$$

In $D=4$, this geometry is a soln. of Einstein's eqn in vacuum with $\Lambda = 3H^2 > 0$ ($8\pi G_N = 1$)

Can be interpreted as geometry generated by interaction with gravity of a system with uniform $\rho = 3H^2$ and $p = -\rho$

Locally can be described by FRW metric in $D=4$ with $a(t) > 0$

$$\frac{\ddot{a}}{a} = \Lambda/3 = H^2 > 0$$

This metric describes the evolution of universe with accelerated expansion favoured by exptl evidence in cosmology $\Rightarrow$

Universe in dS regime ($\Lambda \sim 10^{-120} M_P^4$.

In inflationary scenarios:

Accelerated expn. is triggered by slow evolution of uniform ϕ with energy dominated by positive

$$V(\phi) > 0 \,, \quad V(\phi) \gg \dot{\phi}^2$$

Models can have critical points

ϕ_0 : $\quad \partial_\phi V(\phi)\big|_{\phi_0} = 0$

$$V(\phi_0) = \Lambda > 0$$

Like to have Infl. models from string/M theory!

Consistent only in 10/11 dims

Known <u>Not</u> to admit dS in these dims!

2002: No-go thm (Maldacena, Nunez

No dS in dim. reduction to D=4

⟹ Look for dS in extended ($N \geqslant 2$) SUGRA in D=4 (FT framework close to string/M)

High degree of susy ⟹ only way to deform the action, to allow a non-trivial scalar potential, promote some of global symms to local symms ⟺ $\boxed{\text{GAUGING}}$

⟹ Gauged $N=4$ SUGRA w/ Matter (By us)

Others $N=2$!

- De Sitter Solutions in N=4 matter coupled supergravity ; JHEP 2003

  M. de.Roo, D.B. Westra, S.P.

- Potential and Mass matrix in in N=4 supergravity; JHEP 2003

  M. de.Roo, D.B. Westra, S.P., M.Trigiante

- Group Manifold Reduction of Dual N=1, d=10 supergravity; JHEP 2005

  M.de.Roo, M.G.C. Eenink, D.B.Westra, S.P.

- To appear

  M.de.Roo, D.B. Westra, S.P.

$\boxed{\text{KKLT Model}}$

2003

Gauging:

Gauge Symm is **Abelian**:

$$\delta A^R_\mu = \partial_\mu \epsilon^R \qquad (R = 1,2,\cdots,6+n)$$

Extend to non-Abelian symm acting on subsets of vector fields $\Leftrightarrow$ Gauge Group is product several factors

$\Rightarrow$ $\boxed{\text{yields nontrivial scalar potential}}$

SU(1,1) angle:

All SU(1,1) parameters are <u>not</u> relevant for coupling of vector filds with SU(1,1) scalars!

Let $\quad C = \begin{pmatrix} a & b \\ b^* & a^* \end{pmatrix} \qquad C \in SU(1,1)$

with $\quad a = \frac{1}{2} e^{-i\alpha}(s + \frac{1}{s} - it)$

$\qquad\quad b = \frac{1}{2} e^{i\alpha}(-s + \frac{1}{s} + it)$

$s, t \Leftrightarrow$ non-compact directions total derivative

Formalism

N = 4 theory based on superconformal formulation of gravity

(extn. of super Poincare)

⇒ Simpler to find invariant action with simple transformation rules for fields in SUGRA & with matter

But additional symms have to be broken via imposition of constraints (can be difficult to solve!)

Just as in GTR, formulated with

$e_\mu^{\ a}$, $\omega_\mu^{\ ab}$ one imposes $R_{\mu\nu}^{\ \ a} = 0$

⇒ $\omega_\mu^{\ ab}$ interms of $e_\mu^{\ a}$ and

local translation gauge transfomations

⟶ g.c.t.s

⇒ g.c.t.s & local Lorentz transf.s

For Conformal gravity one assigns a gauge field with each generator M_{ab}, P_a, K_a and D

$$\omega_\mu^{ab} \quad e_\mu^a \quad f_\mu^a \qquad b_\mu$$

$$R_{\mu\nu}^{a}(P) = 0 \ , \quad R_{\mu\nu}^{ab}(M)\, E^{\nu}{}_{b} = 0$$

$\rightarrow$ Reduces d o f and leaves only e_μ^a, b_μ

$$\mathcal{L} = -\tfrac{1}{2}\, e \ \underset{\substack{\uparrow \\ \text{conformal mode}}}{\phi}\, D^a D_a \phi$$ (inv. under local M, D, K & gct!)

$$D_a\phi = E_a^{\ \mu} D_\mu \phi = E_a^{\ \mu}(\partial_\mu - b_\mu)\phi$$

$$\xrightarrow[\text{gauge}]{K} \quad \mathcal{L} = -\tfrac{1}{12}\, e\, R(M)\, \phi\phi \qquad (b_\mu = 0)$$

$$R(M) = R_{\mu\nu}^{ab}(M)\, E_a^{\ \mu} E_b^{\ \nu}$$ (without f_μ^a ter

D-gauge choice ($\phi = \phi_0$ with $\phi_0^2 = \frac{6}{\kappa^2}$)

$\Rightarrow$ Einstein-Hilbert $\mathcal{L}$

Additional 'n' matter Scalars can be coupled :

$$\mathcal{L} = \frac{1}{2}\eta_{RS}\, e\, \phi^R D^a D_a \phi^S \quad (R,S = 1,2,\cdots,n+$$

$$\rightarrow \mathcal{L} = -\frac{1}{2K^2} e\left[R + \frac{1}{2}\partial_\mu \phi^I \partial^\mu \phi^I\right]$$

$$I = 2,3,\cdots\cdots,n+1$$

with $\eta_{RS} = \mathrm{diag}(-,+,+,\cdots,+)$

and $\boxed{\eta_{RS}\,\phi^R\phi^S = -\frac{6}{K^2}}$

Physical scalars described by a n-dim subspace in (n+1)-dim space

$\rightarrow$ σ-model with cosetspace $\frac{SO(1,n)}{SO(n)}$

For $N=4$ theory in superconformal basis:

Extra generators: Q^i, S^i $(i=1,\cdots$

and $SU(4)$ generator for phase rotations of chiral parts of Q^i & S

Same analysis with gauge fields

$\longrightarrow$

$N=4$ Weyl multiplet contains two complex scalar fields ϕ_α $(\alpha=1,2)$ transforming under global $SU(1,1)$ and a local $U(1)$ sym with

$$\phi^\alpha \phi_\alpha = 1 \qquad (\phi^1 = \phi_1^*, \ \phi^2 = -\phi_2^*)$$

$$\Rightarrow \quad \frac{SU(1,1)}{U(1)} \ : \ 2 \text{ real dof}$$

$N=4$ SUGRA multiplet obtained by coupling six $N=4$ vector multiplets to ϕ_α and respecting global $SU(1,1)$ Symmetry.

Arbitrary # of (matter) vector multiplets (n) can be coupled to Weyl multiplet giving a consistent $N=4$ SUGRA coupled to matter theory!

Key Ingradient:

$\alpha_i \leftrightarrow$ for the vectors A_μ in **ith** Subgroup in gauging

$\Longleftrightarrow$ the angles rotate one gauge group w.r.t. another in the product of gauge groups

Taking into account of Scalars(Z^R_a) from the vector multiplets:

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \mathcal{L}^{\phi_\alpha}_{KE} + \mathcal{L}^{Z}_{KE} + \mathcal{L}_{vecto}$$

$$-V(\phi, Z) \leftarrow \text{Scalar potential}$$

$$\boxed{V(\phi, Z) = \left[\frac{1}{4} Z^{RU} Z^{SV} (\eta^{TW} + \frac{2}{3} Z^{TW}) - \frac{i}{36} Z^{RSTUVW}\right] \Phi^*_{(R)} f_{RST} \Phi_{(U)} f_{UR}}$$

$$\eta_{RS} = \text{diag}(-,-,-,-,-,-,\underbrace{+,+,\cdots\cdots,+}_{n})$$

$$\Phi_{(R)} = e^{i\alpha_R}\phi^1 + e^{-i\alpha_R}\phi^2 \qquad Z^{RS} = Z^R_a Z^S_a$$

$$Z^{RSTUVW} = \epsilon^{abcdef} Z^R_a Z^S_b Z^T_c Z^U_d Z^V_e Z^W_f$$

Z_a^R transform under local $SO(6) \times SO(n)$

and under global $SO(6,n)$

with constraints: $\eta_{RS} Z_a^R Z_b^S = -\delta_{ab}$

$$\Rightarrow \boxed{\frac{SU(1,1)}{U(1)} \otimes \frac{SO(6,n)}{SO(6)\times SO(n)}}$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$2 \quad \oplus \quad 6n \text{ physical scalars}$$

<u>Solving the constraints:</u>

$$\phi_1 = \frac{1}{\sqrt{1-r^2}}, \quad \phi_2 = \frac{r e^{i\varphi}}{\sqrt{1-r^2}} \qquad (0<r<1)$$

$\xrightarrow{\text{solves}}$ $\phi^\alpha \phi_\alpha = 1$

For Z- constraints:

Take $\underline{n=6}$ $\Rightarrow$ $\dfrac{SO(6,6)}{SO(6)\times SO(6)}$

$\uparrow$ 36 scalar[s]

Let $X_a^A = Z_a^A$ $\quad A = 1, \cdots, 6$

$Y_a^I = Z_a^I$ $\quad I = 7, \cdots, 12$

$a = 1, \cdots, 6$

Let G : 6×6 Symm $\quad B$: 6×6 antisymm

$\uparrow$ 21 scalars $\quad \uparrow$ 15 scalars

Constraints are solved for

$$X = \frac{1}{2}\left(G + G^{-1} + BG^{-1} - G^{-1}B - BG^{-1}B\right)$$

$$Y = \frac{1}{2}\left(G - G^{-1} - BG^{-1} - G^{-1}B - BG^{-1}B\right)$$

$V = V(r, \varphi, G, B) \leftarrow$ fn. of 38 Scala

For Simplicity: $G = a I_6$, $B = b\begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$

Gauging: $SO(2,1) \times SO(2,1) \times (U(1))^6$

$$\begin{array}{ccc} (A^1_\mu, A^2_\mu, A^7_\mu) & \times (4,5,10) & \times \quad \cdots \\ \uparrow & \uparrow & \\ (g_1, \alpha_1) & (g_2, \alpha_2) & \end{array}$$

$$V(r, \varphi, a, b) = V(r, \varphi)\, V(a, b)$$

$$V(r, \varphi) = (g_1^2 + g_2^2)\frac{1+r^2}{1-r^2} - \frac{2r}{1-r^2}\left(g_1^2 \cos(2\alpha_1 \cdots\right.$$

$$\left. + g_2^2 \cos(2\alpha_2 + \varphi)\right)$$

$$V(a, b) = \frac{1}{16 a^6}(a^2 + b^2 + 1)^2 \left\{ b^2 (a^2 + b^2 + 1)^2 + 2a^2(a^2 - b^2) \right\}$$

Extremise this potential:

$$\partial V = 0$$

$\Rightarrow$ (i) $\partial^2 V > 0$

(ii) $\boxed{V_0 = |g_1 g_2 \sin(\alpha_1 - \alpha_2)|}$

$\Uparrow$

de Sitter with $\Lambda = V_0$

For $\alpha_1 = \alpha_2$ $V_0 = 0$!

But if the other 34 scalars are turned on: the mass matrix for the scalars in G and B contains -ve eigen values !

For $[SO(3)]^4$ gauging:

There are no -ve eigenvalues in the mass matrix <u>but</u> when the 36 scalars in G, B are at the minimum, the 2 $SU(1,1)$ scalars are at <u>max</u> !

$\Rightarrow$ metastable / unstable !

Look for possibility of embedding in String Th.

A group manifold ($SU(2) \times SU(2)$) reduction (generalized Scherk-Schwarz) of $N=1$ SUGRA in $d=10$ (coupled to Y-M multiplets) to $d=4$ $\rightarrow$

(i) Field content same with $N=4$ gauged sugra

(ii) 2 $SU(1,1)$ scalars identified with dilaton and axion

(iii) Potential can be brought to the same form only when the $SU(1,1)$ angles are put to zero!

Are the angles related to some non-perturbative physics? Work in Progress

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