GRAVITATIONAL COLLAPSE IN QUANTUM GRAVITY

* MOTIVATION & APPROACH
* A MODEL : 3 views
  - CLASSICAL
  - SEMI-CLASSICAL
  - QUANTUM GRAVITY
* SUMMARY & OUTLOOK

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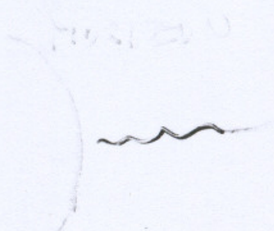
MOTIVATIONS

* WHAT IS A QUANTUM BLACK HOLE?
* HOW DOES IT FORM?
* DOES HAWKING RADIATION SHOW UP IN A SUITABLE APPROXIMATION?
* WHAT IS THE RESULT OF LONG TIME QUANTUM EVOLUTION OF A MATTER-GEOMETRY STATE?

- CLASSICAL COLLAPSE IS WELL UNDERSTOOD AT LEAST IN SPHERICAL SYMMETRY

- SEMI-CLASSICALLY – QUANTUM FIELDS ON A BH BACKGROUND
  HAWKING RADIATION.

TRANSPLANCKIAN MODE PROBLEM:



OBSERVED HAWKING QUANTA ARE REDSHIFTED FROM THE HORIZON.

BACK REACTION!

APPROACH

CLASSICAL THEORY: $g_{\mu\nu}$, φ

METRIC & MATTER FIELD

NON-PERTURBATIVE:

$$g, \varphi \longrightarrow (q, \pi)\ (\varphi, P_\varphi) \qquad H(q, \pi, \varphi, P_\varphi)$$

$$H \rightarrow \hat{H}$$

ATTEMPT TO FOLLOW EVOLUTION OF SUITABLE INITIAL STATE

PERTURBATIVE:

$$g = g_0 + h \qquad \varphi = \varphi_0 + \chi$$

$$h \to \hat{h} \qquad \chi \to \hat{\chi}$$ "small" perturbations

$$\langle \hat{h}\hat{h}\hat{h}\cdots \rangle \qquad \langle \hat{h}\hat{h}\hat{\chi}\hat{\chi}\cdots \rangle$$

AdS/CFT:

OTHER: g, φ ARE "EMERGENT" COLLECTI[VE] DEGREES OF FREEDOM & SHOULDN'T BE QUANTISED .... SO ....:

THE MODEL

$$G_{ab} = 8\pi T_{ab}$$

$$T_{ab} = \partial_a \varphi \partial_b \varphi - \frac{1}{2}(\partial\varphi)^2 g_{ab}$$

$$g_{ab}(r,t) \qquad \varphi(r,t)$$

$$ds^2 = -f^2(r,t)\,dt^2 + g^2(r,t)\,dr^2 + r^2 d\Omega^2$$

$\varphi = 0 \longrightarrow$ flat space or Schwarzschild

$\varphi(r,t)$ IS THE SOURCE OF LOCAL DEGREES OF FREEDOM.

COMPLICATED 2D FIELD THEORY

NO KNOWN ANALYTIC COLLAPSE SOLUTIONS THAT ARE ASYMPTOTICALLY FLAT.

ANALYTIC COLLAPSE MODELS (Opp-Snyder, Vaidya, CGHS & variations] HAVE ONLY MATTER INFLOWS — SCALAR FIELD MODEL IS MUCH RICHER ...

CLASSICAL RESULTS

* THERE ARE 2 CLASSES OF INITIAL DATA (ASYMPTOT. FLAT)

"WEAK" DATA $\longrightarrow$ NO BH FORMATION IN LONG TIME LIMIT

"STRONG" DATA $\longrightarrow$ BH FORMATION ...

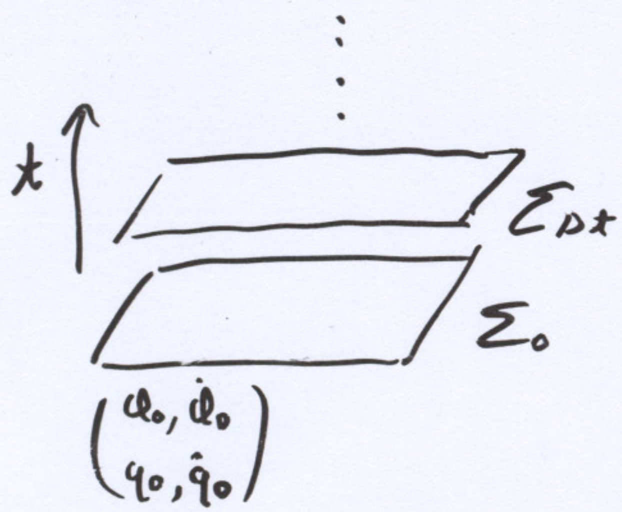
— RESULT OF HARD ANALYSIS (CHRISTODOULO ~1975)

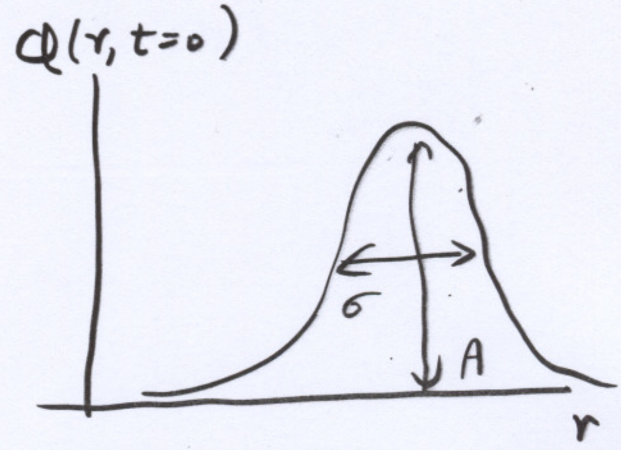
* DETAILS OF TRANSITION WEAK $\to$ STRONG DONE BY NUMERICAL EVOLUTION

(CHOPTUIK ~1993)

WITH Λ (VH, KUNSTATER, OLIVIER, ... ~200

NUMERICAL SIMULATION





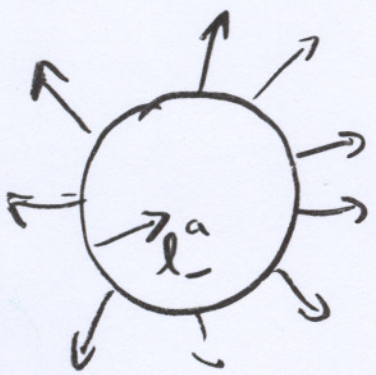
$\dot{\Phi}(r, t=0) = 0$

* GEOMETRY DATA DETERMINED BY CONSTRAINTS

* AT EACH TIME STEP OF THE EVOLUTION TEST FOR TRAPPED SURFACE FORMATION

COMPUTE $\Theta_\pm$.

S^2 embedded in Σ_t

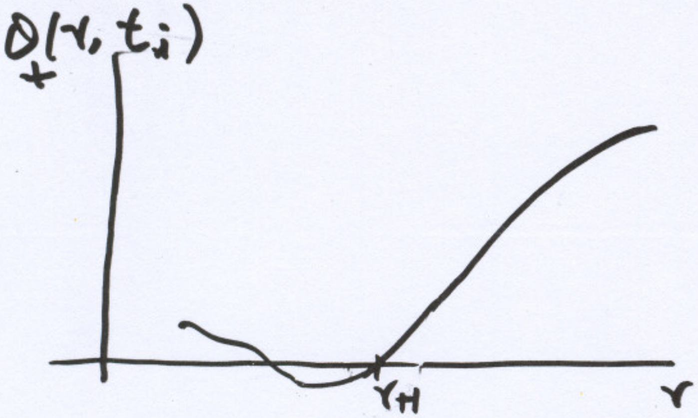


l_+^a : outgoing light direction at each point on S^2.

$$D_a l^a_\pm \Big|_{S^2} \equiv \Theta_\pm \text{ (fn of data on a slice)}$$
$$= \Theta_\pm (r,t)$$

S^2 $\begin{cases} \text{NORMAL: } \Theta_+ > 0, \ \Theta_- < 0 \\ \text{MARGINALLY TRAPPED: } \Theta_+ = 0, \ \Theta_- < 0 \\ \text{TRAPPED: } \Theta_\pm < 0 \end{cases}$

* COMPUTE $\Theta_{\pm}(r,t)$ AT EACH t



SEARCH FOR OUTERMOST ZERO: $\longrightarrow$
RADIUS OF APPARENT HORIZON

$M_{BH} \sim r_{AH}(A,\sigma)$

$A > A_*$: $M_{BH} \sim (A - A_*)^{\gamma}$

$A = A_*$: NAKED SINGULARITY

$A < A_*$: NO HORIZON FORMS

A_*, γ DETERMINED NUMERICALLY

SEMI-CLASSICAL VIEW

$$G_{ab} = 8\pi \langle T_{ab} \rangle$$

HAWKING RADIATION : BH = BLACK BODY

$T \sim \frac{1}{M} \quad P = \sigma T^4$

$$ds^2 = -\left(1 - \frac{2M(t)}{r}\right) dt^2 + \left(1 - \frac{2M(t)}{r}\right)^{-1} dr^2 + r^2 \, d\Omega^2$$

$$\frac{dM}{dt} = -\alpha \left(\frac{M_P}{M}\right)^2 \frac{M_P}{t_P} \qquad (\alpha > 0)$$

EXPECTED TO BREAKDOWN AS $M \to 0$. QG EFFECTS BECOME IMPORTANT.

TRANSPLANCKIAN PROBLEM

QUANTUM GRAVITY

* USE A HAMILTONIAN FORMULATION OF THE $g(r,t)$ $\varphi(r,t)$ EINSTEIN EQNS.

* ADM VERSION : $M \sim \Sigma \times R$

CANONICAL VARIABLES $(q_{ab}(r,t)\,;\,\pi^{ab}(r,t))$

METRIC ON Σ

~EXTRINSIC CURVATURE OF Σ IN M

$$S = \frac{1}{8\pi G}\int_M \sqrt{-g}\, R\, d^4x + \int_M \sqrt{-g}\, g^{ab}\partial_a\varphi\,\partial_b\varphi\, d^4x$$

$$= \int_M d^3x\, dt \left\{ \pi^{ab}\dot{q}_{ab} + P_\varphi\dot{\varphi} - N^a C_a - N\mathcal{H} \right\} + \int_{\partial M} dt\, d^2x\, (\cdots)$$

$C_a(\pi, q\,;\, P_\varphi, \varphi) \simeq 0$ space reparam.

$\mathcal{H}(\;"\quad"\;) \simeq 0$ time "

$$S^{RED} = \int dr\,dt \left[\dot{R}P_R + \dot{\Lambda}P_\Lambda + \dot{\varphi}P_\varphi - N^r C_r - N\mathcal{H} \right] + \int dt\,(\cdots)$$

$$ds^2 = \Lambda^2(r,t)\,dr^2 + R^2(r,t)\,d\Omega^2$$

* GAUGE FIXING

FULL

$r = R$ $\qquad$ $t = f(q, \Pi)$

PARTIAL

$t = f(q, \Pi)$

USEFUL TO EVOLVE WITH A UNKNOWN TIME VARIABLE

* WE WORK WITH ONLY A TIME GAUGE FIXING.

$$S^{RED}_{GF} = \int dr\,dt \left(\dot{R} P_R + \dot{Q} P_Q - N^r C_r \right) + \int dt \, (\cdots) .$$

E CONSTRAINT REMAINS.

THIS PROBLEM WAS POSED BY UNRUH (1976) TO UNDERSTAND HAWKING RADIATION.

→ FULL GAUGE FIXING TO GET A REDUCED THEORY FOR THE SCALAR FIELD:

$$H[Q, P_Q] = \int_0^\infty dr\, f(Q, P_Q)\, e^{\int_\infty^r dr'\, g(Q, P_Q)}$$

"I PRESENT IT HERE IN THE HOPE THAT SOMEONE ELSE MAY BE ABLE TO DO SOMETHING WITH IT"

GOAL FOR QUANTISATION:

A FRAMEWORK IN WHICH ONE CAN OBTAIN A COMPUTATIONAL PROCEDURE FOR STUDYING GRAV. COLLAPSE IN QG.

INITIAL DATA $\longrightarrow$ INITIAL STATE $|\Psi_0\rangle$

$\theta_\pm \longrightarrow \hat{\theta}_\pm$

CURVATURES $\longrightarrow$ $\widehat{\text{CURVATURE}}$

A PROCEDURE FOR STEP BY STEP TIME EVOLUTION OF $|\Psi_0\rangle$.

QUANTISATION

POISSON ALGEBRA OF "BASIC" CLASSICAL VARIABLES

$\longrightarrow$ COMMUTATOR ALGEBRA ON HILBERT SPACE

$$\mathcal{Q}_f = \int_0^\infty f\,\mathcal{Q}\,dr \qquad U_\lambda(P_\phi) = \exp(i\lambda P_\phi)$$

translation.

$$\{\mathcal{Q}_f, U_\lambda\} = i\lambda f(r)\,U_\lambda$$

HILBERT SPACE

$$\underset{r_1}{\bullet}\ \bullet \quad \bullet\bullet \quad \bullet\bullet \quad \bullet \quad \bullet \quad \bullet\bullet\bullet\bullet \quad \bullet\bullet\underset{r_N}{\bullet}$$

$\{r_i\}$ a sample of points from half line $r \in [0,\infty)$

$|a_1 \cdots a_N ; b_1 \cdots b_N\rangle$

a_i : "excitations" of field $R(r)$

b_i : " " " $\mathcal{Q}(r)$

$$\hat{\mathcal{Q}}_f\,|\,\rangle = \sum_{k=1}^{N} f(r_k)\,a_k\,|\,\rangle$$

$$\hat{U}_\lambda(r_l)\,|\,\rangle = |a_1 \cdots a_l - \lambda \cdots a_N ; b_1 \cdots b_N\rangle$$

CURVATURE OPERATORS

CLASSICALLY ALL CURVATURES DIVERGE AS $\frac{1}{R^n}$. $n>0$

SO WE NEED $\widehat{\frac{1}{R}}$.

A PRESCRIPTION:

$$R_f = \int_0^\infty f R \, dr$$

$\rightarrow R(r_i)$ if f is a narrow Gaussian peaked at r_i

$$\frac{1}{|R_f|} = \left[\frac{2}{i f(r)} e^{-i\lambda P_R} \left\{ \sqrt{|R_f|}, e^{i\lambda P_R} \right\} \right]^2$$

— an identity.

RHS IS A FN OF BASIC OPERATORS

DEFINE $\widehat{\frac{1}{R}}$ USING RHS $\rightarrow \widehat{RHS}$

SPECTRUM OF $\widehat{\frac{1}{R}}$ BOUNDED ABOV

TRAPPING TEST OPERATORS

* $\theta_\pm (P_\varphi, \varphi, R, P_R) \longrightarrow \hat{\theta}_\pm$

  HAVE WELL DEFINED ACTION ON BASIS STATES $|a_1 \cdots a_N; b_1 \cdots b_N\rangle$

* STATES THAT SATISFY QUANTUM TRAPPING CONDITIONS ~~HAVE~~ HAVE BEEN CONSTRUCTED

COHERENT STATES PEAKED ON CLASSICAL SOLN: OF CONSTRAINTS

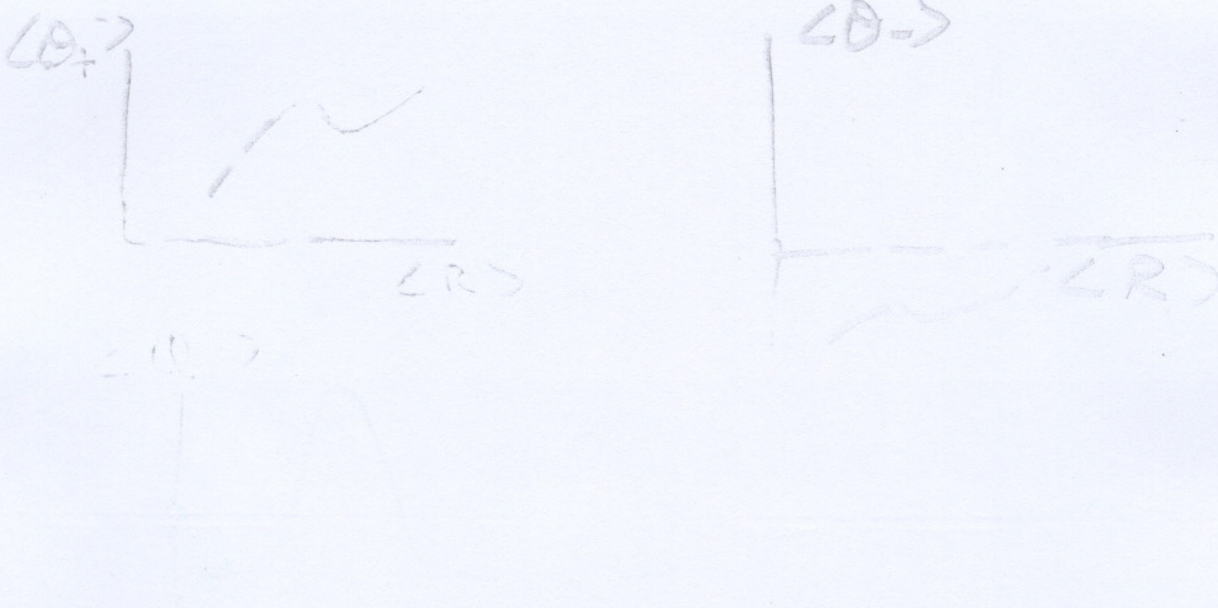
EVOLUTION (IN PROGRESS)

GOAL: TO EVOLVE INITIAL STATE

$$|\Psi\rangle_{t_0+\Delta t} = (I + i\,\Delta t\,\hat{H})\,|\Psi\rangle_{t_0}$$

STEP BY STEP, CHECKING THE TRAPPING CONDITION AS WE GO.

FOLLOW IN TIME THE VARIABLES



ENSURE THAT IF $\langle\Psi_0|\hat{C}_r|\Psi_0\rangle = 0 + \mathcal{O}(\ell_p)$

THEN $\langle\Psi_{\Delta t}|\hat{C}_r|\Psi_{\Delta t}\rangle = 0 + \mathcal{O}(\ell_p)$

SUMMARY

A FRAMEWORK FOR EXPLICIT GRAVITATIONAL COLLAPSE CALCULATIONS IN QG.

. . . . BUT

MUCH WORK STILL TO BE DONE.