Quantum ElectroDynamics III

Feynman diagram

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Human Instinct

What?

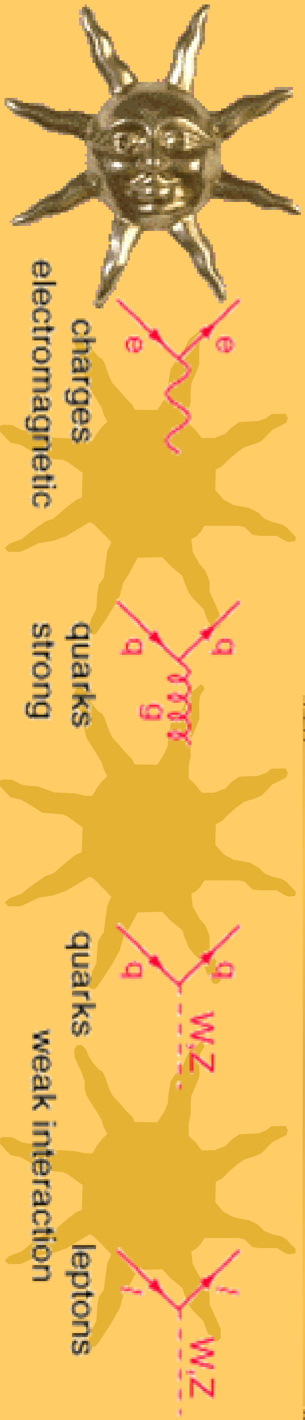
Feynman diagrams

Why?

Feynman diagrams

Feynman diagrams

How?



What?

- *Graphic way to represent exchange forces*
- *Describing a variety of particle interactions*

Developed by
Richard Feynman
when working on the development of QED

1942

Why ?

To calculate

Decay rates and Scattering cross section

Calculation of Decay rates and Scattering cross sections

Need two information

- ***Dynamical***
  - *Evaluation of relevant Feynman diagram to determine the amplitude M for the process.*
- ***Kinematical***
  - *The **phase space** factor, it depends on the masses, energies, and momenta of the participants.*

How?

- *Ingredients (Rules)*
- *Recipe (Structure)*

- ***Feynman rules:***

Electron (e^-) & Positron (e^+)

(Dirac eq.)

Photons

(Maxwell eq.)

Dirac Equation in electromagnetic field

(concept of gauge symmetry)

The Feynman rules for

★ Electron (e^-)		★ Positron (e^+)
$\psi(X) = a e^{\frac{-i}{\hbar}(X.P)} u^s(P)$	**Free**	$\psi(X) = a e^{\frac{i}{\hbar}(X.P)} v(P)$
$(\gamma^\lambda P_\lambda - mc)u = 0$	**Dirac**	$(\gamma^\lambda P_\lambda + mc)v = 0$
$\bar{u}(\gamma^\lambda P_\lambda - mc) = 0$	**Adjoint**	$\bar{v}(\gamma^\lambda P_\lambda + mc) = 0$
$\bar{u}^1 u^2 = 0$	**Orthogonal**	$\bar{v}^1 v^2 = 0$
$\bar{u}u = 2mc$	**Normalized**	$\bar{v}v = -2mc$
$\sum_{s=1,2} u^s \bar{u}^s = (\gamma^\lambda p_\lambda + mc)$	**Completeness**	$\sum_{s=1,2} v^s \bar{v}^s = (\gamma^\lambda p_\lambda - mc)$

The Feynman rules for (γ)

★ Photon (γ)

$$A^{\mu}(X) = a e^{\frac{-i}{\hbar}(X.P)} \varepsilon^{\mu}(s)$$ **Free**

$$\varepsilon^{\mu} P_{\mu} = 0$$ **Lorentz condition**

$$\varepsilon_{(1)}^{\mu *} \varepsilon_{\mu(2)} = 0$$ **Orthogonal**

$$\varepsilon^{\mu *} \varepsilon_{\mu} = 1$$ **Normalized**

$$\sum_{s=1,2} \left(\varepsilon_{(s)}\right)_i \left(\varepsilon_{(s)}\right)_j = \delta_{ij} - \hat{p}_i \hat{p}_j$$ **Completeness**

Complete Lagrangian for f & γ

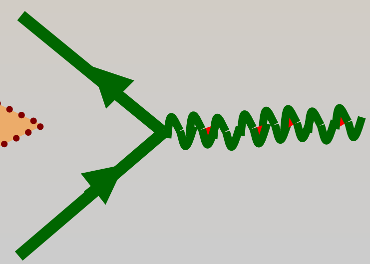
- *Lagrangian density describing the fermionic field in the presence of an electromagnetic field is*

$$\mathcal{L} = \overline{\Psi}(X)\left[\gamma^{\lambda}\left(i\partial_{\lambda} - qA_{\lambda}\right) - m\right]\Psi(X) - \frac{1}{4}F^{\lambda\nu}F_{\lambda\nu} - J^{\lambda}A_{\lambda}$$

$$\mathcal{L} = \overline{\Psi}(X)\left[i\gamma^{\lambda}\partial_{\lambda} - m\right]\Psi(X) - \frac{1}{4}F^{\lambda\nu}F_{\lambda\nu} - \left(J^{\lambda} + q\overline{\Psi}(X)\gamma^{\lambda}\Psi(X)\right)A_{\lambda}$$

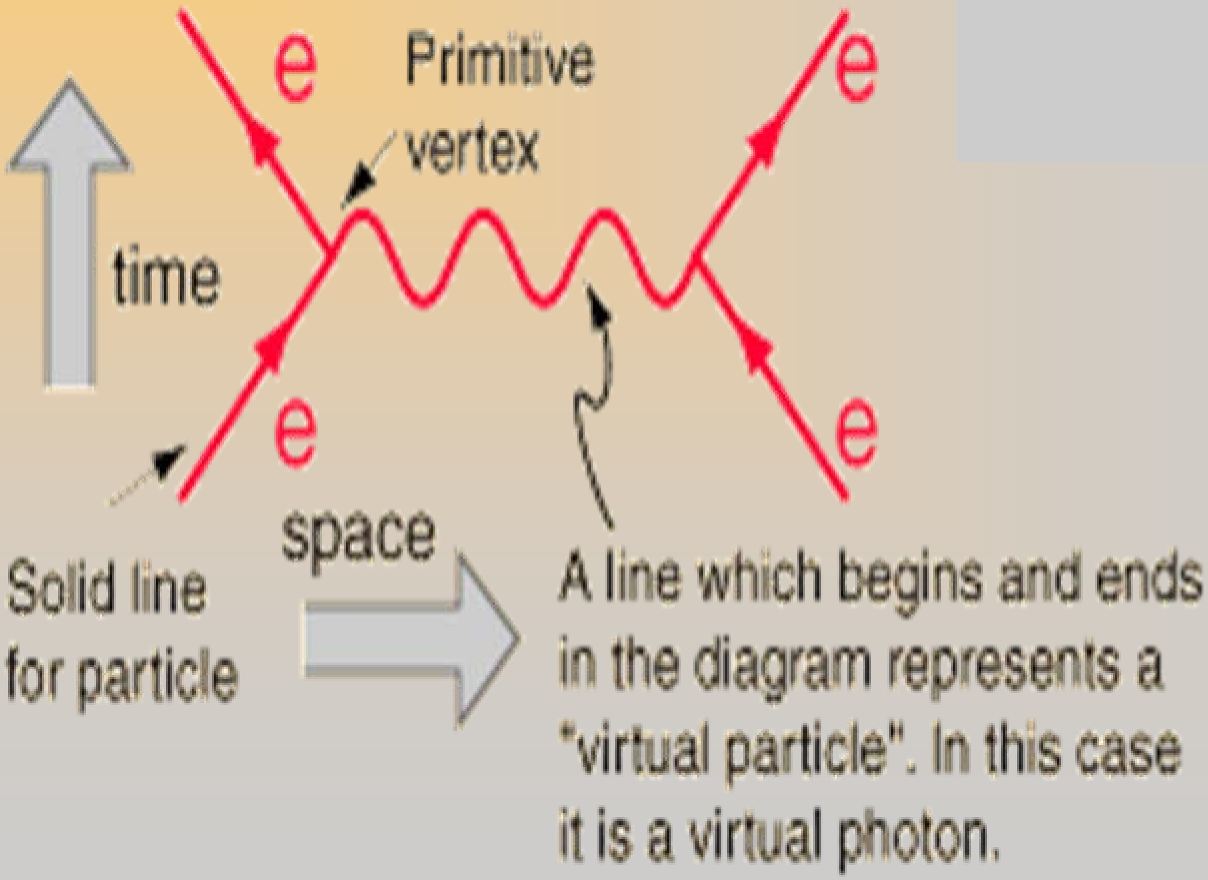
- *Current coupled to A_{λ}, describe the interaction vertex.*

Feynman diagram



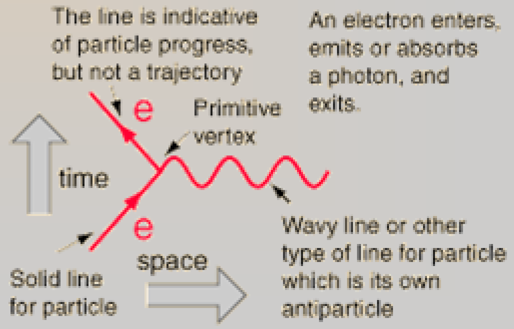
Current produce by Dirac particle

Structure of Feynman Diagrams



Structure of Feynman Diagrams

- Fermions represented by straight lines with arrows pointing in direction of time flow
- Forward-facing arrows represent particles
- Backward-facing arrows represent antiparticles
- Photons and weak bosons, W^- and W^+ and Z^0 are squiggly lines
- Gluons are curly lines

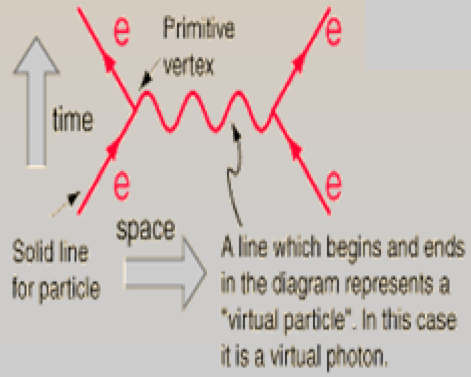


Line types

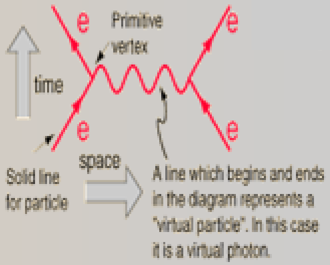
- ***External lines***
  - ***Enter*** *and* ***leave*** *diagram*
  - *Represent real* ***"observable"*** *particles*
- *real particles and must have*

$$E^2 = p^2 + m^2$$

- *Referred to as particle being* ***on*** ***"mass shell"***



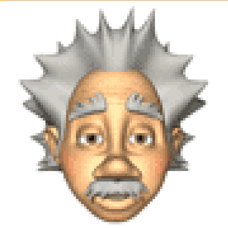
Line types



- ***Internal lines***

  Connect vertices, called propagators

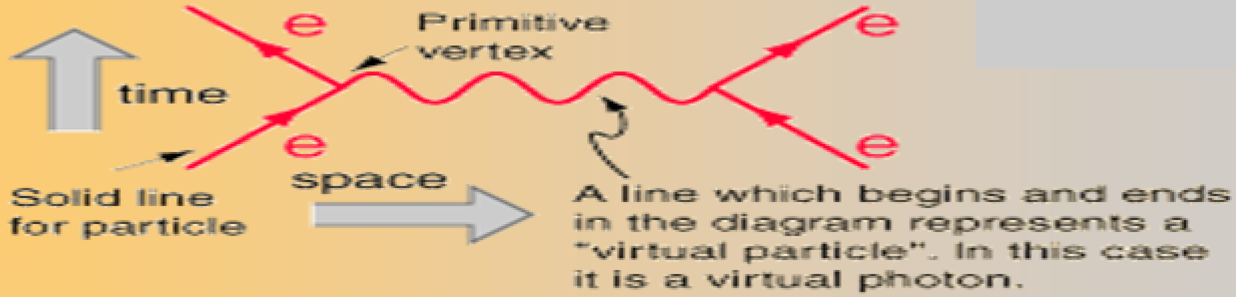
  *Represent **"virtual"** particles that **cannot** be **observed***



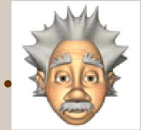
  ***Do not** have to **obey** relativistic mass, energy, momentum relationship:*

  $$(mc^2)^2 = E^2 - (pc)^2$$

  *Referred to as particle being **off "mass shell"***



Virtual

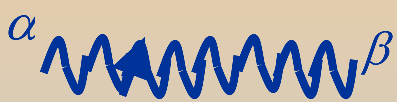
- *At the point of emission (or absorption) the vertices gives a contradiction with Einstein relation between energy and mass.* 

Vertices

- Conserve: energy, momentum, & charge for all types of interactions
- *Determine order of perturbation contributes to the particular calculation*
- Same number of arrows enter as leave

How to write currents by using F.D

- For each QED vertex, write factor $ie\gamma^{\mu}$
- For each internal photon line having momentum k write factor



$$iD_{F\alpha\beta}(k) = i\frac{g_{\alpha\beta}}{k^2 + i\varepsilon}$$

- For each internal lepton line having momentum p write factor 

$$iS_F(p) = i\frac{1}{\not{p} - m + i\varepsilon} = \frac{\not{p} + m}{p^2 - m^2 + i\varepsilon}$$

- *For each external (initial) fermions*

$u_r(p)$ 

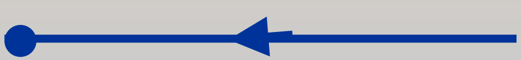
- For each external (final) fermions

$\bar{u}_r(p)$ 

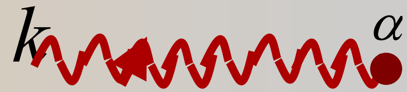
- For each external (initial) anti-fermions

$\bar{v}_r(p)$ 

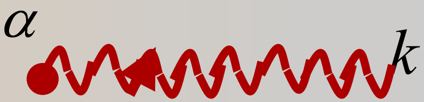
- For each external (final) anti - fermions

$v_r(p)$ 

- For each initial photon $\varepsilon_{r\alpha}(k)$



- For each final photon $\varepsilon^{*}{}_{r\alpha}(k)$

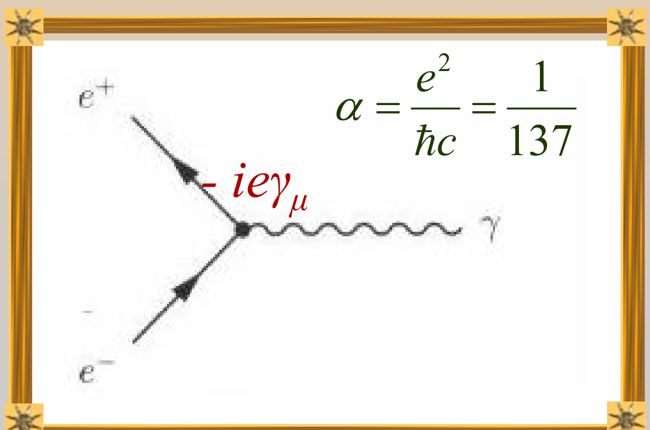


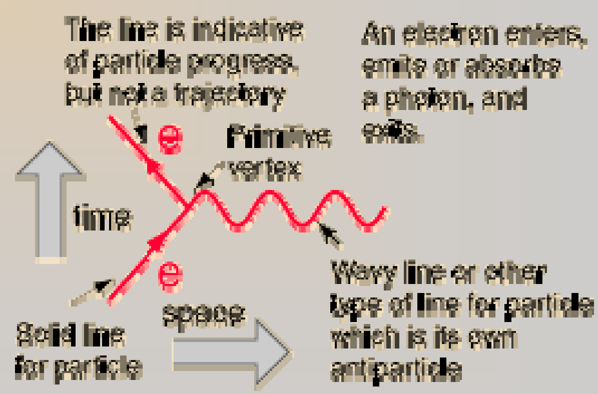
Examples

- **Basic vertices**

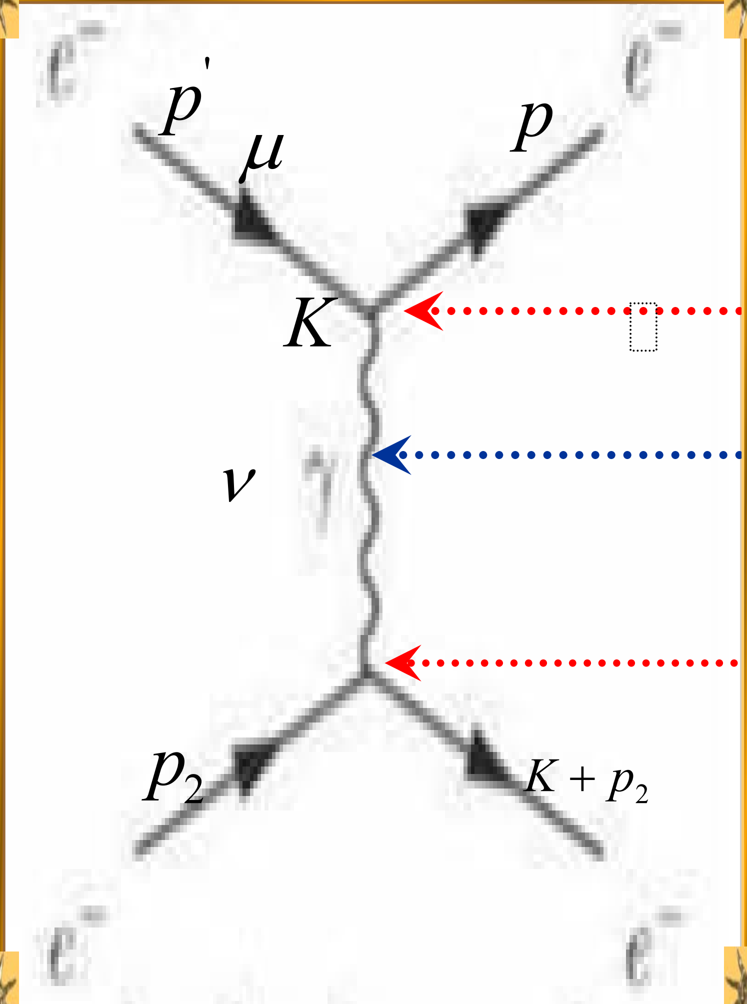
  Electromagnetic

  Charge particle enters, emits (or absorbs) a photon and exits.





Current



$$L = J_\mu J^{\mu^+}$$

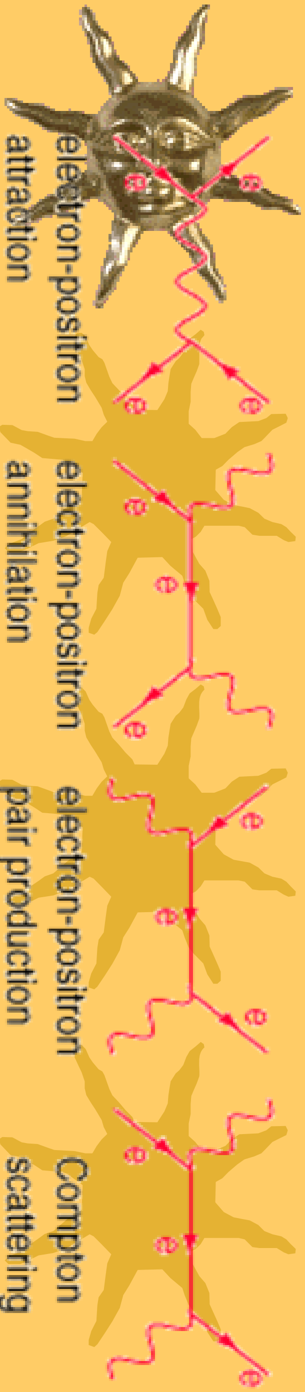
$$J_\mu = \overline{u_r}(p)\, i\, e\, \gamma_\mu\, u_r(p')$$

$$iD^{\mu\nu}(k) = i\frac{g^{\mu\nu}}{k^2 + i\varepsilon}$$

$$J_\nu = \overline{u_r}(K + p_2) ie\gamma_\nu u_r(p_2)$$

$$M_{fi} = \overline{u_r}(p) ie\gamma_\mu u_r(p')\, i\frac{g^{\mu\nu}}{K^2 - i\varepsilon}\overline{u_r}(K + p_2) ie\gamma_\nu u_r(p_2)$$

$$= -i\frac{e^2}{K^2 - i\varepsilon}\overline{u_r}(p)\gamma_\mu u_r(p')\,\overline{u_r}(K + p_2)\gamma^\mu u_r(p_2)$$



Lowest Order Fundamental Processes

Scattering

- **Elastic**
  - **Mott** $e\mu \rightarrow e\mu$
  - **Møller** $ee \rightarrow ee$
  - **Bhabha** $e^- e^+ \rightarrow e^- e^+$
- **Inelastic**
  - **Pair production** $\gamma\gamma \rightarrow e^- e^+$
  - **Compton** $e\gamma \rightarrow e\gamma$
  - **Pair annihilation** $e^- e^+ \rightarrow \gamma\gamma$
  - **Muon production** $e^- e^+ \rightarrow \mu^- \mu^+$

Scattering

$e^- + e^+ \rightarrow e^- + e^+$

$e^- + e^+ \rightarrow \gamma + \gamma$

$\gamma + \gamma \rightarrow e^- + e^+$

$e^- + \gamma \rightarrow e^- + \gamma$

electron-positron attraction

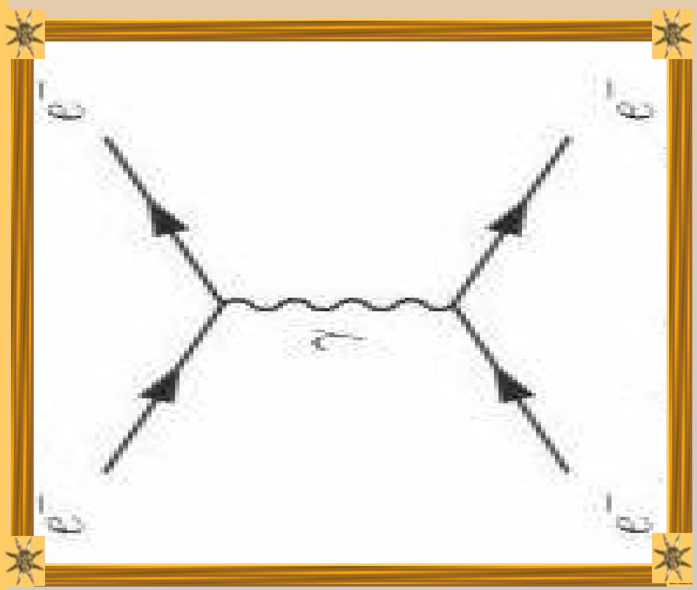
electron-positron annihilation

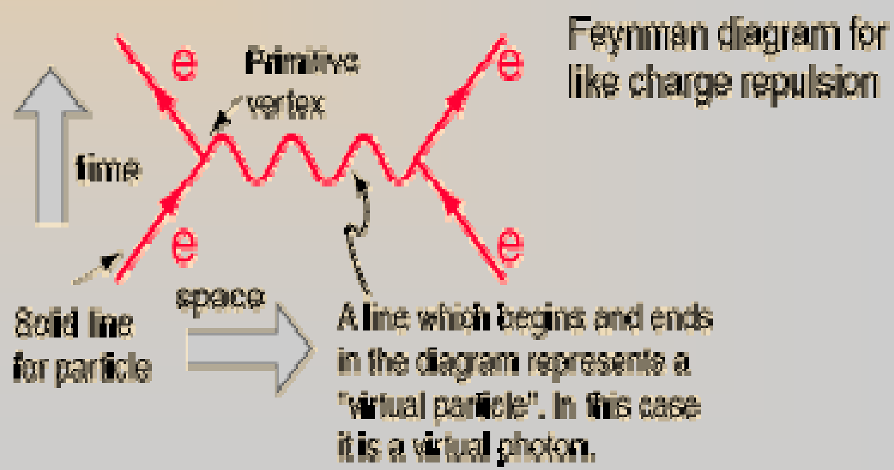
electron-positron pair production

Compton scattering

Møller scattering $ee \rightarrow ee$

- Electron-electron scattering (*Moller scattering*)
- Two electron enters, a photon passes between them



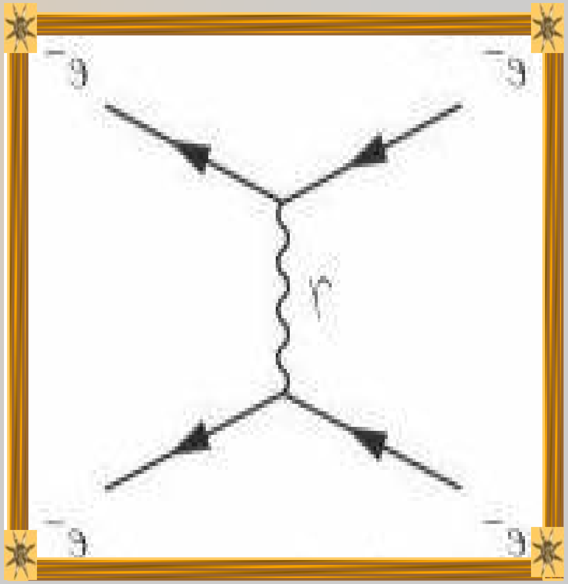


Bahabha scattering $e^- e^+ \rightarrow e^- e^+$

Twist into any topological configuration

Rule:

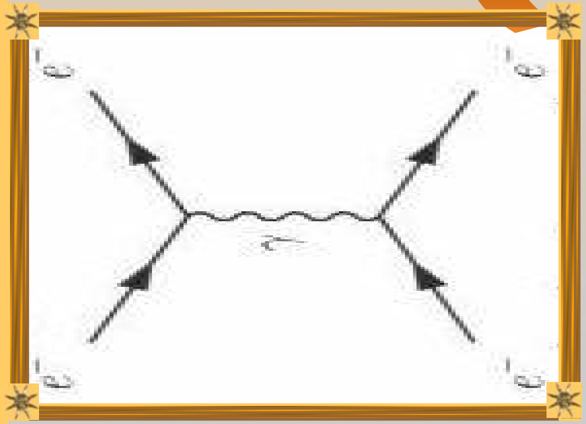
Particle line running backward in time is interpreted as the corresponding antiparticle going forward



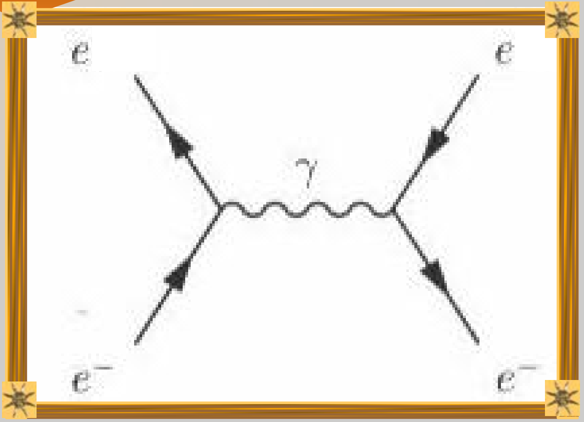
e^-e^+ annihilate to form a photon which produces a new e^-e^+

Bhabha & Moller scattering are related by cross symmetry

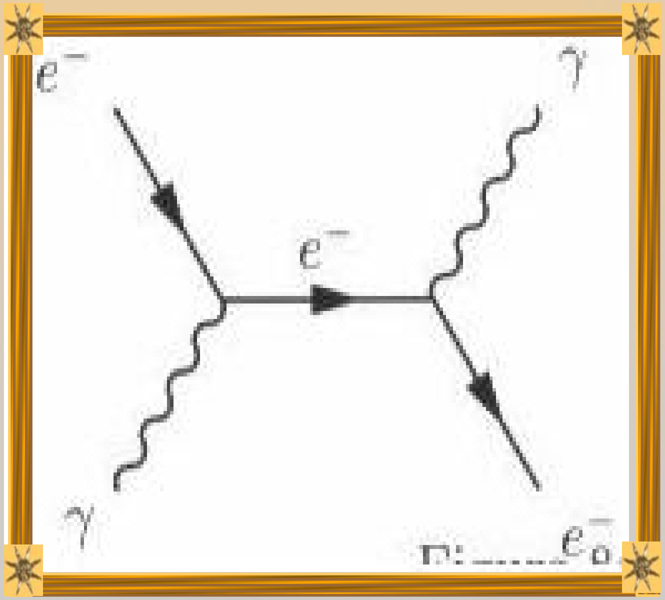
$$e_1^- + e_2^+ \rightarrow e_3^- + e_4^+$$

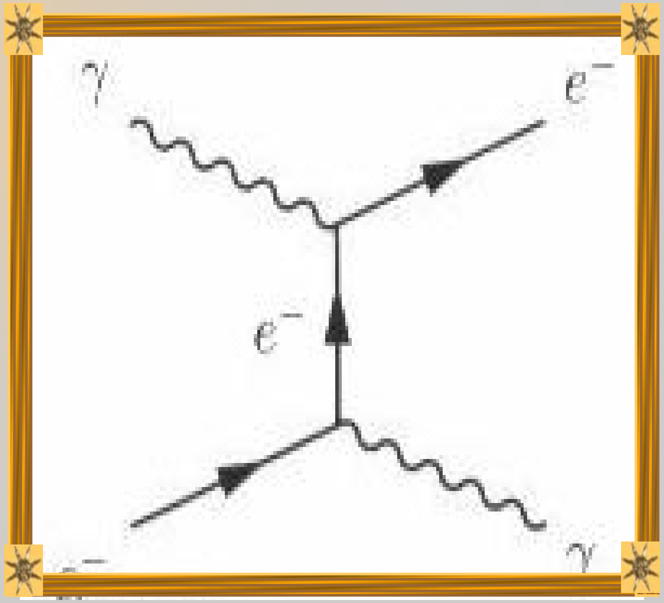


$$A + B \rightarrow C + D$$
$$A + \overline{C} \rightarrow \overline{B} + D$$
$$\overline{C} + \overline{D} \rightarrow \overline{A} + \overline{B}$$



Compton scattering





Remember

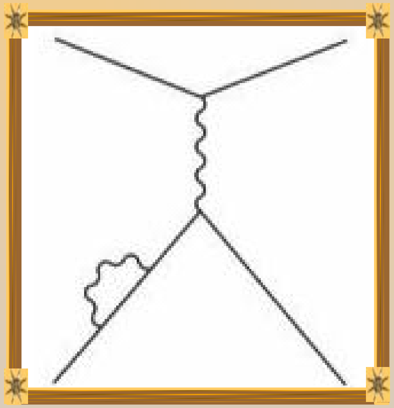
- *Dominant contribution comes from tree level*

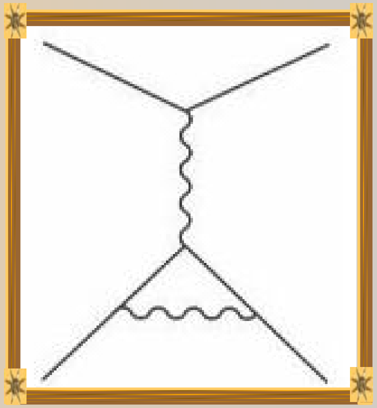
$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$$

- *Ignore higher order contribution*

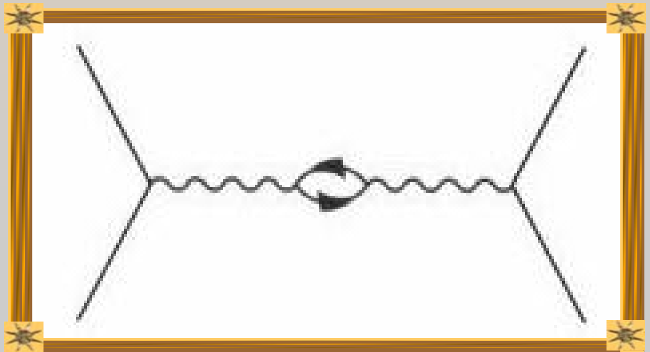
QED Interactions

- *Higher order diagrams*





- *Charge screening*



THANK YOU!

Recall

Forces	coupling	Strength	Range	Particles
Strong	α_s	1	10^{-15}	Gluons; m=0
Electromagnetic	α	1/137	∞	Photon; m=0
Weak	α_w	10^{-6}	10^{-18}	W &Z boson
Gravity	α_g	10^{-39}	∞	graviton; m=0