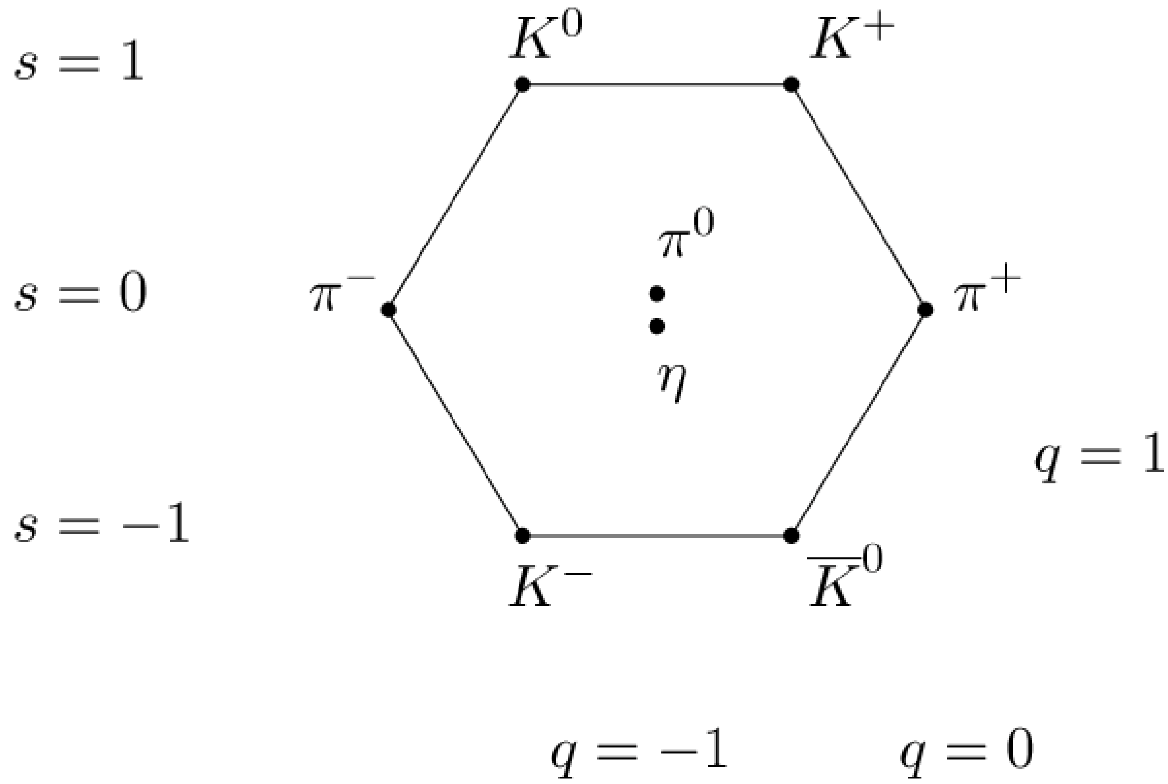
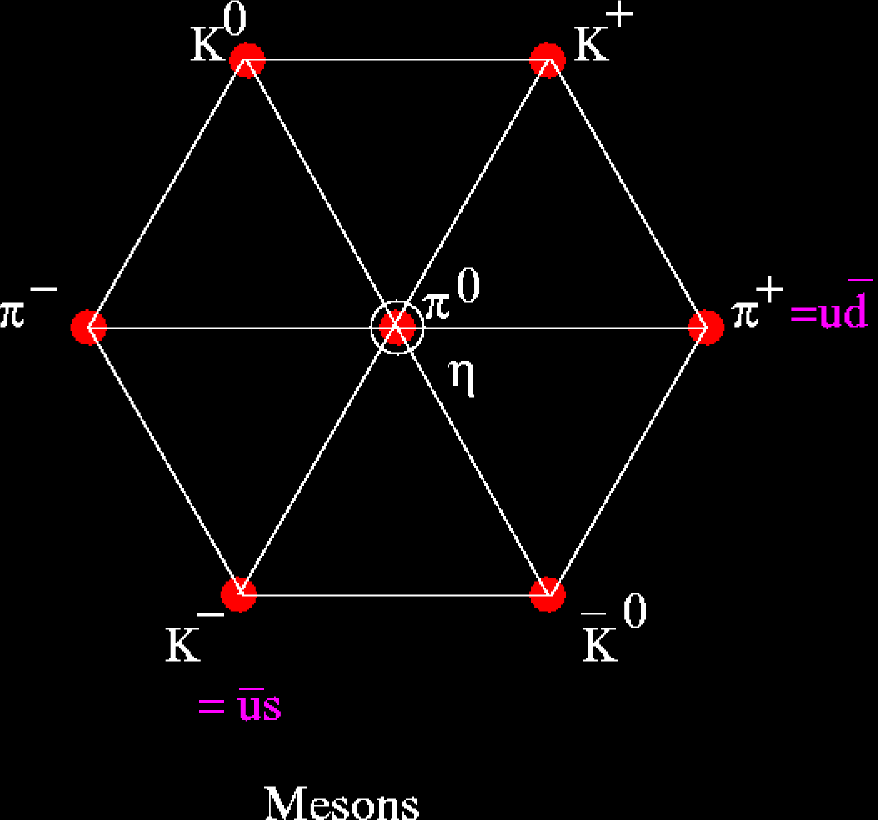
QCD, a practical introduction.

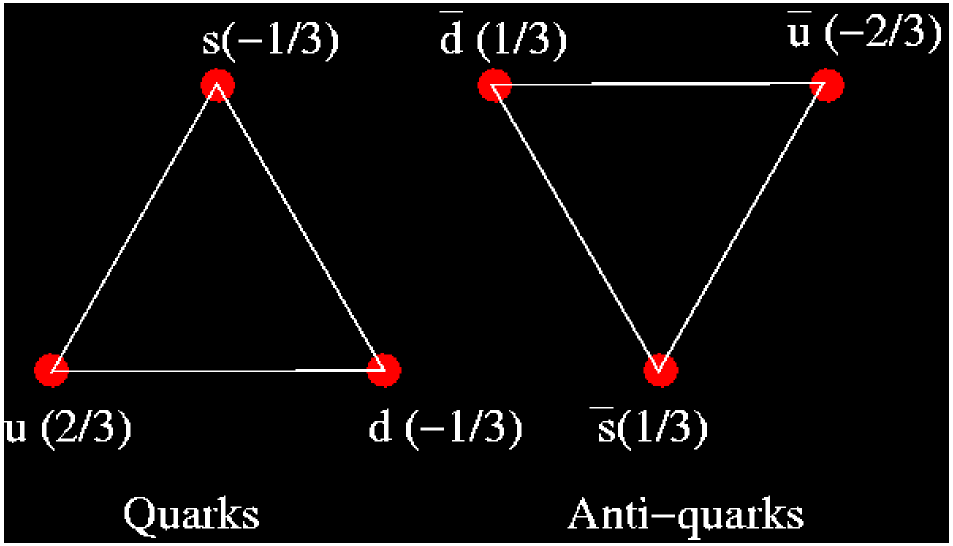
Bilal Masud
Centre for High Energy Physics
Punjab University



(with “strange” *strangeness* conserved experimentally as “associated production”) was explained as…..



with $8 \oplus 1 = 3 \otimes \bar{3}$



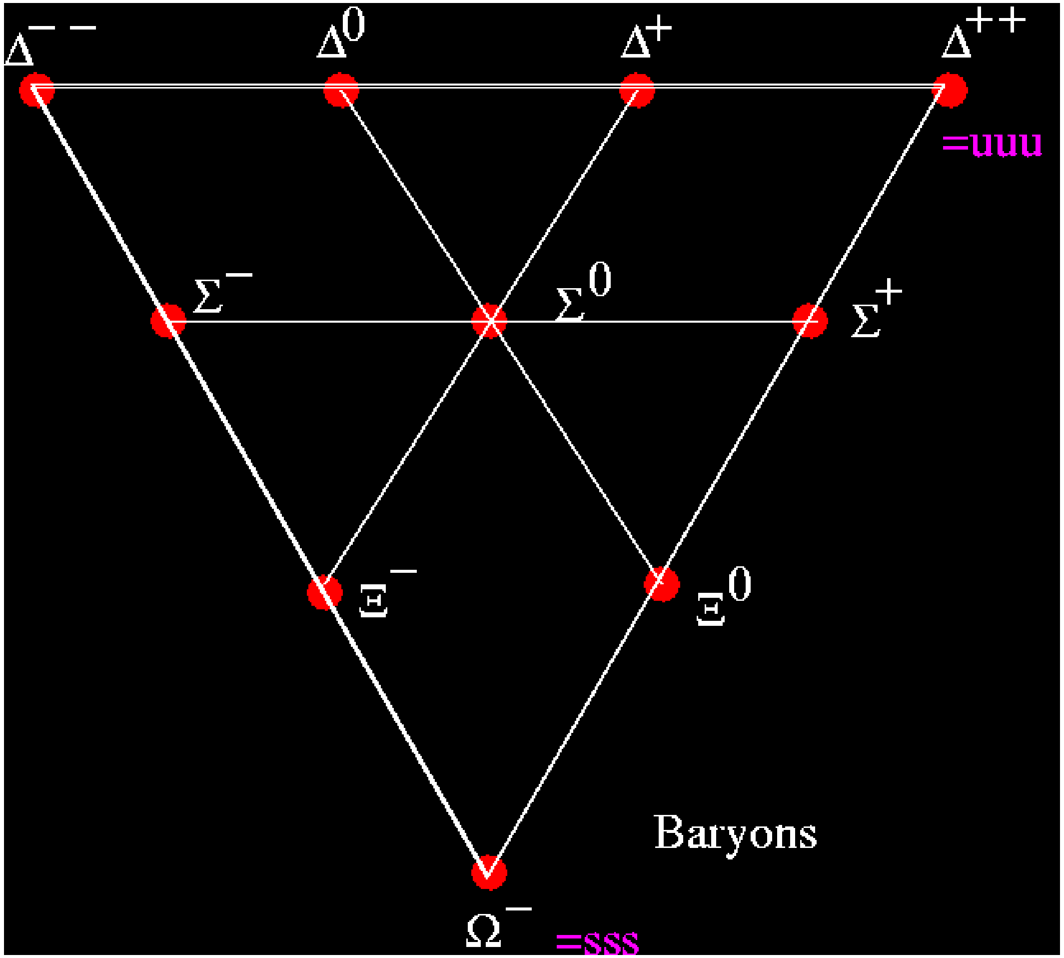
Associated Production….

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

Explained as

$$d\overline{u} + uud \rightarrow d\overline{s} + sud$$

Early problems, 3 identical Fermions….



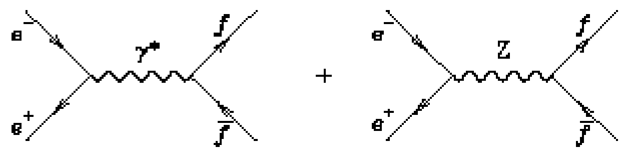
Colour Degree of Freedom…

$$\sigma_0 = \frac{4\pi\alpha^2}{3s} \cdot \sum_{\text{color}} \cdot \sum_f e_f^2 = \frac{4\pi\alpha^2}{3s} \cdot 3 \sum_f e_f^2$$

$$\sigma_0 = \frac{4\pi\alpha^2}{3Q^2} \sum_q e_q^2; \qquad R = \frac{\sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_q e_q^2 = \frac{2}{3}$$

Motivation for Colour SU(3)

- Consider the ratio R of the e^+e^- total hadronic cross section to the cross section for the production of a pair of point-like, charge-one objects such as muons.
- The virtual photon excites all electrically charged constituent-anticonstituent pairs from the vacuum.



- At low energy the virtual photon excites only the u, d and s quarks, each of which occurs in three colours.

$$
\begin{aligned}
R &= N_c \sum_i Q_i^2 \\
&= 3\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2\ .
\end{aligned}
$$

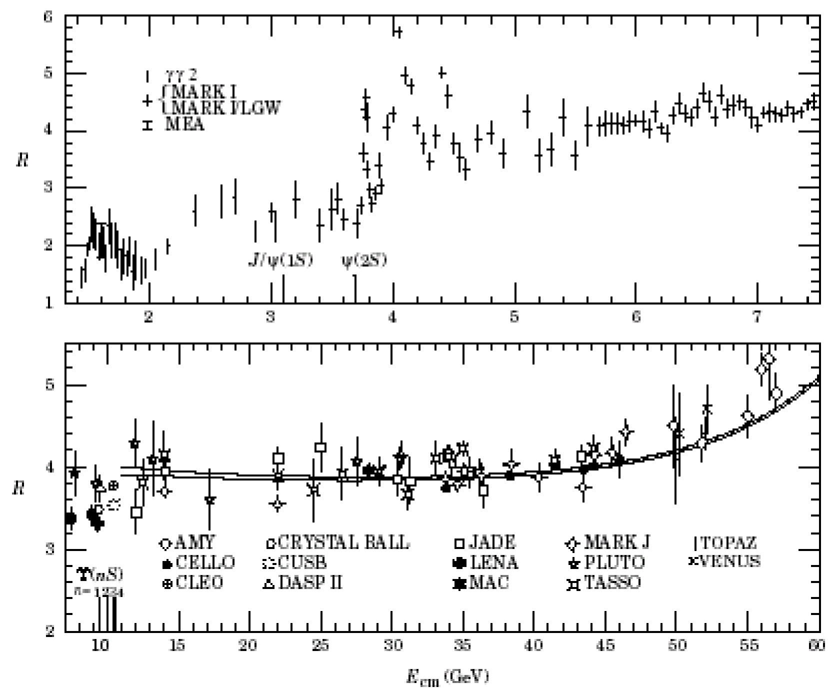
- For centre-of-mass energies $E_{\rm cm} \geq 10$ GeV, one is above the threshold for the production of pairs of c and b quarks, and so

$$
R = 3\left[2 \times \left(\frac{2}{3}\right)^2 + 3 \times \left(-\frac{1}{3}\right)^2\right] = \frac{11}{3}\ .
$$

Data

The data on R are in reasonable agreement with the prediction of the three colour model.

$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



The present situation….

Colour SU(3) and spectroscopy

- The observed baryons are interpreted as three-quark states.
- The quark constituents of the baryons are forced to have half-integral spin in order to account for the spins of the low-mass baryons.
- The quarks in the spin-$\frac{3}{2}$ baryons are then in a symmetrical state of space, spin and $SU(3)_f$ degrees of freedom.
- However the requirements of Fermi-Dirac statistics imply the total antisymmetry of the wave function.
- We introduce the colour degree of freedom: a colour index a with three possible values (usually called red, green, blue for $a = 1, 2, 3$) is carried by each quark.
- The baryon wave functions are totally antisymmetric in this new index.

Quark	Charge	Mass	Baryon Number	Isospin
u	$+\frac{2}{3}$	~ 4 MeV	$\frac{1}{3}$	$+\frac{1}{2}$
d	$-\frac{1}{3}$	~ 7 MeV	$\frac{1}{3}$	$-\frac{1}{2}$
c	$+\frac{2}{3}$	$\sim 1.5\ \mathrm{GeV}$	$\frac{1}{3}$	0
s	$-\frac{1}{3}$	~ 135 MeV	$\frac{1}{3}$	0
t	$+\frac{2}{3}$	$\sim 172\ \mathrm{GeV}$	$\frac{1}{3}$	0
b	$-\frac{1}{3}$	$\sim 5\ \mathrm{GeV}$	$\frac{1}{3}$	0

in the Summary Table, the $f_0(1500)$, $f_1(1510)$, $f_2(1950)$, $f_2(2300)$, $f_2(2340)$, and one of the two peaks in the $\eta(1440)$ entry are not in this table. Within the $q\bar{q}$ model, it is especially hard to find a place for the first two of these f mesons and for one of the $\eta(1440)$ peaks. See the "Note on Non-$q\bar{q}$ Mesons" at the end of the Meson Listings.

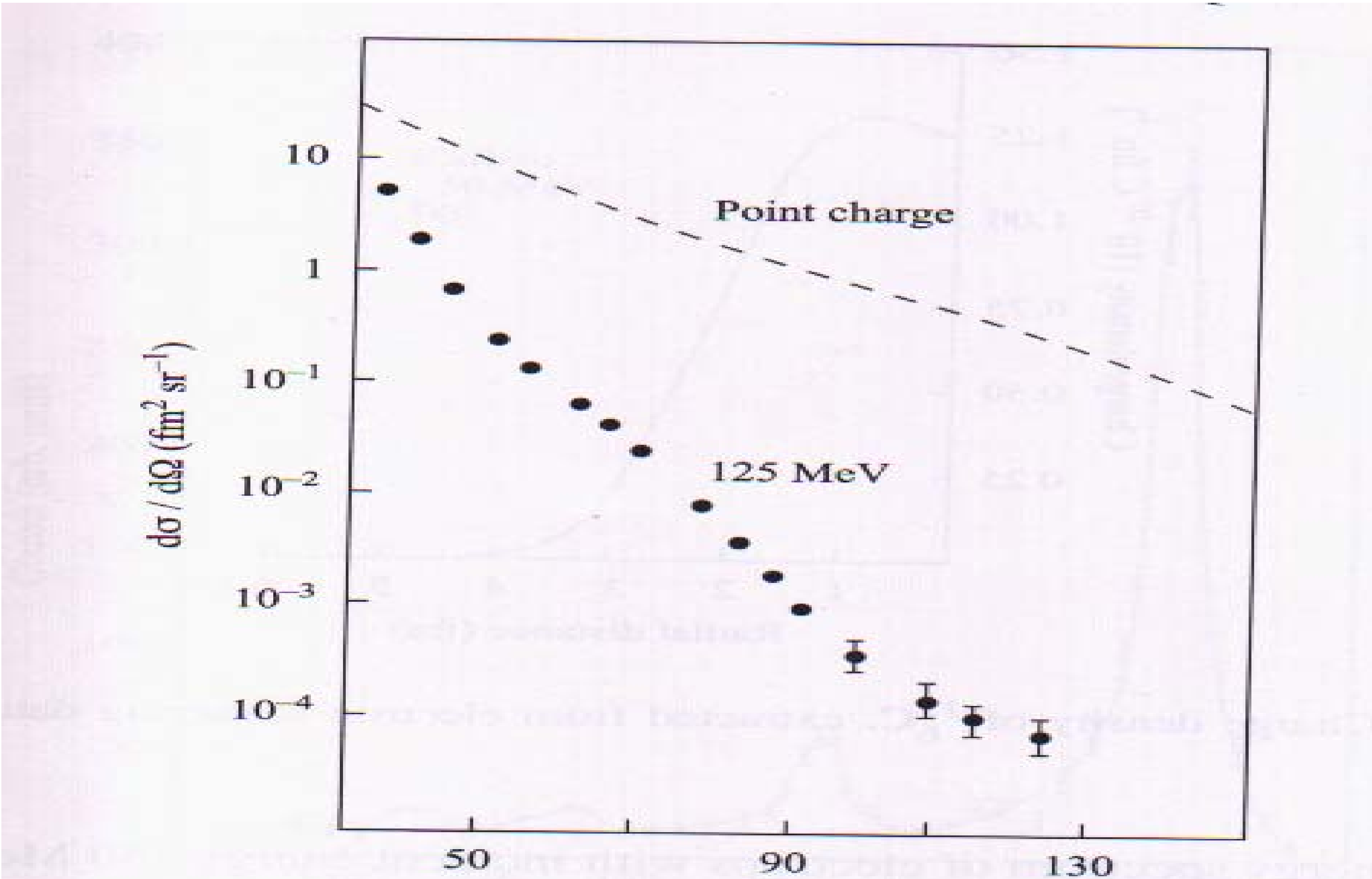
$N\ ^{2S+1}L_J$	J^{PC}	$u\bar{d}, u\bar{u}, d\bar{d}$ $I=1$	$u\bar{u}, d\bar{d}, s\bar{s}$ $I=0$	$c\bar{c}$ $I=0$	$b\bar{b}$ $I=0$	$\bar{s}u, \bar{s}d$ $I=1/2$	$c\bar{u}, c\bar{d}$ $I=1/2$	$c\bar{s}$ $I=0$	$\bar{b}u, \bar{b}d$ $I=1/2$	$\bar{b}s$ $I=0$	$\bar{b}c$ $I=0$
$1\ ^1S_0$	0^{-+}	π	η, η'	$\eta_c(1S)$	$\eta_b(1S)$	K	D	D_s	B	B_s	B_c
$1\ ^3S_1$	1^{--}	ρ	ω, ϕ	$J/\psi(1S)$	$\Upsilon(1S)$	$K^*(892)$	$D^*(2010)$	D_s^*	B^*	B_s^*	
$1\ ^1P_1$	1^{+-}	$b_1(1235)$	$h_1(1170), h_1(1380)$	$h_c(1P)$		K_{1B}†	$D_1(2420)$	$D_{s1}(2536)$			
$1\ ^3P_0$	0^{++}	$a_0(1450)$*	$f_0(1370)$*, $f_0(1710)$*	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$K_0^*(1430)$					
$1\ ^3P_1$	1^{++}	$a_1(1260)$	$f_1(1285), f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	K_{1A}†					
$1\ ^3P_2$	2^{++}	$a_2(1320)$	$f_2(1270), f_2'(1525)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$K_2^*(1430)$	$D_2^*(2460)$				
$1\ ^1D_2$	2^{-+}	$\pi_2(1670)$	$\eta_2(1645), \eta_2(1870)$			$K_2(1770)$					
$1\ ^3D_1$	1^{--}	$\rho(1700)$	$\omega(1650)$	$\psi(3770)$		$K^*(1680)$‡					
$1\ ^3D_2$	2^{--}					$K_2(1820)$					
$1\ ^3D_3$	3^{--}	$\rho_3(1690)$	$\omega_3(1670), \phi_3(1850)$			$K_3^*(1780)$					
$1\ ^3F_4$	4^{++}	$a_4(2040)$	$f_4(2050), f_4(2220)$			$K_4^*(2045)$					
$2\ ^1S_0$	0^{-+}	$\pi(1300)$	$\eta(1295), \eta(1440)$	$\eta_c(2S)$		$K(1460)$					
$2\ ^3S_1$	1^{--}	$\rho(1450)$	$\omega(1420), \phi(1680)$	$\psi(2S)$	$\Upsilon(2S)$	$K^*(1410)$‡					
$2\ ^3P_2$	2^{++}	$a_2(1700)$	$f_2(1950), f_2(2010)$		$\chi_{b2}(2P)$	$K_2^*(1980)$					
$3\ ^1S_0$	0^{-+}	$\pi(1800)$	$\eta(1760)$			$K(1830)$					

* See our scalar minireview in the Particle Listings. The candidates for the $I=1$ states are $a_0(980)$ and $a_0(1450)$, while for $I=0$ they are: $f_0(600)$, $f_0(980)$, $f_0(1370)$, and $f_0(1710)$. The light scalars are problematic, since there may be two poles for one $q\bar{q}$ state and $a_0(980)$, $f_0(980)$ may be $K\bar{K}$ bound states.

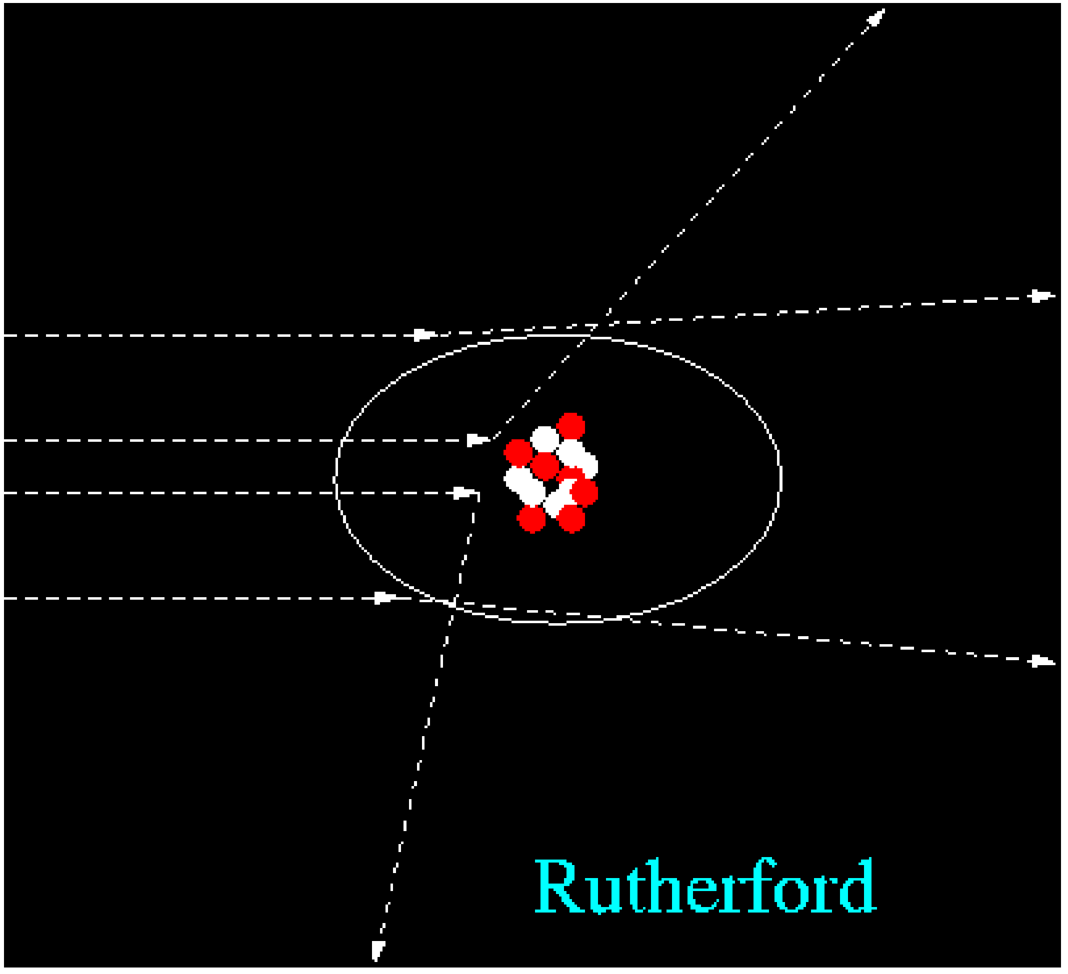
† The K_{1A} and K_{1B} are nearly equal (45°) mixes of the $K_1(1270)$ and $K_1(1400)$.

Not only history: Parton model

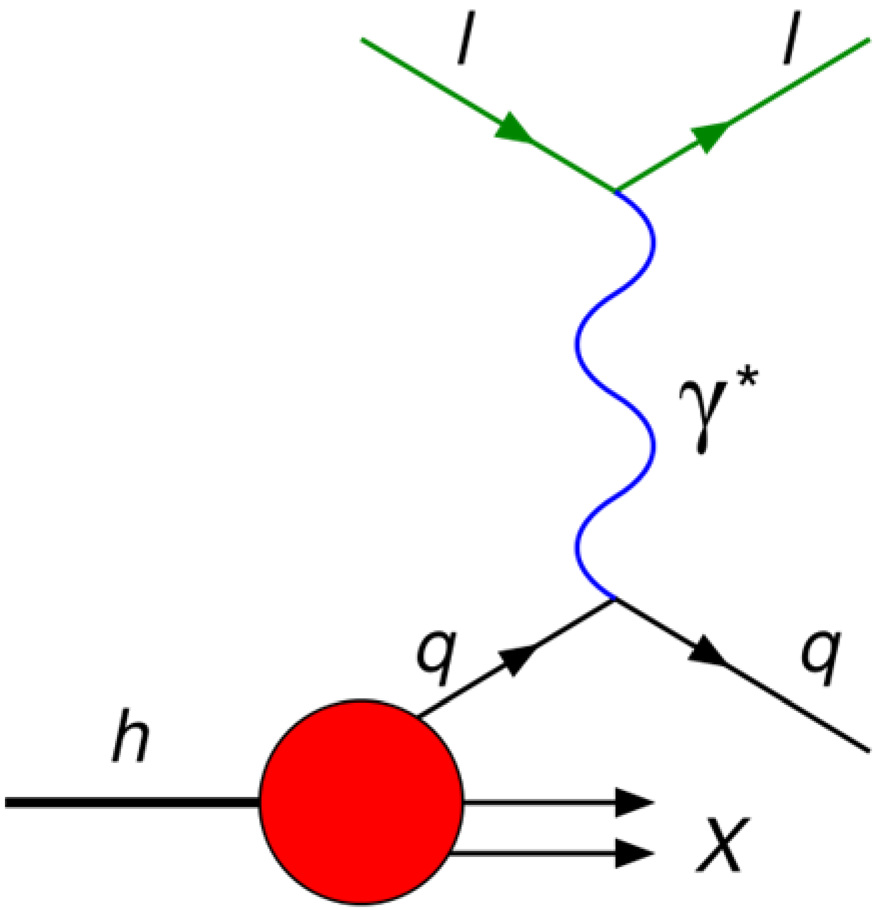
- sub-structure and related differential cross section



was (earlier) interpreted as…

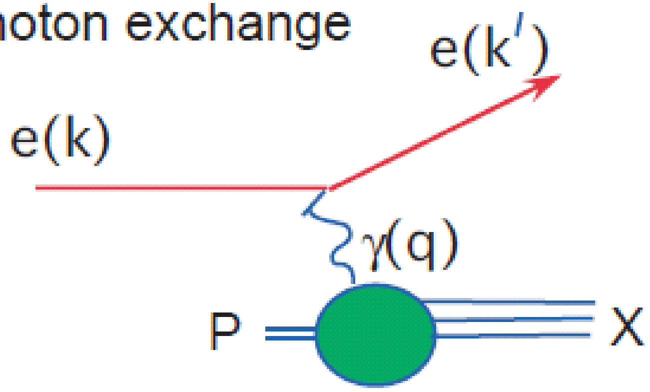


Lepton Hadron (Electron-Proton) Scattering (to be compared, *through QED,* with point-like electron-muon scattering)

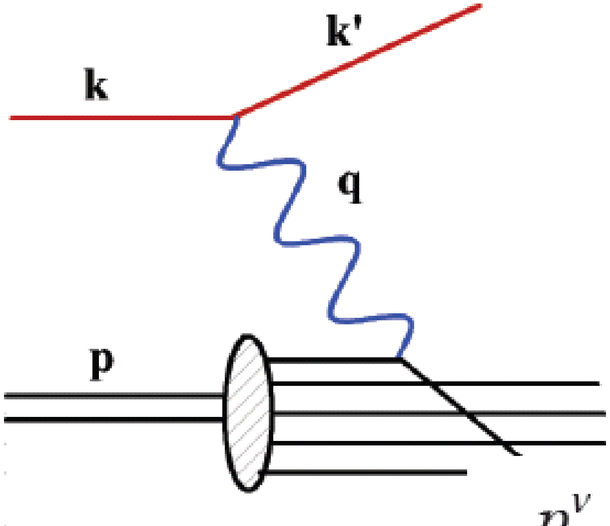


2. DIS: Structure Functions and Scaling

Photon exchange



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k) \times \frac{-ig^{\mu\mu'}}{q^2} \times \langle X|\, eJ^{\rm EM}_{\mu'}(0)\,|p,\sigma\rangle$$



$$Q^2 = -q^2 = 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$$

$$p^\nu = \left(m, \vec{0}\right) \; ,$$

$$k^\mu = \left(E, \vec{k}\right) = \left(E, 0, 0, k\right) \; ,$$

$$k'^\mu = \left(E', \vec{k}'\right) = \left(E', k'\sin\theta, 0, k'\cos\theta\right)$$

$$\nu = \frac{p \cdot q}{m} = \left(E - E'\right)$$

$$x = \frac{-q^2}{2m\nu} \equiv \frac{Q^2}{2m\nu} = \frac{Q^2}{2m\left(E - E'\right)}$$

The leptonic tensor:

$$L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda,\lambda'} (\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k))^* \, (\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k))$$

$$= \frac{e^2}{2\pi^2} \left(k^\mu k'^{\,\nu} + k'^{\,\mu} k^\nu - g^{\mu\nu} k \cdot k' \right)$$

- **The hadronic tensor:**

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J_\mu|p,\sigma\rangle^* \langle X|J_\nu|p,\sigma\rangle \; (2\pi)^4 \, \delta^4(p_X - p - q)$$

And the cross section:

$$2\omega_{k'} \frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2} \; L^{\mu\nu} W_{\mu\nu}$$

$W_{\mu\nu}$ **has sixteen components,**

but known properties of the strong interactions constrain $W_{\mu\nu}$ **. . .**

$$\partial^\mu J_\mu^{\mathrm{EM}}(x) = 0$$

$$\Rightarrow \langle X | \, \partial^\mu J_\mu^{\mathrm{EM}}(x) \, | p \rangle = 0$$

$$\Rightarrow (p_X - p)^\mu \langle X | \, J_\mu^{\mathrm{EM}}(x) \, | p \rangle = 0$$

$$\Rightarrow q^\mu W_{\mu\nu} = 0$$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1(\nu, Q^2) + \left(p_\mu - q_\mu \frac{p.q}{q^2} \right)\left(p_\nu - q_\nu \frac{p.q}{q^2} \right) W_2(\nu, Q^2)$$

$$F_1 = M W_1 \qquad\qquad F_2 = \nu W_2$$

Point-Like Scattering

$$p_X{}^2 = p^2 = M^2$$ implies $$\frac{Q^2}{2M\nu} = 1$$

Q^2 and ν are *not* independent

And if there is a point-like *parton i* inside a proton carrying a fraction *f* of the proton's four-momentum

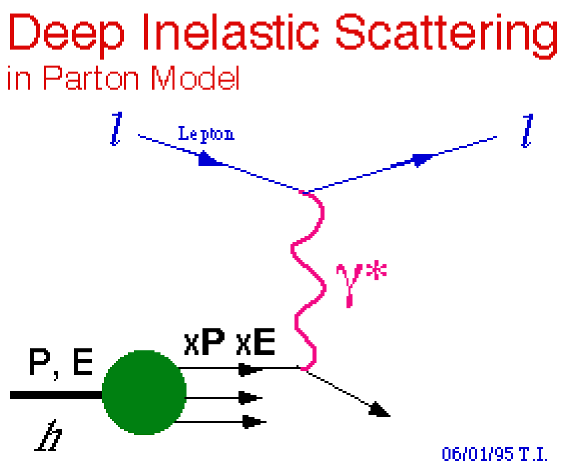
$$p_i{}^\mu \approx fp^\mu$$

Q^2 and ν are *not* independent

$$m_i \approx fM$$

$$(q + fp)^2 = m_i{}^2$$

$$f = \frac{Q^2}{2M\nu} \equiv x$$



Probability distributions *f(x)* of the fraction *x*

$$W_2(\nu, Q^2) = \sum_i \int_0^1 dx f_i(x) e_i^{\,2} \delta\left(\nu - \frac{Q^2}{2Mx}\right)$$

W_2 for a point-like parton (like a muon in QED) is calculated, using the following, to be $e_i^{\,2}\delta\left(\nu - \frac{Q^2}{2Mx}\right)$

In the lab frame:

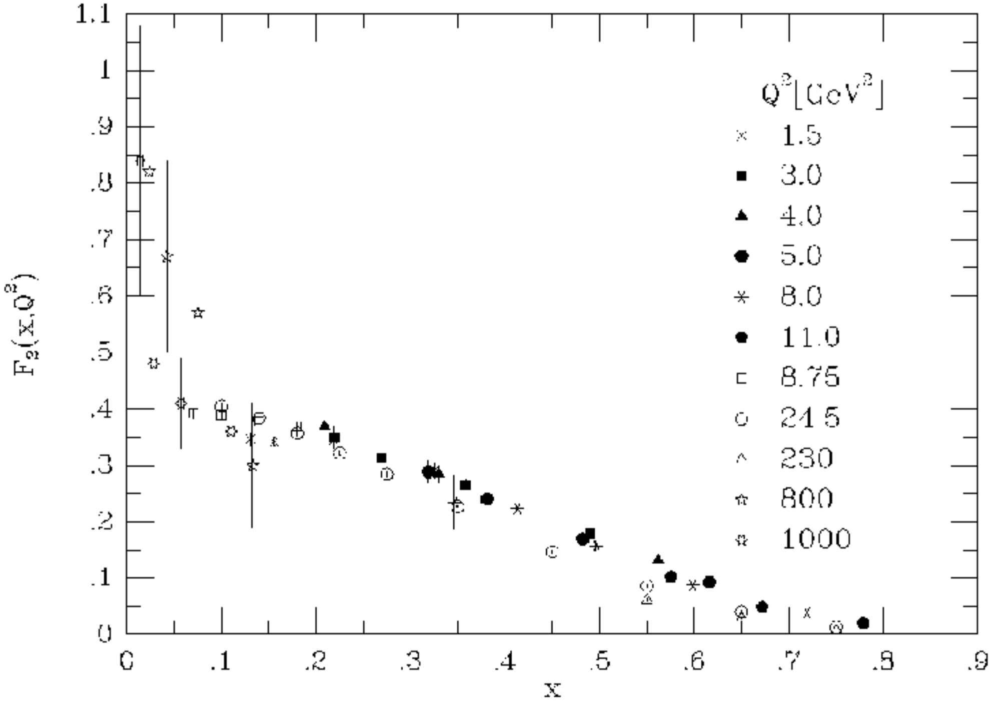
$$L_{\mu\nu}W^{\mu\nu} = 4EE'\cos^2\theta/2\left[W_2(\nu, q^2) + 2W_1(\nu, q^2)\tan^2\theta/2\right]$$

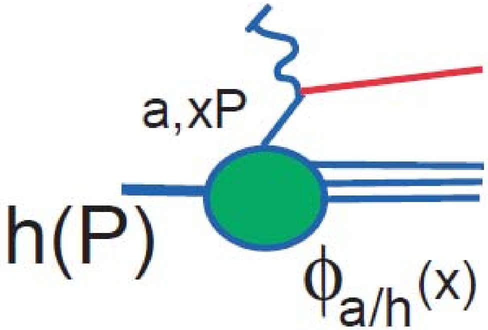
electron-muon elastic scattering:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\left[1 - \frac{q^2}{2M^2}\tan^2\theta/2\right]\delta\left(\nu + \frac{q^2}{2M}\right)$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2}\left[W_2(\nu, q^2) + 2W_1(\nu, q^2)\tan^2\theta/2\right]$$

The above means the structure functions W_2 and F_2 *are functions of the Bjorken* x only. This is called the *Bjorken scaling*





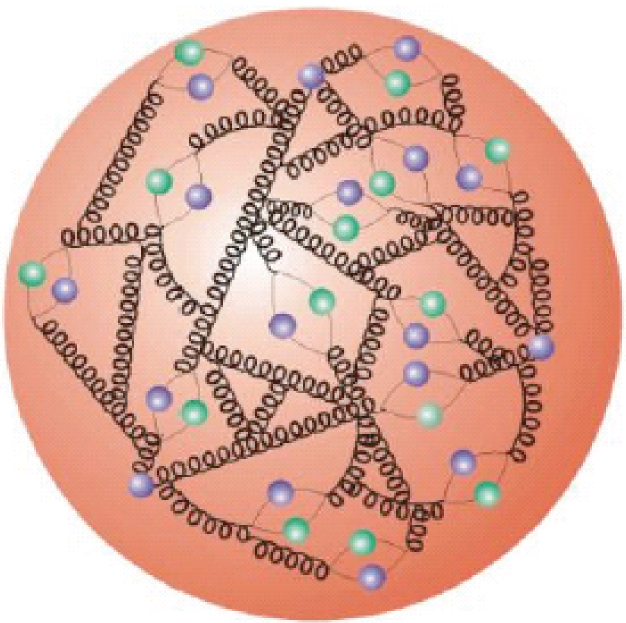
Basic Parton Model Relation

$$\sigma_{\rm eh}(p,q) = \sum_{\text{partons } a} \int_0^1 d\xi\, \hat{\sigma}^{\rm el}_{ea}(\xi p, q)\, \phi_{a/h}(\xi)$$

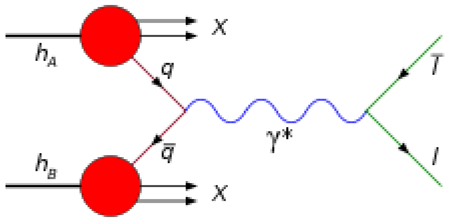
- **where**: σ_{eh} is the cross section for $e(k) + h(p) \to e(k' = k - q) + X(p + q)$
- **and** $\hat{\sigma}^{\rm el}_{ea}(xp, q)$ is the elastic cross section for $e(k) + a(\xi p) \to e(k' - q) + a(\xi p + q)$ which sets $(\xi p + q)^2 = 0 \to \xi = -q^2/2p\cdot q \equiv x$.

- **and** $\phi_{a/h}(x)$ is the distribution of parton a in hadron h, the "probability for a parton of type a to have momentum xp".

Artist View of a Proton



The proton is not an elementary particle. It's a mess!



When we collide protons on anti-protons at the Tevatron or soon protons on protons at the LHC, we need to know the momentum distributions (parton distribution functions) of the quarks and gluons inside the proton.

$$u^p(x) = d^n(x) \equiv u(x) \qquad \bar{u}^p(x) = \bar{d}^n(x) \equiv \bar{u}(x)$$

$$d^p(x) = u^n(x) \equiv d(x) \qquad \bar{d}^p(x) = \bar{u}^n(x) \equiv \bar{d}(x)$$

$$s^p(x) = s^n(x) \equiv s(x) \qquad \bar{s}^p(x) = \bar{s}^n(x) \equiv \bar{s}(x)$$

Sea Quarks

There is a sea of virtual quark - antiquark pairs inside the proton.

$$u_{\text{sea}}(x) = \bar{u}_{\text{sea}}(x), \quad d_{\text{sea}}(x) = \bar{d}_{\text{sea}}(x), \quad s_{\text{sea}}(x) = \bar{s}_{\text{sea}}(x)$$

The c, b and t quarks are too heavy to contribute much.

$$u(x) = u_{\text{val}}(x) + u_{\text{sea}}(x) \qquad d(x) = d_{\text{val}}(x) + d_{\text{sea}}(x)$$

Sum Rules:

$$\int \left[u(x) - \bar{u}(x)\right] \mathrm{d}x = 2$$

$$\int \left[d(x) - \bar{d}(x)\right] \mathrm{d}x = 1$$

$$\int \left[s(x) - \bar{s}(x)\right] \mathrm{d}x = 0$$

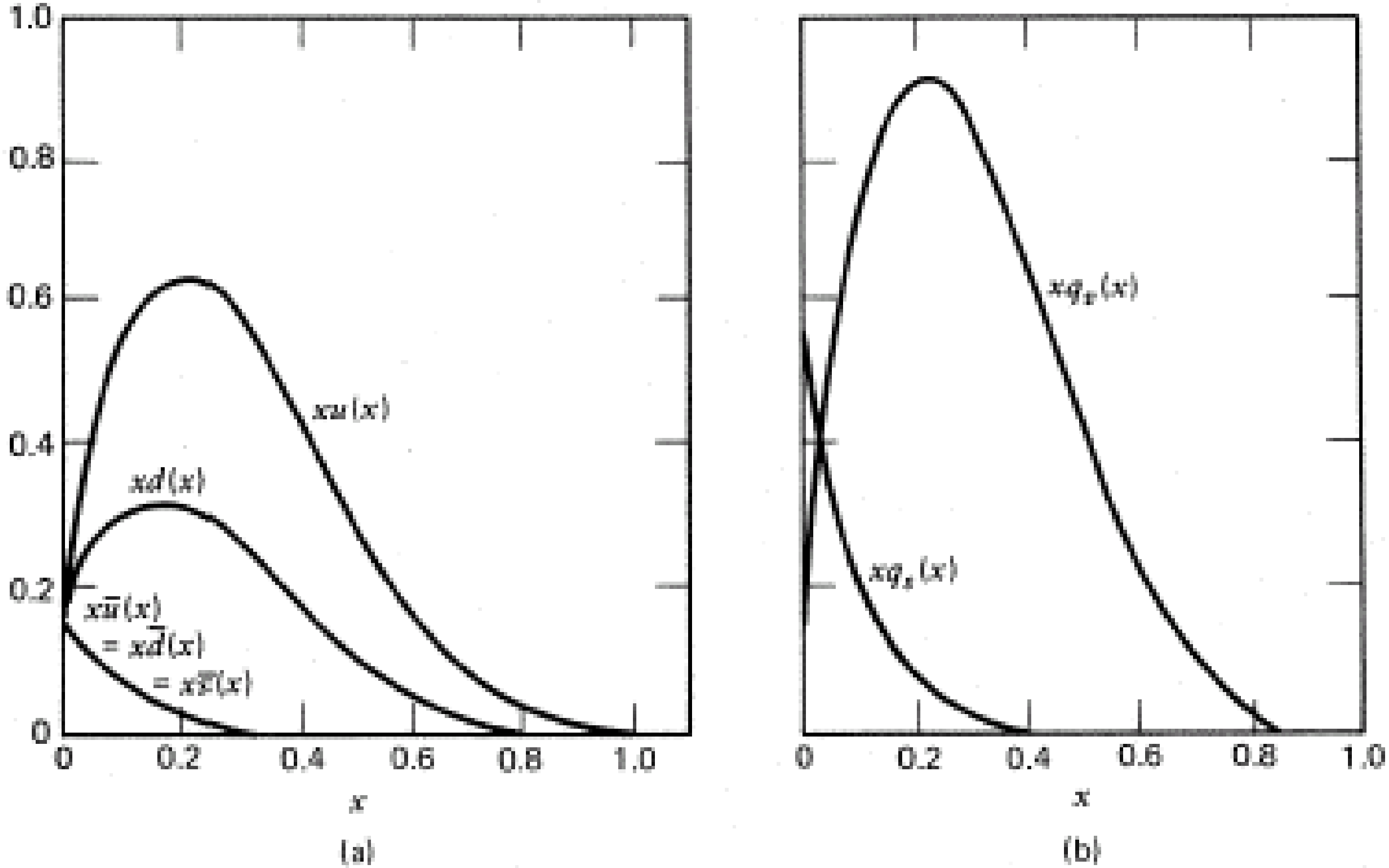
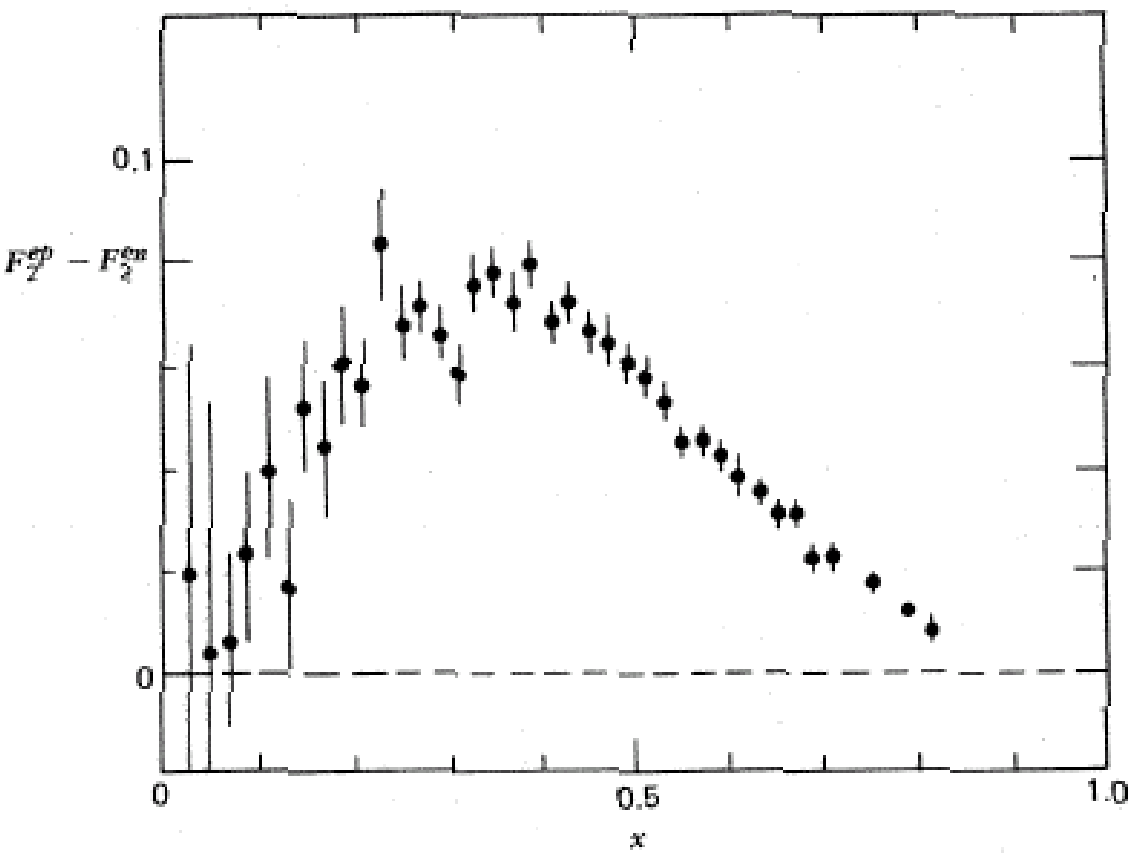
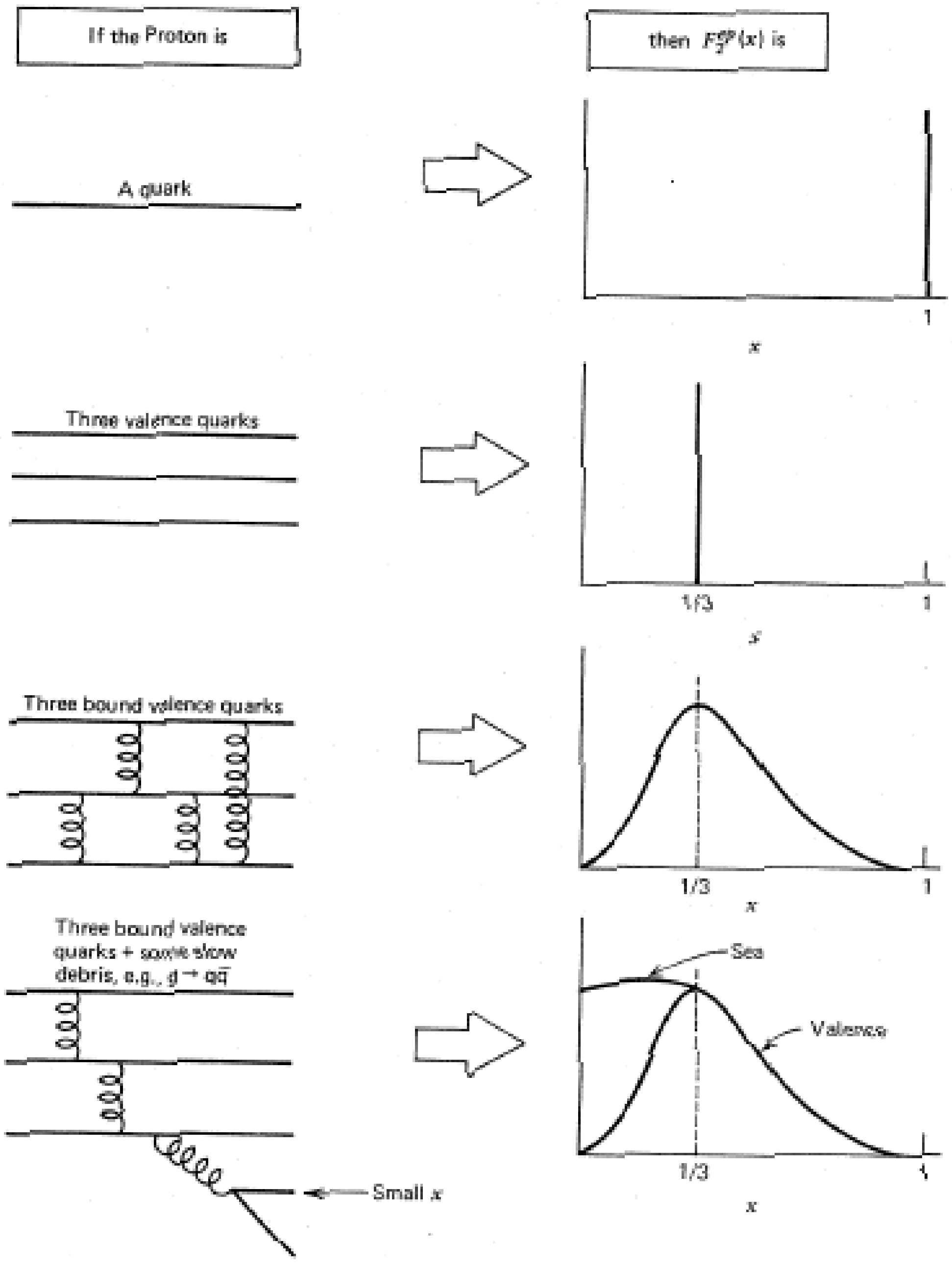


Fig. 9.9 The quark structure functions extracted from an analysis of deep inelastic scattering data. Figure (b) shows the total valence and sea quark contributions to the structure of the proton.

When sea contribution cancels….





Only QED!

H*ere*, we need QCD!

QCD effects modify all above as…

- 1-Quark-gluon vertex (like electron-photon vertex but multiplied with “colour factors”)

The 3-fold colour degree of freedom for quarks and antiquarks, combined with 8-fold “bicolour” degree of freedom for a gluon means that the strength of the Quark-gluon vertex is to be chosen from 3*3*8=72 numbers, read as elements of 8 SU(3) generator lambda matrices, each of order 3*3

$$\left(D_\mu\right)_{ab} = \partial_\mu \delta_{ab} + ig\left(t^B G^B_\mu\right)_{ab}$$

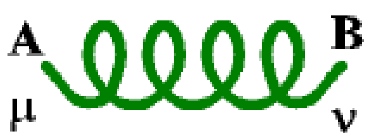
$$\left[t^B, t^C\right] = if^{BCD} t^D$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^B_{\alpha\beta}F^{B,\alpha\beta} + \sum_f \bar{q}_{f,a}\left(iD_\mu\gamma^\mu - m_f\right)_{ab} q_{f,b}$$

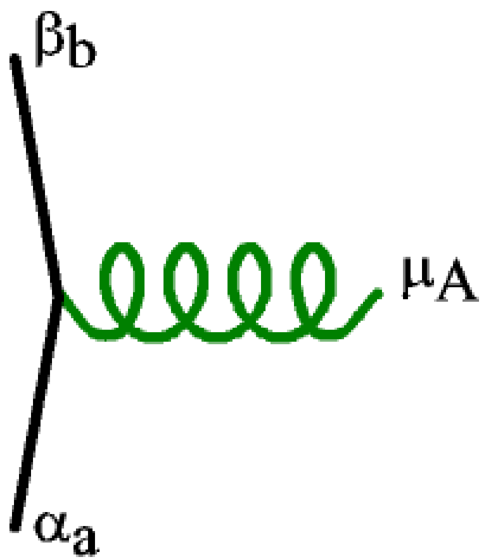
$$F^B_{\alpha\beta} = \left[\partial_\alpha G^B_\beta - \partial_\beta G^B_\alpha - gf^{BCD}G^C_\alpha G^D_\beta\right].$$

Quark 

$$\frac{i\delta_{ab}}{(\gamma^\mu q_\mu - m)_{\alpha\beta}} = \frac{i(\gamma^\mu q_\mu + m)_{\alpha\beta}\,\delta_{ab}}{q^2 - m^2}$$

Gluon 

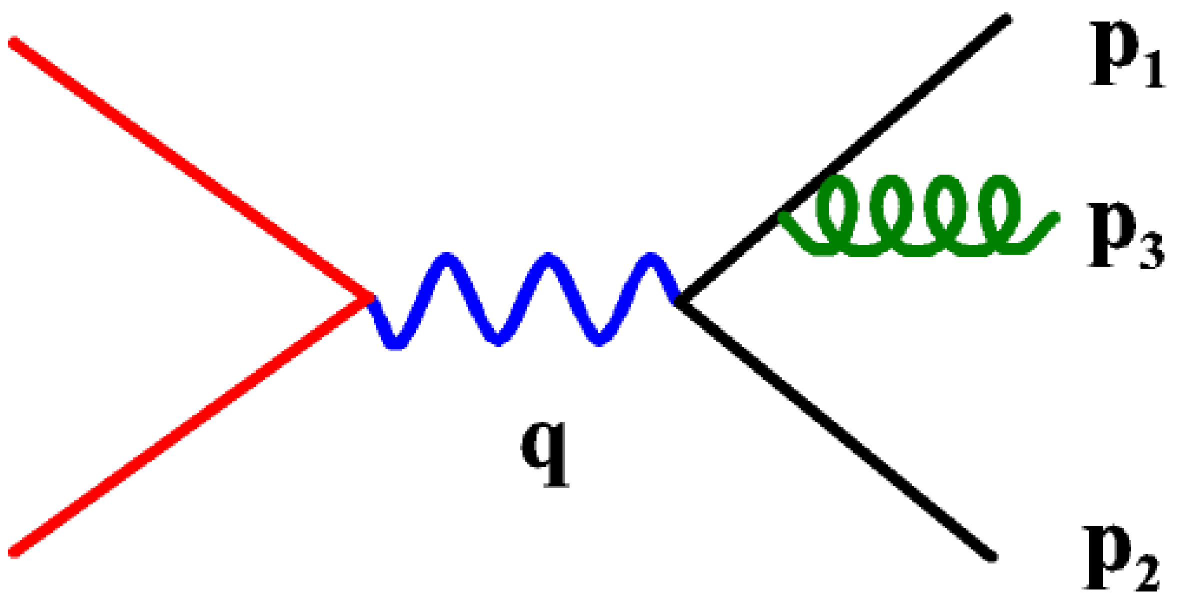
$$\frac{-i}{q^2}\delta_{AB}\left[g^{\mu\nu}-(1-\lambda)\frac{q^\mu q^\nu}{q^2}\right]$$



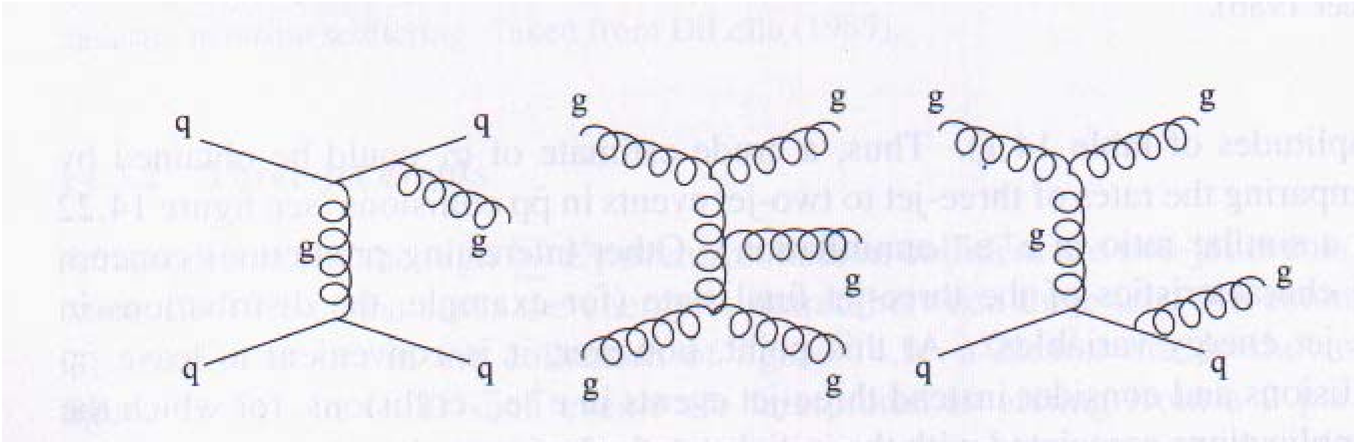
$$-ig\gamma^\mu_{\alpha\beta}\frac{\lambda^A_{ab}}{2}$$

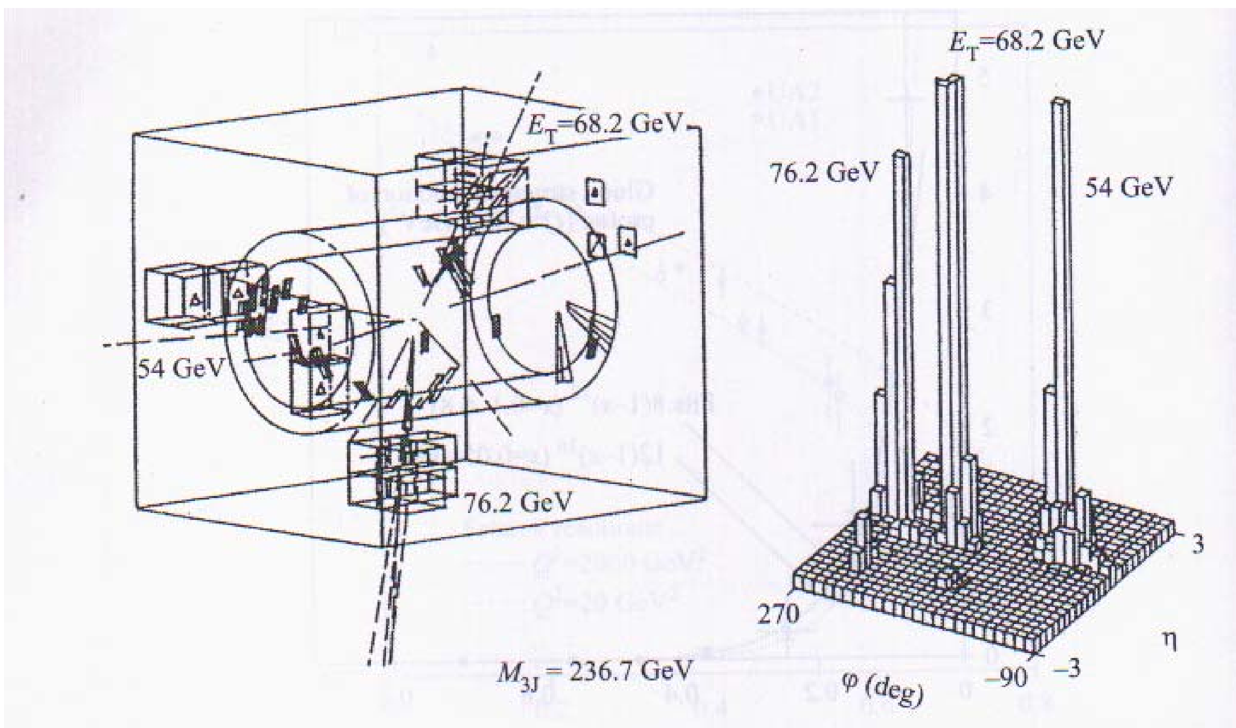
Effects of Quark-Gluon vertex

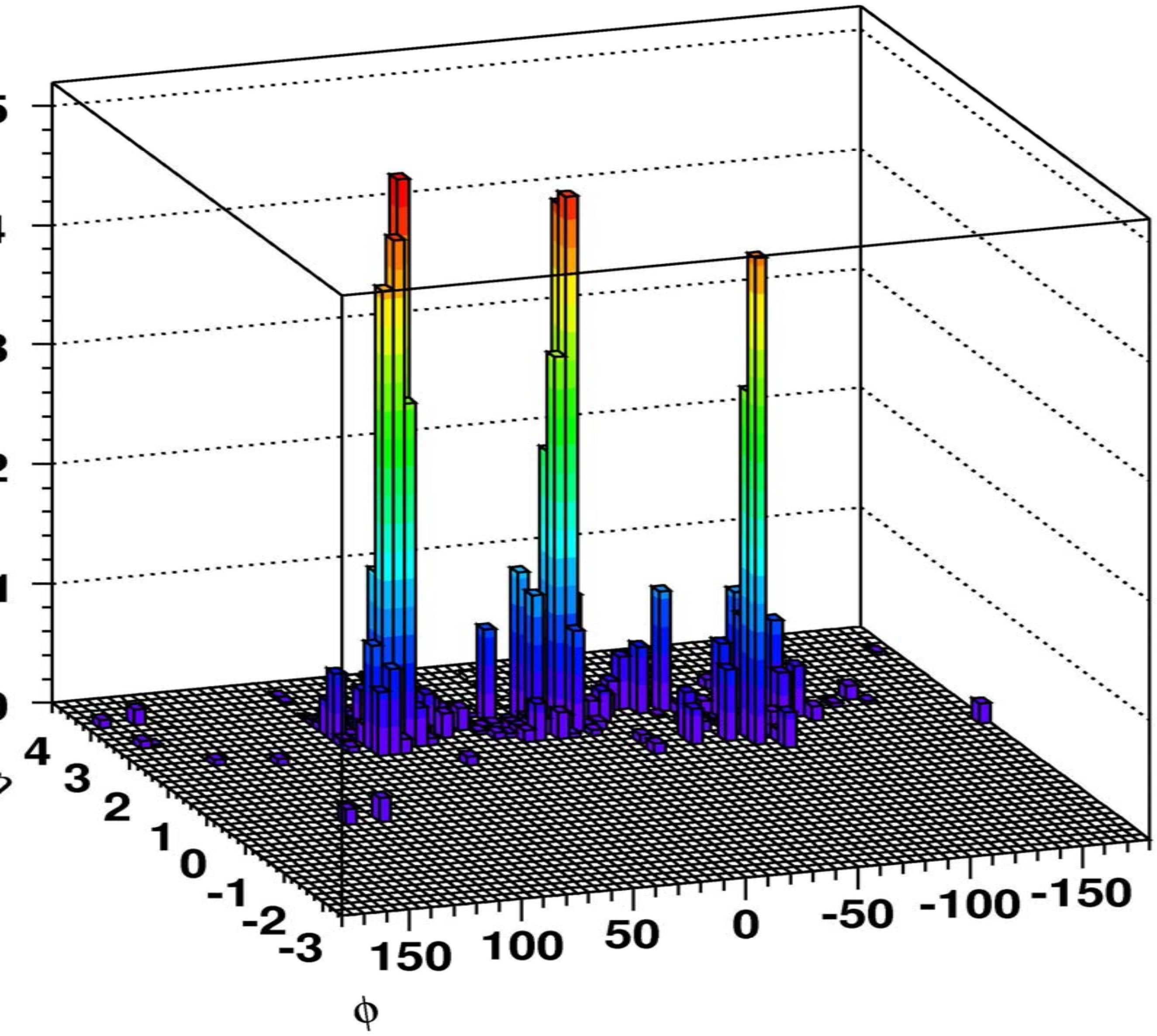
Meaning a) three jet events in addition to the two-jet events expected from the “QED portion”….



Or….

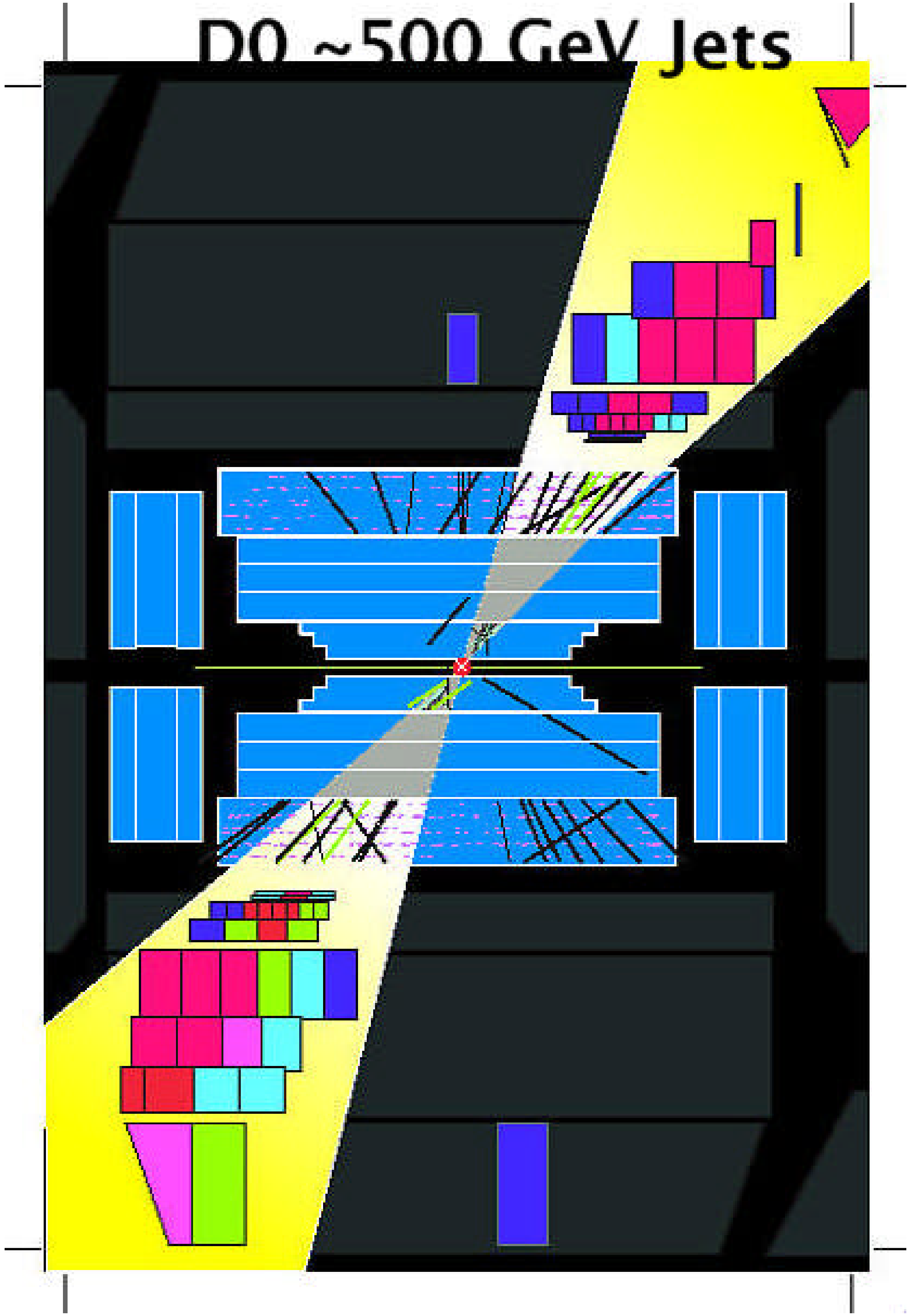

q
q
g
g
q
q
g
g
g
g
g
g
g
g
g
q
q


E_T=68.2 GeV
54 GeV
76.2 GeV
M_{3J} = 236.7 GeV
E_T=68.2 GeV
76.2 GeV
54 GeV
270
φ (deg)
−90
3
η
−3

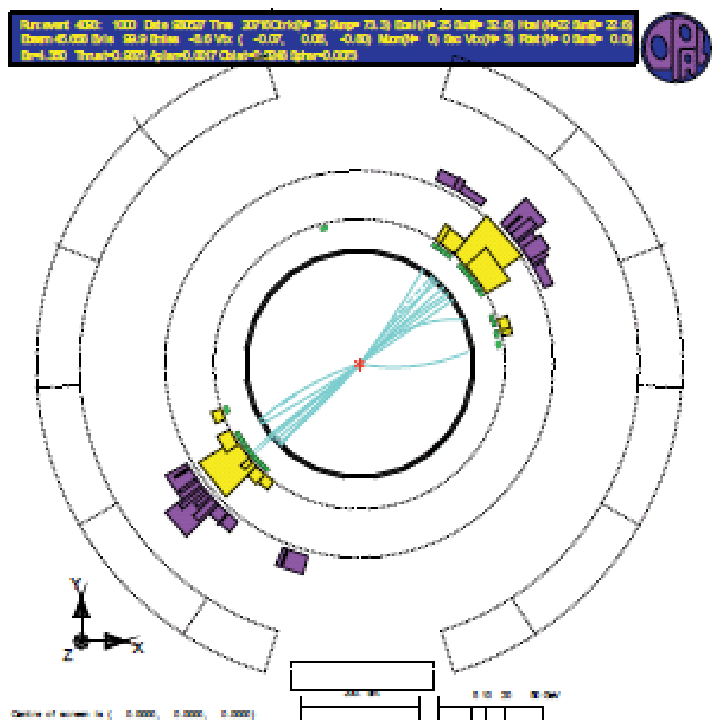


A QCD effect

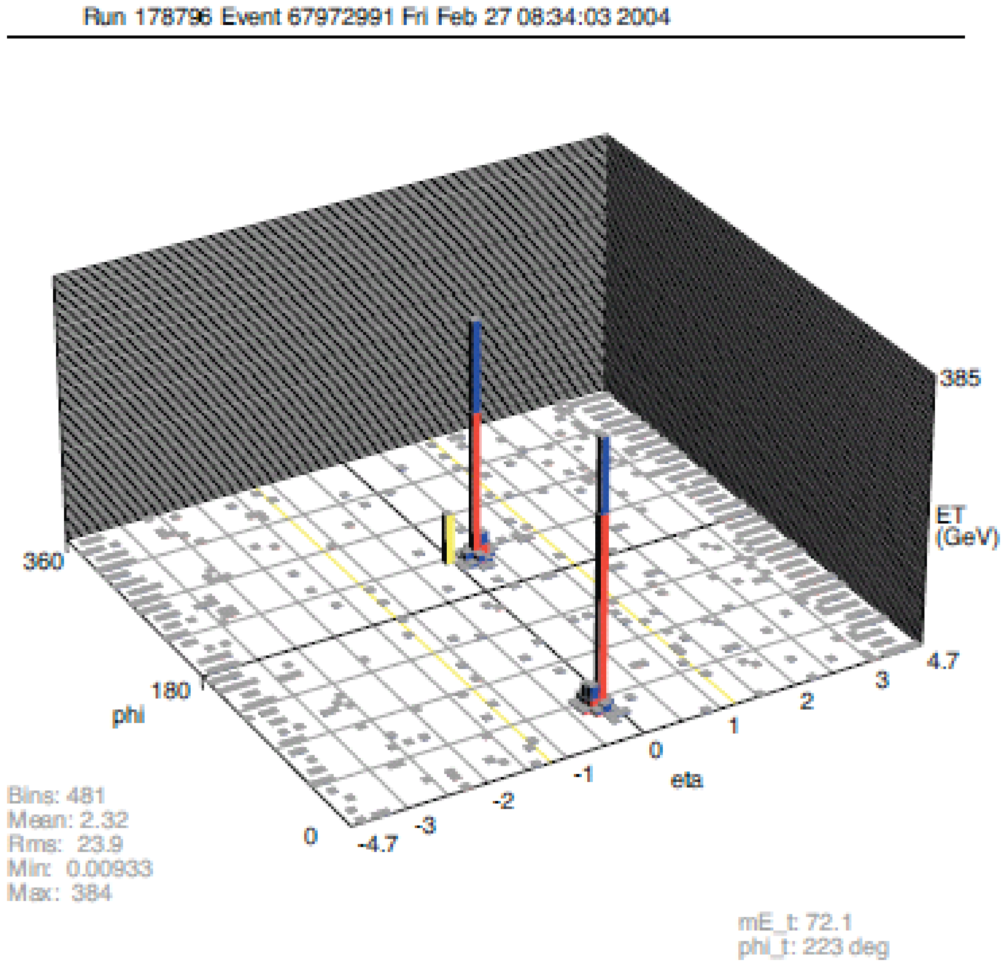
For a comparison: The QED portion



For e^+e^-:



And in nucleon-nucleon collisions:



b) diagrams like

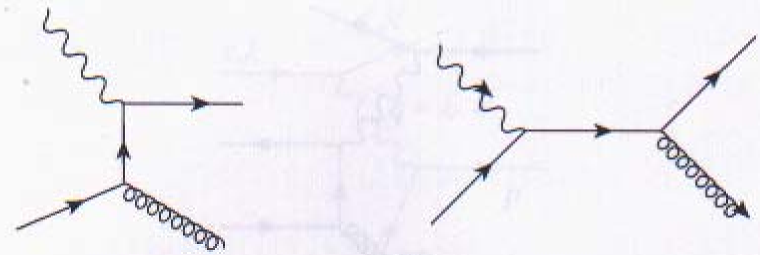


Figure 15.12. Virtual photon processes entering into figure 15.9.

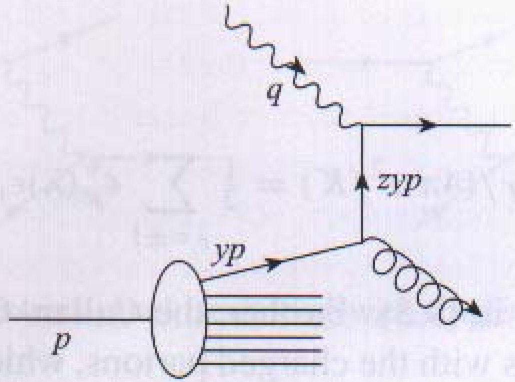
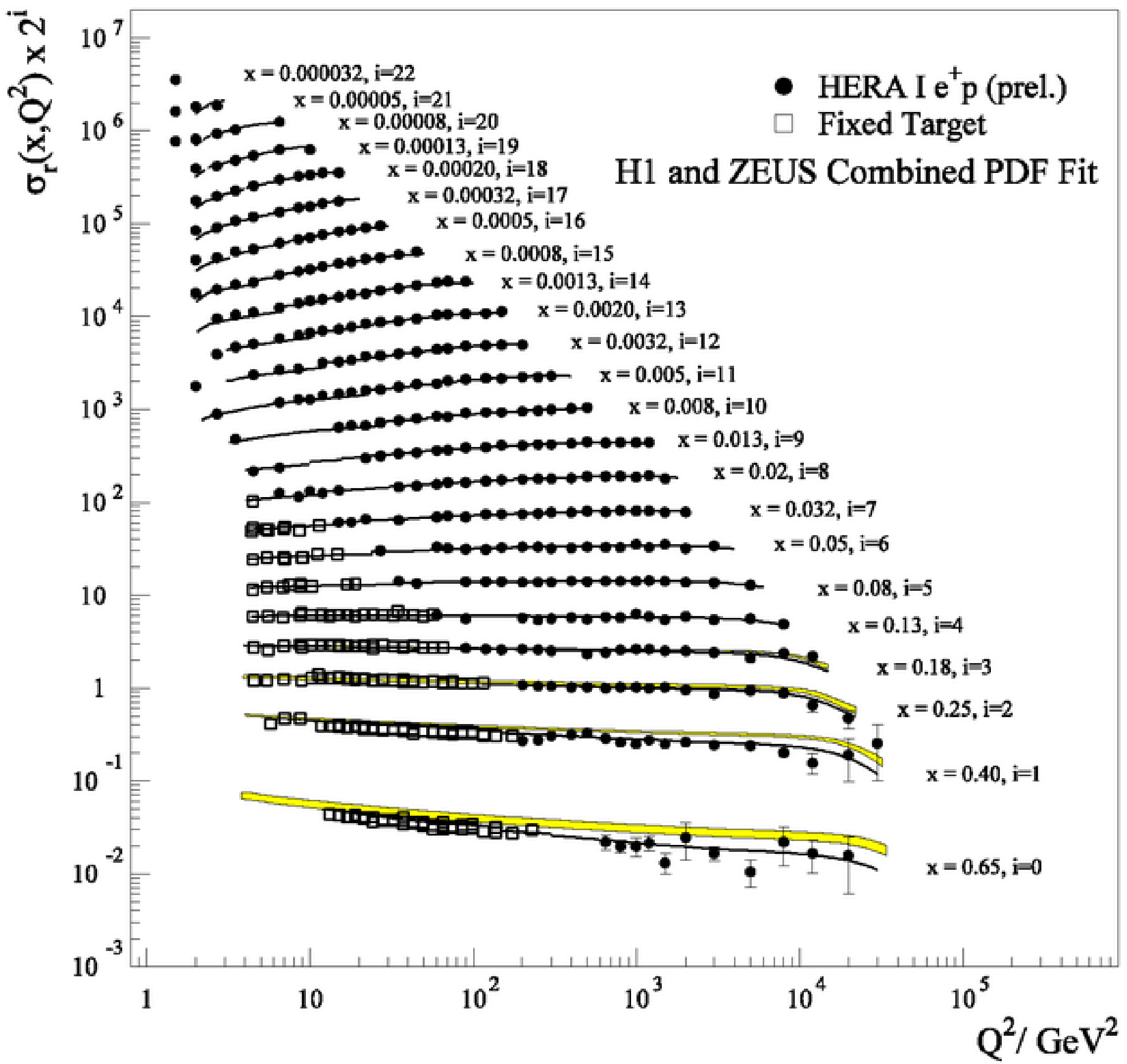
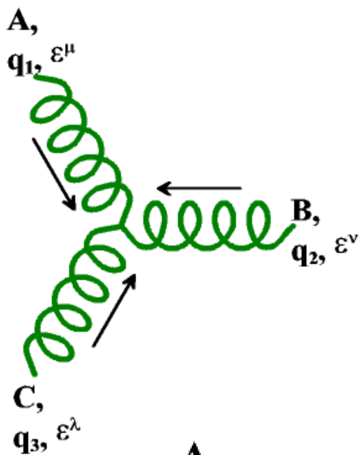


Figure 15.13. The first process of figure 15.12, viewed as a contribution to e$^-$–nucleon scattering.

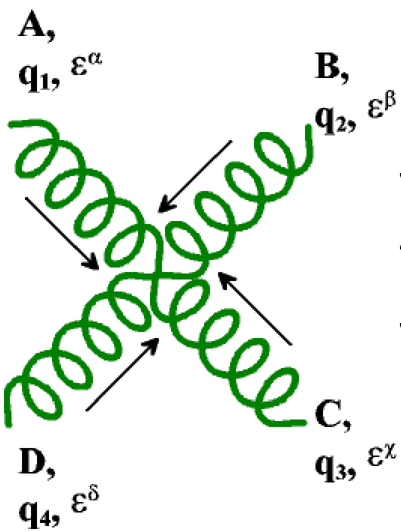
explain “*violation* of Bjorken scaling”



Also Gluon-Gluon vertices



$$-g f^{ABC}[(q_1-q_2)^{\lambda} g^{\mu\nu} + (q_2-q_3)^{\mu} g^{\nu\lambda} + (q_3-q_1)^{\nu} g^{\mu\lambda}]$$

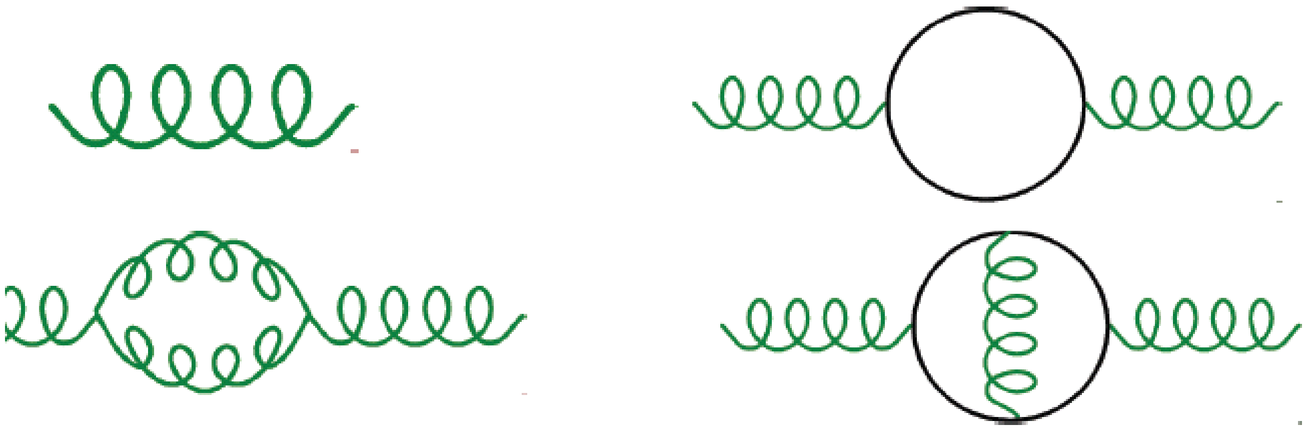


$$-ig^2 f^{EAD} f^{EBC}[g^{\alpha\beta} g^{\chi\delta} - g^{\alpha\chi} g^{\beta\delta}]$$

$$-ig^2 f^{EAC} f^{EBD}[g^{\alpha\beta} g^{\chi\delta} - g^{\alpha\delta} g^{\beta\chi}]$$

$$-ig^2 f^{EAB} f^{ECD}[g^{\alpha\chi} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\chi}]$$

Reversing signs of loop contribution to the "running of coupling constant" resulting from RENORMALIZATION: in QCD coupling constant *decreases* with larger momentum transfers….

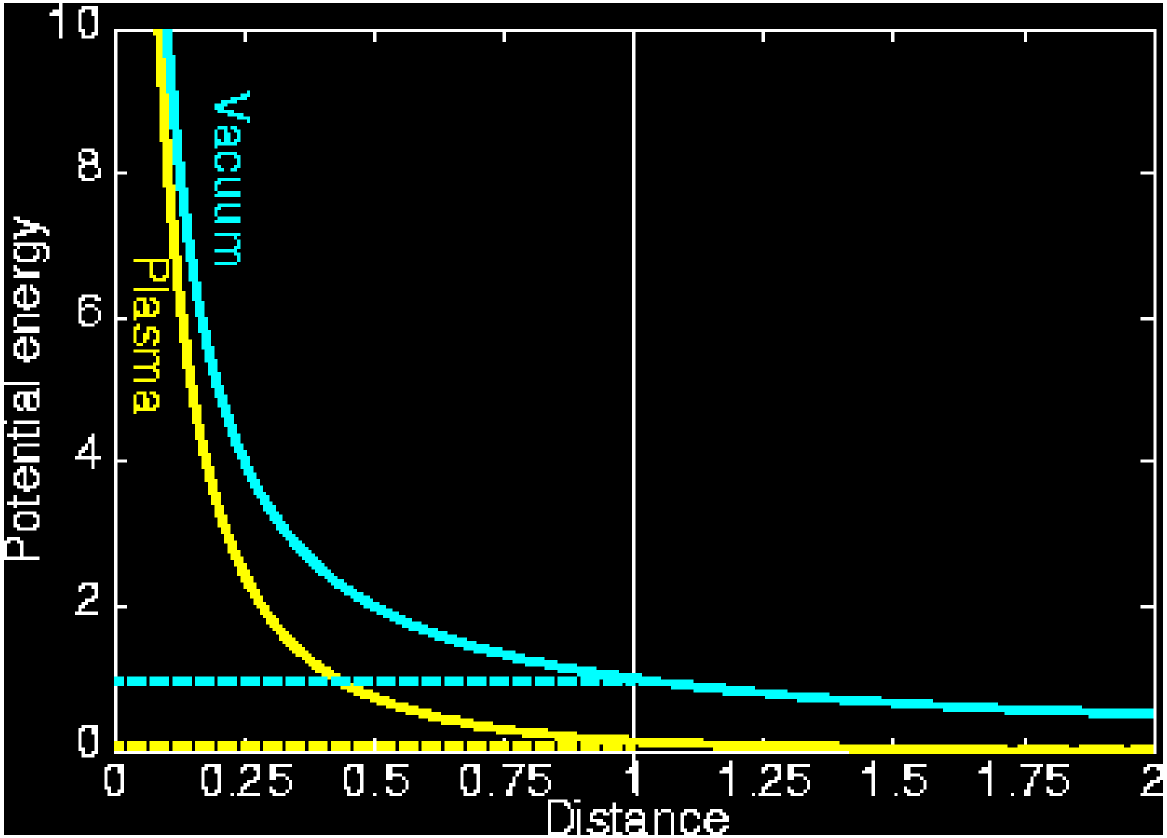


$$\alpha_s(\mu) \sim \alpha_s(M) - \frac{\beta_0}{4\pi}\ln\left[\frac{\mu^2}{M^2}\right]\alpha_s^{\,2}(M) + \left(\frac{\beta_0}{4\pi}\right)^2 \ln^2\left[\frac{\mu^2}{M^2}\right]\alpha_s^{\,3}(M) + \ldots,$$

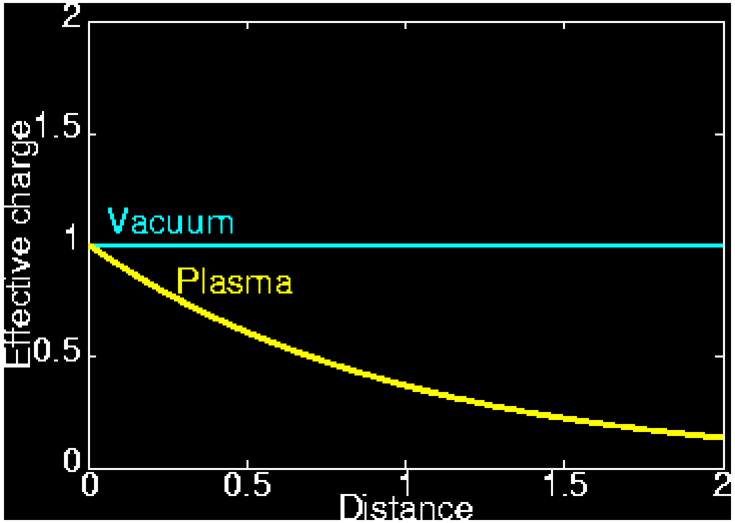
$$\beta_0 = \frac{11C_A - 2n_f}{3} = 11 - \frac{2}{3}n_f\,.$$

$$\alpha_s(\mu) = \frac{\alpha_s(M)}{1 + \left(\beta_0/4\pi\right)\alpha_s(M)\ln\left[\frac{\mu^2}{M^2}\right]}$$

Compare with QED or electric Plasmas…(trend opposite to QCD)



Again electric charge effects, *not* of the Colour charge



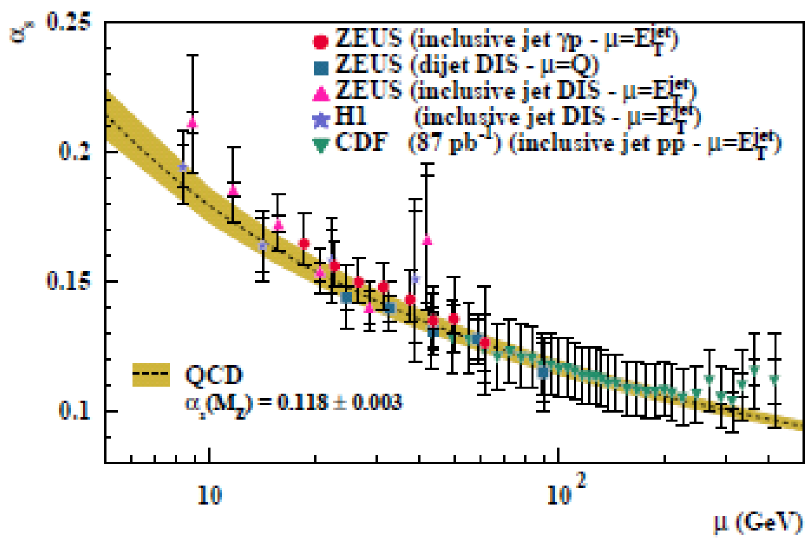
Back to QCD….

The running $\alpha_S(Q^2)$

- non-Abelian character of theory leads to :

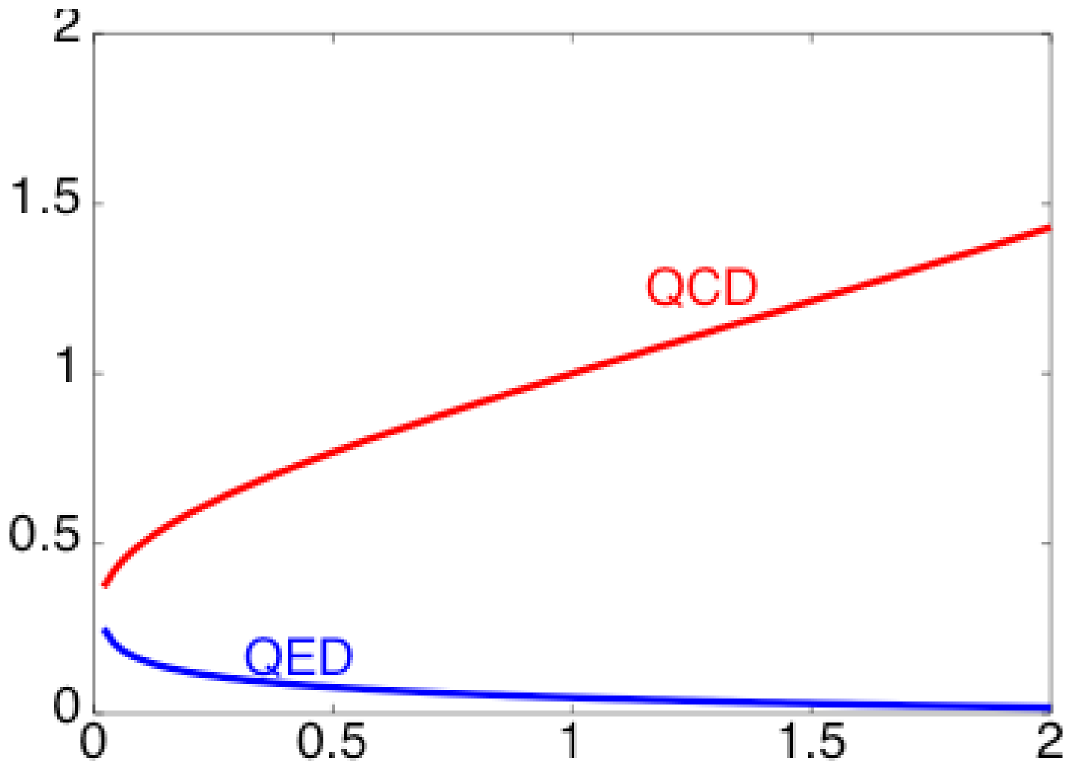
$$\alpha_S(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\ln(Q^2/\Lambda^2)}$$

- this exhibits **asymptotic freedom** as long as $N_f < 17$
- on the other side... **confinement**



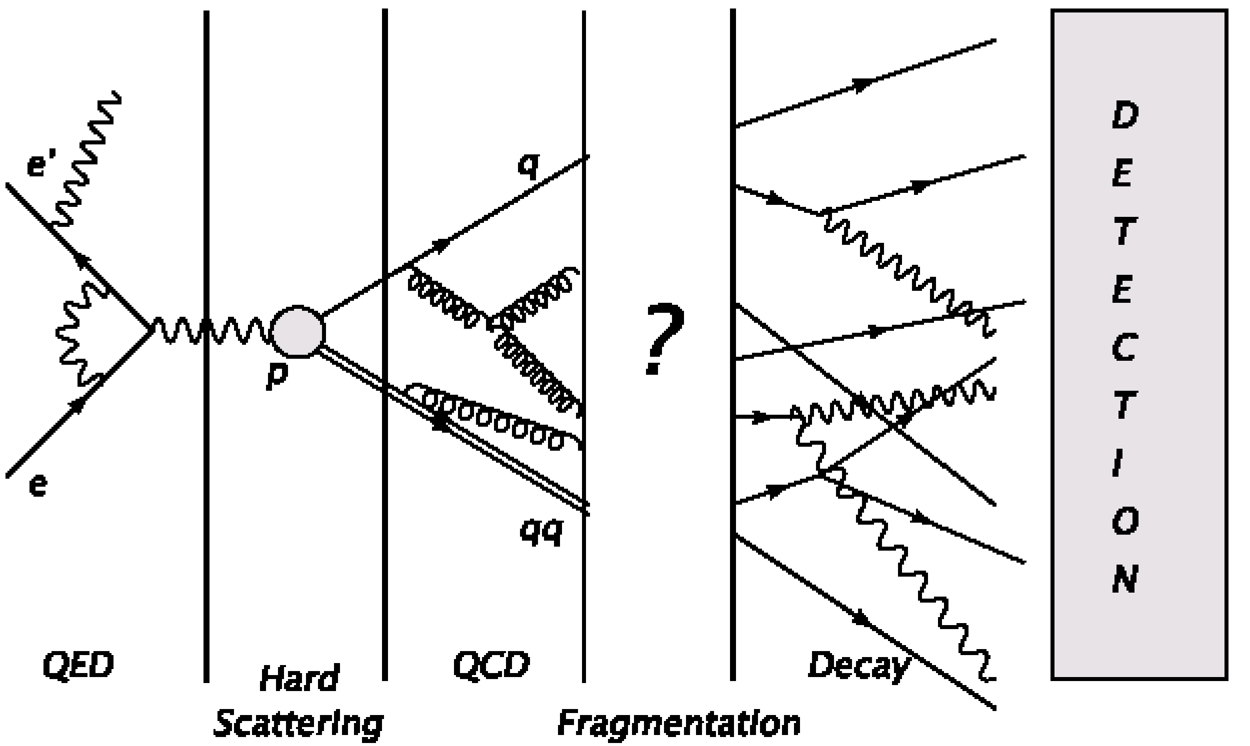
Bino Maiheu

Horizontal is the distance scaled probed and vertical is the Charge strength….



Continued: QCD effects modify all above (QED portion) as…

- 2-Quarks and gluons do not reach detector but only hadrons….

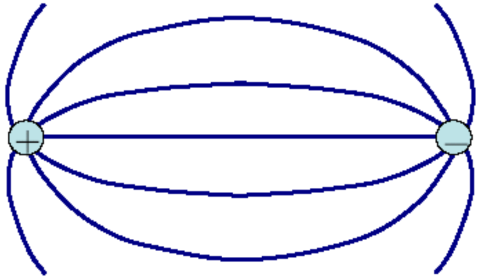


2a-Quarks and gluons remain inside hadrons:

Confinement

Asymptotic freedom: $Q\bar{Q}$ becomes increasingly QED-like at short distances.

QED:



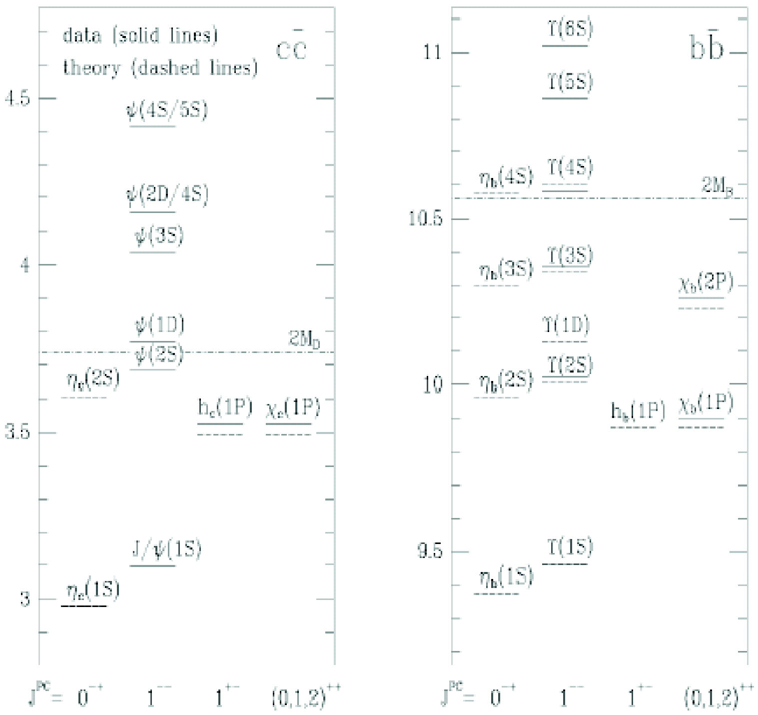
but at long distances, gluon self-interaction makes field lines attract each other:

QCD:

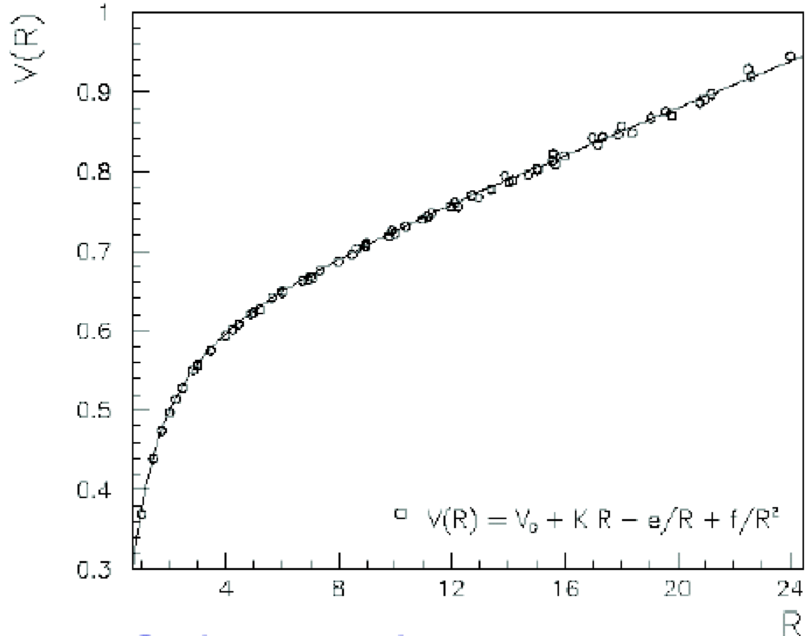
→linear potential → confinement

Interquark Potential

Can measure from quarkonia spectra:



or from lattice QCD:



→ String tension

$\kappa \approx 1 \ \mathrm{GeV/fm}.$

$$V(r) = \kappa r$$

The Lund String Model

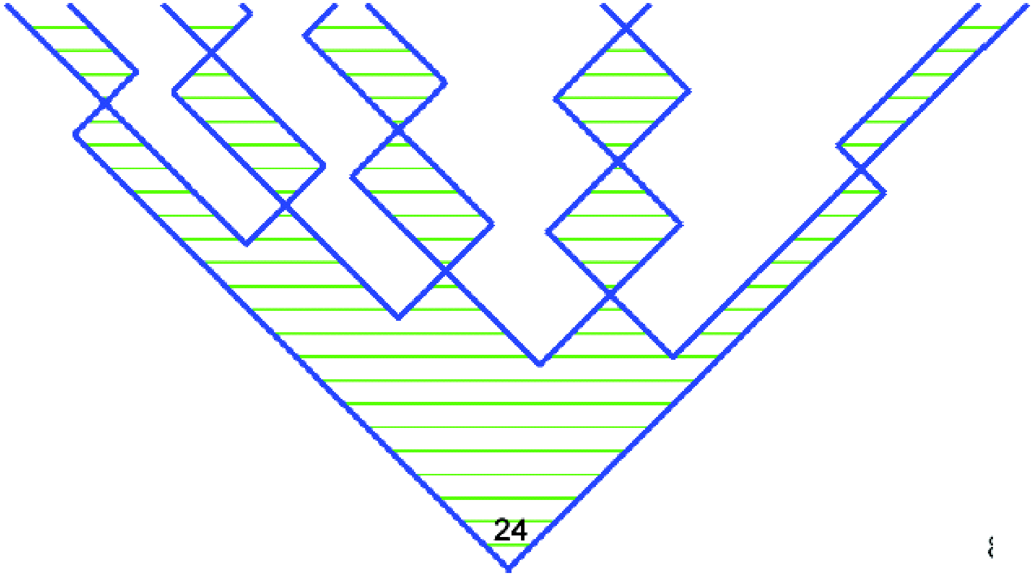
Start by ignoring gluon radiation:

e^+e^- annihilation = pointlike source of $q\bar{q}$ pairs

Intense chromomagnetic field within string → $q\bar{q}$ pairs created by tunnelling. Analogy with QED:

$$\frac{d(\text{Probability})}{dx\ dt} \propto \exp(-\pi m_q^2/\kappa)$$

Expanding string breaks into mesons long before yo-yo point.



How can you calculate with large coupling….

The S-matrix expansion in powers of coupling or H_I

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int \dots \int d^4x_1\, d^4x_2 \dots d^4x_n\, T\{\mathscr{H}_I(x_1)\mathscr{H}_I(x_2)\dots\mathscr{H}_I(x_n)\}, \quad (6.23)$$

Can be written as

$$S = T\exp\left[-i\int d^4x\, \mathcal{H}_I(x)\right]$$

And remains well defined no matter how large is H_I

Path Integrals…

$$< f|i >= \int [dx(t)] e^{iS/\hbar} = \int \left(\prod dx_i\right) e^{iS/\hbar}$$

Challenges for QCD: why only colour singlets and why clustering….