Supersymmetry and its Implications

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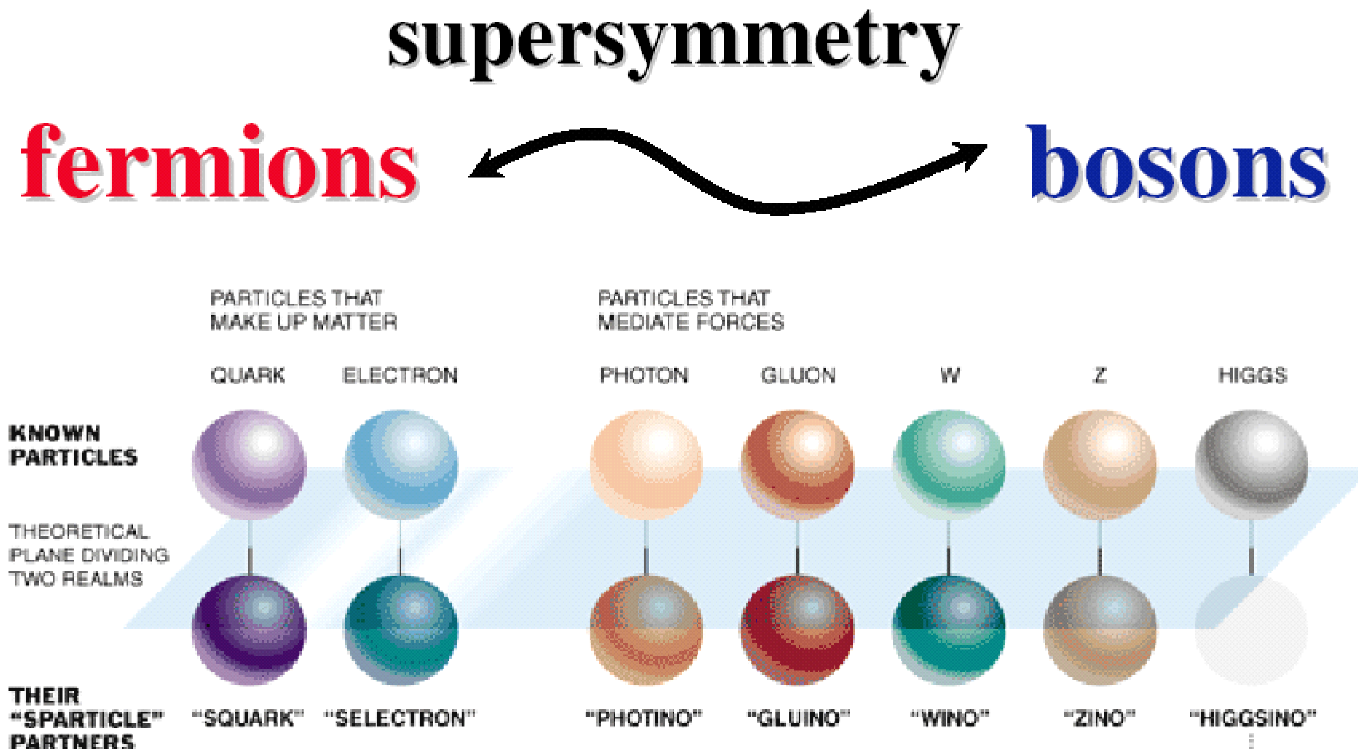
Layout

- What is Supersymmetry and Why:
  - a. Hierarchy problem:
  - b. Gauge unification.
  - c. Extension of Symmetries, No Go Theorem and idea of Superfields.
  - d. SUSY Algebra and Framework.
- MSSM- Wess-Zumino Model.
- R-parity, R-parity conservation
- R-parity violations and implications
- LSP as Dark Matter candidate
- References

LECTURE I

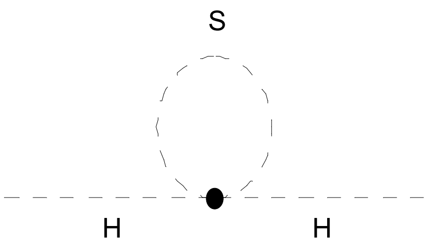
WHAT IS SUPERSYMMETRY?

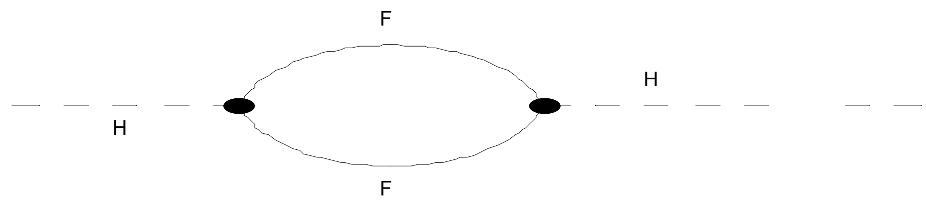


Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

Motivation: a. Hierarchy Problem- Higgs Self mass





$$\Delta(m_H)_S^{\ 2} = \frac{\lambda_S}{16\pi^2}[\Lambda^2 - m_S^{\ 2}\ln(\frac{\Lambda^2}{m_S^{\ 2}}) + ...]$$

$$\Delta(m_H)_F^{\ 2} = \frac{\left|\lambda_f\right|^2}{8\pi^2}[-\Lambda^2 + 3m_f^{\ 2}\ln(\frac{\Lambda}{m_f}) + ...]$$

$$2\Delta(m_H)_S^{\ 2} + \Delta(m_H)_F^{\ 2} = \frac{\lambda}{8\pi^2}[-2m_S^{\ 2}\ln(\frac{\Lambda^2}{m_S^{\ 2}})] + 3m_f^{\ 2}\ln(\frac{\Lambda}{m_f})]]$$

$$\lambda_S = |\lambda_f|^2 = \lambda$$

- Following possibility exists for the resolution of the Higgs selfmass:

1. New mechanism for electroweak symmetry breaking.
2. Fine tuning is present. Couplings / masses are equal in SUSY limit.
3. Fermions and bosons are related by a new symmetry (Super symmetry)-a new scale ~TeV exists.

   LHC is expected to throw light on Higgs.

b. Gauge Coupling Unification

The gauge coupling constants are given as

$\alpha_i = \frac{g_i^{\,2}}{4\pi}$, i=1,2,3 corresponding to electromgnetic, weak and strong coupling

constants using the renormalization group equations (RGE), whose 1-loop result can be summarized as :

$$\frac{d}{dt} g_i = \frac{1}{16\pi^2} b_i g_i^{\,3},\ \text{or alternatively},\ \frac{d}{dt}(\alpha_i^{-1}) = 4\pi(-2)g_i^{\,3}, \frac{d}{dt} g_i = -\frac{b_i}{2\pi}, i = 1,2,3$$

where $t = ln[\frac{q}{q_o}]$, with q the RGE scale.

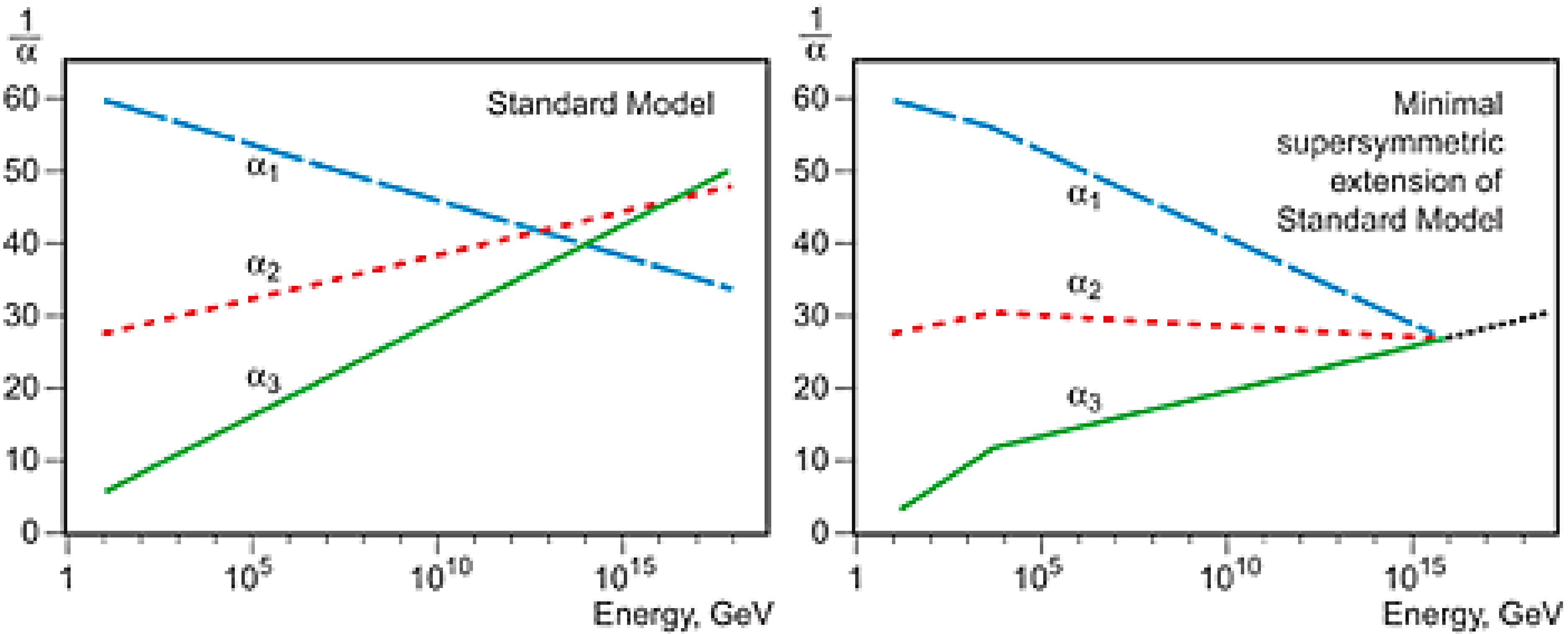
In the SM one takes

$b_i^{SM} = (\frac{41}{10}, \frac{-19}{6}, -7)$ while in the MSSM, $b_i^{MSSM} = (\frac{33}{5}, 1, -3)$.

The MSSM set of cofficients have larger values as a result of extra MSSM particles in the loops. The normalization g_i is choosen to correspond to the covaiant derivative for grand unification of SM gauge group $SU(3)_c \otimes SU(2) \otimes U(1)$ into SU(5) or SO(10). Thus in terms of conventional electroweak gauge couplings g & g' with

$e = g\sin\theta_W = g'\cos\theta_W$, one has $g_2 = g$ & $g_1 = \sqrt{\frac{5}{3}}g'$.

The above defined squared couplings α_i have the property that their reciprocal runs linearly with RG scale at 1-loop level.


1/α
60
50
40
30
20
10
0
1
10^5
10^10
10^15
Energy, GeV
Standard Model
α1
α2
α3
Minimal
supersymmetric
extension of
Standard Model

Extension of Symmetries

Internal Symmetries:
e.g, SU(2),SU(3),SU(6),

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

External Symmetries:

Poincare (Space-Time) Symmetries

- The idea was introduced in 1973 to extend the symmetries of quantum field theories

  It also provides a framework for unification of gravity with other gauge forces at the Planck scale.

Coleman-Mandula Theorem-
(No Go Theorem):

- It was shown by Coleman & Mandula(1967) that it was not possible to combine internal symmetries (isospin SU(2) and flavor SU(3)) with external symmetries (SL(2,C),Poincare) into a larger symmetry group. Extension of internal symmetry groups was later shown to be possible only after introducing additional fermionic coordinates θ^μ to x^μ. This led to the idea of superspace and superfields given by:

- Salam and Strathdee(1974) resolved the problem posed by the No Go theorem in an elegant way, by introducing super fields:

$$\Phi = \Phi(x, \theta, \bar{\theta}^{\cdot})$$

$$\Phi(x, \theta, \bar{\theta}^{\cdot}) = C(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta M(x) + \overline{\theta\theta} N(x) + \theta\sigma^{\mu}\bar{\theta}V_{\mu}(x) +$$
$$\theta\theta\bar{\theta}\,\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}D(x)$$

where $C(x)$, M(x), N(x), ϕ(x), $\bar{\chi}$(x),$\bar{\lambda}$(x), ψ(x), $V_{\mu}(x), d(x)$ are component fields with 16 bosonic and 16 fermionic degrees of freedom.i.e. a superfield in 8-dimensions is equivalent to 16-component set of ordinary fields in 4-dimensions.

SUSY ALGEBRA

P_μ : translation

Q : SUSY transformation

$$\{Q_\alpha, Q^\dagger{}_\beta\} = 2\delta^{\alpha\beta}(\sigma^\mu)_{\alpha\beta} P_\mu$$

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger{}_\alpha, Q^\dagger{}_\beta\} = 0$$

$$\{Q, P^\mu\} = \{Q^\dagger, P^\mu\} = 0$$

$$Q\,|\,boson >=|\,fermion>$$

$$Q^\dagger\,|\,fermion>=|\,boson>$$

If there are “N” SUSY generators Q, the framework is “N” dimensional

SUSY Models

Minimal Supersymmetric Standard Model, MSSM (N=1 and two Higgs doublets)

Non-Minimal MSSM (may have more super-fields than one, shall not be discussed in these lectures).

MSSM

Table 1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model.

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks	Q	$(\tilde{u}_L \;\; \tilde{d}_L)$	$(u_L \;\; d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(×3 families)	$\overline{u}$	$\tilde{u}^*_R$	$u^\dagger_R$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\tilde{d}^*_R$	$d^\dagger_R$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu} \;\; \tilde{e}_L)$	$(\nu \;\; e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\tilde{e}^*_R$	$e^\dagger_R$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	H_u	$(H^+_u \;\; H^0_u)$	$(\tilde{H}^+_u \;\; \tilde{H}^0_u)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	H_d	$(H^0_d \;\; H^-_d)$	$(\tilde{H}^0_d \;\; \tilde{H}^-_d)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^{\pm}\ \ \widetilde{W}^{0}$	$W^{\pm}\ \ W^{0}$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\mathbf{1}, \mathbf{1}, 0)$

Standard Model Particles		SUSY Partners		
Particles	States	Sparticles	States	Mixtures
Quarks(q) (Spin-1/2)	$\begin{pmatrix} u \\ d \end{pmatrix}_L$, u_{R}, d_R $\begin{pmatrix} c \\ s \end{pmatrix}_L$, c_{R}, s_R $\begin{pmatrix} t \\ b \end{pmatrix}_L$, t_{R}, b_R	Squarks($\tilde{q}$) (Spin-0)	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L, \tilde{u}_R, \tilde{d}_R,$ $\begin{pmatrix} \tilde{c} \\ \tilde{s} \end{pmatrix}_L, \tilde{c}_R, \tilde{s}_R$ $\begin{pmatrix} \tilde{t} \\ \tilde{b} \end{pmatrix}_L, \tilde{t}_R, \tilde{b}_R$	
Leptons(l) (spin-1/2)	$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, e_R, \nu_{e_R}$ $\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \mu_R, \nu_{\mu_R}$, $\begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L, \tau_R, \nu_{\tau_R}$	Sleptons($\tilde{l}$) (spin-0)	$\begin{pmatrix} \tilde{e} \\ \tilde{\nu}_e \end{pmatrix}_L, \tilde{e}_R, \tilde{\nu}_{e_R}$ $\begin{pmatrix} \tilde{\mu} \\ \tilde{\nu}_\mu \end{pmatrix}_L, \tilde{\mu}_R, \tilde{\nu}_{\mu_R}$ $\begin{pmatrix} \tilde{\tau} \\ \tilde{\nu}_\tau \end{pmatrix}_L, \tilde{\tau}_R, \tilde{\nu}_{\tau_R}$	
Gauge/ Higgs bosons (spin-1, spin-0)	g,Z,h,H,A,γ $W^\pm$, $H^\pm$	Gauginos/Higgsinos (spin-1/2)	$\tilde{g}, \tilde{Z}, \tilde{\gamma}, \tilde{H}_1^0$ $\tilde{W}^\pm, \tilde{H}^\pm$	$\tilde{\chi}^0_{1,2,3,4}$ $\tilde{\chi}^\pm_{1,2}$
Graviton	G	Gravitino (spin-3/2)	$\tilde{G}$	

Section 1:SUSY Framework; 15 to 28 from

Supersymmetry by M. Carena, Fermilab, USA..

- Four component vs. two component Weyl spinors

- A Dirac spinor is a four component object having components:

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \psi_D^c = \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{1.1}$$

- Whereas a Majorana spinor is a 4-component object with components

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \qquad \psi_M^c = \psi_M \tag{1.2}$$

- Gamma matrices for Weyl (2 components) fermions are:

$$\gamma^\mu = \begin{pmatrix} 0, \sigma^\mu \\ \bar{\sigma}^\mu, 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \tag{1.3}$$

- Usual Dirac contraction may then be expressed in terms of 2-component contraction

$$\bar{\psi}_D \psi_D = \psi\chi + h.c \qquad \text{with} \qquad \bar{\psi}_D = \left(\psi^\alpha, \bar{\chi}_{\dot{\alpha}}\right)$$

SECTION 2:

Examples: In particular, notice:

$$\bar{\psi}_D \gamma^\mu \psi_D = \bar{\psi}_D \sigma^\mu \psi_D + \bar{\chi}\sigma^\mu \chi = -\bar{\psi}\bar{\sigma}^\mu \psi + \bar{\chi}\bar{\sigma}^\mu \chi \tag{2.1}$$

- Since $\psi_{D,L}$=♏, therefore, Majorana particles lead to vanishing vector currents. Hence they must be neutral under e.m. interactions. On the other hand,

$$\bar{\psi}_D\gamma^\mu\gamma^5\psi_D = \bar{\psi}\sigma^\mu\psi - \bar{\chi}\sigma^\mu\chi = -\bar{\psi}\bar{\sigma}^\mu\psi - \bar{\chi}\bar{\sigma}^\mu\chi \qquad (2.2)$$

- Chiral currents instead do not vanish, i.e.,

- They may therefore couple to the Z-boson.

- Other relations may be found in literature.

Superspace-Grassmann Algebra

- As seen, in order to describe Supersymmetric theory, it is convenient to introduce the concept of super-space.
- In this framework, one must introduce new coordinates in addition to x^μ called the Grassmannian coordinates, which are spinors.
- These anti-commute as:

$$\left.\begin{array}{l}\left\{\theta_\alpha,\theta_\beta\right\}=\theta_\alpha\theta_\beta+\theta_\beta\theta_\alpha=0; \qquad \theta\theta\theta=0;\\ \left[\theta Q,\bar{\theta}\bar{Q}\right]=2\theta\bar{\theta}\sigma^\mu P_\mu\end{array}\right\} \qquad (2.3)$$

Define derivatives: $\partial_\alpha=\dfrac{\partial}{\partial\theta^\alpha};\ \partial_\alpha\theta^\beta=\delta_\alpha^\beta;\ \partial_\alpha(\theta^\beta\theta_\beta)=2\theta_\alpha$

Sec.3: SUSY TRANSFORMATIONS

- SUSY fields representations are governed by the principle of invariance under SUSY transformations.

- SUSY operators may be represented as derivative operators:

$$Q_\alpha = i[-\partial_{\theta_\alpha} - i\sigma^\mu \overline{\theta}_\alpha \partial_\mu] \qquad (3.1)$$

Ex. 1). Check that they satisfy SUSY algebra given before.

2). Next check that the operator

$$\overline{D}_{\dot\alpha} = -\partial_{\dot\alpha} + i\theta_\alpha \sigma^\mu \partial_\mu \qquad (3.2)$$

commutes with the SUSY transformations.

3). So, if a field depends on the variable

$y^\mu = x^\mu - i\overline{\theta}\sigma^\mu\overline{\theta}$, Since $\overline{D}y^\mu = 0$ satisfies $\overline{D}\Phi = 0$, the SUSY transformation of such field depends only on y.

$$\Phi(x,\theta,\overline{\theta}=0) = A(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) \qquad (3.3a)$$

SUSY field is represented as:

$$\Phi(x,\theta,\overline{\theta}) = \exp(-i\partial_\mu \theta\sigma^\mu\overline{\theta})\Phi(x,\theta,\overline{\theta}=0) \qquad (3.3b)$$

The SUSY transformation of a chiral field is chiral.

Thus, the field in full may be represented as:

$$4)\ \Phi(x,\theta,\overline{\theta}) = A(x) + i\partial^\mu A(x)\theta\sigma_\mu\overline{\theta} - \frac{1}{4}\partial^2 A(x)\bar{\theta}^2\,\overline{\theta}^2 + \sqrt{2}\theta\psi(x)$$

$$+ i\frac{\theta^2}{2}\partial^\mu\psi(x)\sigma_\mu\overline{\theta} + F(x)\theta^2 \qquad (3.4)$$

Sec. 4: Vector Superfields

(i) Vector super fields are generic hermitian fields. Minimal irreducible representations may be obtained by:

$$V(x,\theta,\overline{\theta}) = -(\theta\sigma^{\mu}\overline{\theta})V_{\mu} + i\theta^{2}\overline{\theta}\overline{\lambda} - i\overline{\theta}^{2}\theta\lambda + \frac{1}{2}\theta^{2}\overline{\theta}^{2}D \qquad (4.1)$$

Where vector super-field V_{μ} in the first term on the R.H.S is defined as a regular gauge vector field with fermions super-partner $(\lambda,\overline{\lambda})$ and auxiliary scalar field. Under SUSY transformations the components of V_{μ} transform like:

$$\delta V_\mu^a = -\bar{\xi}\bar{\sigma}_\mu \lambda^a - \bar{\lambda}^a \bar{\sigma}_\mu \xi, \quad \delta\lambda_\alpha^a = -\frac{i}{2}(\sigma^\mu \bar{\sigma}^\nu \xi)_\alpha F_{\mu\nu}^a + \xi_\alpha D^a \qquad (4.2)$$

$$\delta D^a = i(\bar{\xi}\bar{\sigma}^\mu \nabla_\mu \lambda^a - \nabla_\mu \bar{\lambda}^a \bar{\sigma}^\mu \xi), \qquad (4.3)$$

with $\nabla_\mu = \partial_\mu + igV_\mu^a T^a$ (covariant derivative),

The D component of a vector field transforms as a total derivative.

(ii) Dimensionality count of various field components:

Since the total (integrated) lagrangian over the lagrangian density has to be dimensionless (why? explain), the dimensionalities of various components can be estimated as:

D=[V]+2; $[V_\mu]$=[V]+1; [λ]=[V]+3/2 (4.4a)

SUSY FRAMEWORK-continued

(iii) Super-fields strength and gauge transformations:

- Chiral field derived from V is given by

$$(W = -\bar{D}\bar{D}DV / 4):$$

$$W^{\alpha}(x,\theta,\theta = 0) = -i\lambda^{\alpha} + (\theta\sigma_{\mu\nu})^{\alpha} F^{\mu\nu} + \theta^{\alpha} D - \theta^{2}(\bar{\sigma}^{\mu} D_{\mu}\bar{\lambda})^{\alpha} \qquad (4.4b)$$

- Ex. 3: Construct $W^{\alpha}W_{\alpha}$ *and show that it is invariant under SUSY transformations.*

A SUSY Lagrangian

- Aim/Objective: To construct a SUSY lagrangian invariant under SUSY and gauge transformations

- The variation of lagrangian δ**O** should be a total derivative such that the action is invariant and minimum a la variation principle.

$$S = \int d^4 x \, \text{¢}$$

The dimensionality of all fields components, i.e. F-component, D-component and vector field component (See eqs. (3.3a-b,3.4)), follows:

$$[\text{¢}_{\text{int}}] \leq 4$$

Field dimensions-continued

- Whereas, the dimensions of chiral & vector fields are

$$[\Phi] = 1, \quad [W_\alpha] = 3/2, \quad [V] = 0, \qquad \text{[See eqs (4.4a,b)]}$$

Sec. 5 SUSY Lagrangian

- Once the foregoing SUSY framework is introduced, the total lagrangian is then given by:

$$\mathcal{L}_{SUSY} = \frac{1}{4g^2}(Tr[W^{\alpha}W_{\alpha}]_F + h.c) + \sum_i(\bar{\Phi}\exp[gV]\Phi) + ([P(\Phi)]_F + h.c), \qquad (5.1)$$

where W^{α} is given by (4.4), Φ by (3.3a,3b) and where P(Φ) is the most generic dimension 3, gauge invariant, polynomial function of the chiral fields Φ and is called SUPERPOTENTIAL. It has the general expression

$$P[\Phi] = c_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{\lambda_{ijk}}{3!} \Phi_i \Phi_j \Phi_k \qquad (5.2)$$

- Ex: 2: Substitute (3.3) and (3.4) into (5.2) to find super-potential in terms of field components *A(x),ψ(x) and F (x).*

Ex: 3: Show that the D-terms of V^a(See eq. (4.1)) and the F-terms of Φ_i do not receive any derivative contribution. Thereby show that auxiliary fields can be integrated out by the equation of motion.

MSSM Super-potential

- The word “minimal” means that we want to keep the number of interactions as small as possible.

$$W = W_{R_p} + W_{\not R_p}$$

$$W_{R_p} = \varepsilon_{ab}\left[\frac{1}{2} h^E_{ij} H^a_1 L^b_i E^C_j + h_{ij} H^a_1 Q^b_i D^C_j + h^U_{ij} H^a_2 Q^b_i U^C_j + \mu H^a_1 H^b_2\right]$$

$$W_{\not R_p} = \varepsilon_{ab}\left[\frac{1}{2} \lambda_{ijk} L^a_i L^b_j E^C_k + \lambda'_{ijk} L^a_i Q^b_j D^C_k + \kappa_j H^a_j H^b_2\right] + \frac{1}{2} \lambda''_{ijk} U^C_i D^C_j D^C_k$$

$$\varepsilon_{12} = -\varepsilon_{21}, \varepsilon^{12} = -\varepsilon^{21},$$

$$\varepsilon^{11} = \varepsilon^{22} = \varepsilon_{11} = \varepsilon_{22} = 0$$

Supersymmetry Breaking

- Super-symmetry is a broken symmetry in the vacuum state.
- Soft super-symmetry breaking mechanism was introduced to understand SUSY breaking.

$$○ \;▪\; ○_{SUSY} \;≡\; ○_{soft}$$

- This results in correction to Higgs mass as

$$\Delta(m_H)^2 = (m_{Soft})^2[\frac{\lambda}{16\pi^2}\ln(\frac{\Lambda}{m_{soft}})+.....]$$

- Lightest Higgs boson mass must be less than 135 GeV.

- GMSB-ordinary gauge interactions are responsible for the appearance of soft terms in the MSSM. Gravitino is the LSP. Chiral super-multiplets couple to a gauge singlet chiral supermultiplet *S* through a super-potential. $q, \overline{q}, l$ and $\overline{l}$

$$W = y_2 S l \overline{l} + y_3 S q \overline{q}$$

$q, \overline{q}$ get mass $y_3 < S >$ and $l, \overline{l}$ get mass $y_2 < S >$, while the scalars got masses $|y_2 <S>|^2 \pm |y_3 <F_S>|^2$. F_S is auxiliary component of S.

Superstring implies Supersymmetry

- Superstring is a fundamental and wider theory and incorporates the idea of supersymmetry.

- String theory extends space-time structure by introducing super fields defined over d-dimensional space-time coordinates, $X^{\mu}(\sigma, \tau)$,

  where σ represents the position on the string and

  ♌ is the proper time

Action of String Theory

$\mathrm{I}[X] = T\int d\sigma^+ \int d\sigma^- [\eta_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^+} \frac{\partial X^\nu}{\partial \sigma^-}]$, where T=constant called string tension μ=0,1,2,3,...d-1.

$\sigma^\pm$ are 2-dimensional string coordinates, $\sigma^\pm = \tau \pm \sigma$

Neveu & Schwartz in 1971 added fermionic field doublets $\psi_1{}^\mu(\sigma,\tau)$ & $\psi_2{}^\mu(\sigma,\tau)$

by introducing action for this theory:

$\mathrm{I}[X,\psi] = \int d\sigma^+ \int d\sigma^- [T \frac{\partial X^\mu}{\partial \sigma^+} \frac{\partial X_\mu}{\partial \sigma^-} + i\psi_2{}^\mu \frac{\partial}{\partial \sigma^+}(\psi_2)_\mu + i\psi_1{}^\mu \frac{\partial}{\partial \sigma^+}(\psi_1)_\mu]$,

Gervais & Sakita pointed out that in addition to two dimensional conformal invariance and d-dimensional lorentz invariance for appropriate boundary conditions this theory has a symmetry under infinitesimal transformations that interchange the bosonic field X^μ with the fermionic fields $\psi_r{}^\mu (r=1,2)$,

$$\delta\psi_2{}^\mu(\sigma^+,\sigma^-) = iT\alpha_2(\sigma^-)\frac{\partial X^\mu(\sigma^+,\sigma^-)}{\partial \sigma^-},$$

$$\delta\psi_1{}^\mu(\sigma^+,\sigma^-) = iT\alpha_1(\sigma^+)\frac{\partial X^\mu(\sigma^+,\sigma^-)}{\partial \sigma^+},$$

$$\delta X^\mu(\sigma^+,\sigma^-) = \alpha_1(\sigma^+)\psi_1{}^\mu(\sigma^+,\sigma^-) + \alpha_2(\sigma^-)\psi_2{}^\mu(\sigma^+,\sigma^-)$$

where α_1, α_2 are a pair of fermionic functions of σ^+, σ^- respectively, like the Grassmann variables. This was an example of a symmetry connecting bosons with fermions or what is called the Supersymmetry.

Wess-Zumino Model

Wess & Zumino introduced the idea in quantum field theory by taking Majorana(self charge Dirac field ψ, a pair of real scalar & pseudoscalar bosonic conjugate) auxiliary fields F & G with invariance under the infinitesimal transformation.

$$\delta \mathrm{A} = \bar{\alpha}\psi,\ \delta B = -i\bar{\alpha}\gamma^5\psi, \delta\psi = \partial_\mu(A + i\gamma_5 B)\gamma^\mu\alpha + (F - i\gamma_5 G)\alpha, \delta F = \bar{\alpha}\gamma^\mu\partial_\mu\psi,$$

$$\delta G = -i\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\psi$$

where α is an arbitrary constant representing infinitesimal fermion c-number parameter. The lagrangian density for this system can be defined:

$$\text{¢} = -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{1}{2}\partial_\mu B\partial^\mu B - \frac{1}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{1}{2}(F^2 + G^2) + m(FA + GB - \frac{1}{2}\bar{\psi}\psi)$$
$$+ g[F(A^2 + B^2) + 2GAB - \bar{\psi}(A + i\gamma_5 B)\psi]$$

As the bosonic or auxiliary fields (F&G) to the real scalars (A&B) through their field equations to a redefined lagrangian density as:

$$\text{¢} = -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{1}{2}\partial_\mu B\partial^\mu B - \frac{1}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{1}{2}m^2(A^2 + B^2) - \frac{1}{2}m\bar{\psi}\psi -$$
$$gmA(A^2 + B^2) - \frac{1}{2}g^2(A^2 + B^2)^2 - g\bar{\psi}(A + i\gamma_5 B)\psi$$

This Lagrangian exhibits relations between scalars A&B and fermion ψ masses but also between Yukawa interactions and scalar self couplings characteristics of SUSY theories

- Summary of most relevant spartners decays for the 5 MSSM models, whose SUSY mass spectra has been given before. The columns against each decay underneath the MSSM model concerned give decay ratio (in particularly relative to the total decay rate).

Lecture II: SUSY Signals at LHC/Dark Matter candidate (WIMP)

- The SUSY signals for LHC depends on various models.

- Five significant (MSSM) models are discussed in “LHC Discoveries Unfolded” by Joseph Lykken and Maria Spiropulu,, Ch.7, in the World Scientific Publication entitled “Perspectives on LHC Physics”, p.109, (2008). These particles appearing as missing energy with 5σ and first 100 pb^{-1} cross section. See relevant Figure and Table (pp 124-127 of Lykken & Maria’s above article) with captions below reproduced below:

SUSY signals-contd.

- The various (five) MSSM models predict SUSY (new) particles denoted with “tilde”, which are summarized in the following figure.

SUSY MASS Spectrum

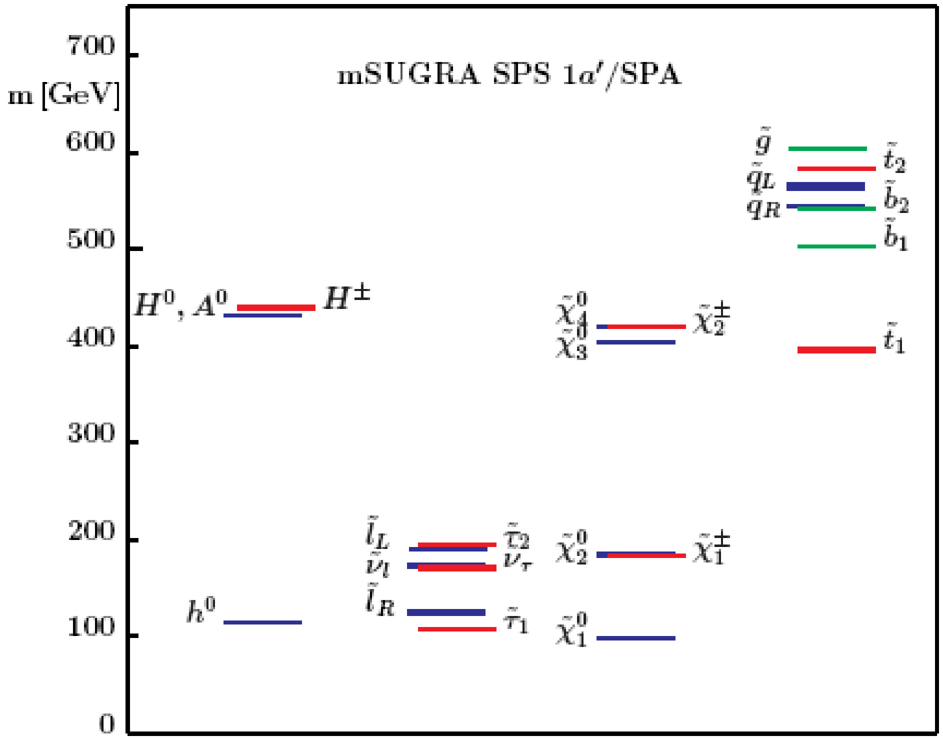


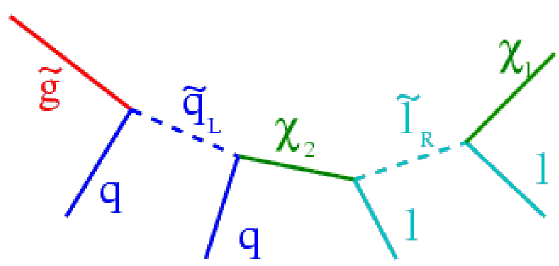
Figure 1:Mass spectrum of sparticles and Higgs bosons for the mSUGRA reference point SPS 1a’.

SUSY Studies, by Jan Kalinowski

Supersymmetry signals at LHC

- LHC is expected to give a copious production of squarks, gluinos.

- Multi-jets and large missing energy may be the signatures of gluino/squark decay chain.

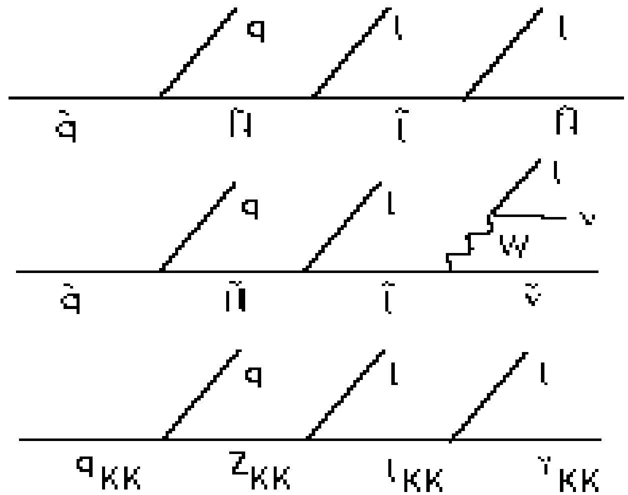


$\tilde{g}$ = gluino

$\tilde{q}_L$ = squark

χ = neutralino

Some Sparticle Decays



neutralino

sneutrino

KK photon

R-parity

$SUSY$ and SM particles are distinguished by a multiplicative quantum number called R-parity defined as follows.

$$R_p = (-1)^{2S}(-1)^{3B+L} \begin{cases} +1 \text{ for SM particle} \\ -1 \text{ for their Spartner} \end{cases}$$

Also equivalently,

$$R_p = (-1)^{2S}(-1)^{3(B-L)}$$

which allows for R-parity conservation as long as (B-L) is conserved (modulus2) through, individually B and L may separately be violated.

As a result sparticles no longer mediate interactions, sparticles are always pair produced (SM particle decay) and a sparticles eventually decays to a lightest and hence stable sparticle.

R-parity Conservation

- Supersymmetry super potential predicts Baryon and Lepton number violation.

- Proton decay becomes possible.

- An additional symmetry is defined which gives -1 phase to the s-particles and leaves the standard particles invariant.

$$\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$$

Lepton-number violating coupling

Baryon-number violating coupling

R-Parity Violation

- It predicts Baryon, Lepton number and flavor violation

$$\mathcal{L}_{\not R_p} = \lambda_{ijk}\left[\tilde{\nu}_{iL}\overline{e_{kR}}e_{jL} + \tilde{e}_{jL}\overline{e_{kR}}\nu_{iL} + \tilde{e}^{*}_{kR}\overline{(\nu_{iL})^{c}}e_{jL}\right]$$

$$+\lambda'_{ijk}\left[\tilde{\nu}_{iL}\overline{d_{kR}}d_{jL} + \tilde{d}_{jL}\overline{d_{kR}}\nu_{iL} + \tilde{d}^{*}_{kR}\overline{(\nu_{iL})^{c}}d_{jL}\right.$$

$$\left.-\tilde{e}_{iL}\overline{d_{kR}}u_{PL} - \tilde{u}_{PL}\overline{d_{kR}}e_{iL} - \tilde{d}^{*}_{kR}\overline{(e_{iL})^{c}}u_{pL}\right] + h.c$$

SUSY Lagrangian in terms of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermions fields. It also contains generalized Yukawa interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{SUSY} = {} & (D_\mu A_i)^\dagger (D A_i) + (\frac{i}{2}\bar{\psi}\bar{\sigma}^\mu D_\mu \psi_i + h.c.) \\
& -\frac{1}{4}(G^a_{\mu\nu})^2 + (\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a + h.c.) \\
& -(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i \psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.) \\
& -V(F_i, F_i^*, D^a)
\end{aligned}
$$

- The last term is a scalar potential term that depends only on the auxiliary fields.

The Scalar Potential

$$V(F_i, F_i^*, D^a) = \sum_i F_i^* F_i + \frac{1}{2}\sum_a (D^a)^2$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \qquad D^a = -g\sum_i A_i^a T^a A_i$$

Observe that the quartic couplings are governed by the gauge couplings and that scalar potential is positive definite! The latter is not a surprise. From the SUSY algebra, one obtains,

$$H = \frac{1}{4}\sum_{a=1}^{2}(Q_a^\dagger Q_a + Q_a Q_a^\dagger)$$

If for a physical state the energy is zero, this is a ground state.

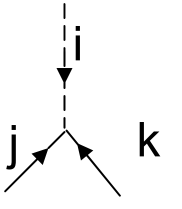
The SUSY is broken spontaneously if the vacuum energy is not zero!

SUSY Couplings

- The scalar part of the super-potential

$$P(A) = \frac{m_{ij}}{2} A_i A_j + \frac{\lambda_{ijk}}{6} A_i A_j A_k$$

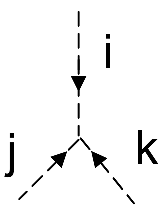
- The Yukawa couplings between scalar fermion fields

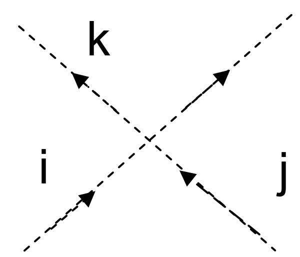


$$\frac{1}{2} \frac{\partial^2 P(A)}{\partial A_i \partial A_j} \psi_i \psi_j + h.c. \rightarrow \lambda_{ijk} \psi_i \psi_j A_k$$

Couplings are derivable from the same scalar potential represented by:

$$\left(\frac{\partial P(A)}{\partial A_i}\right)^2 \rightarrow m^*_{ml}\lambda_{mjk}\, A^*_i A_j A_k \text{ and } \lambda_{mjk}\lambda_{mil} A_j A_k A^*_i A^*_l$$





Cosmology: Need to go Beyond SM(BSM)- SUSY included

- Neutrino Physics:

  $$m_{\nu} \neq 0, \theta_{\nu} \neq 0$$

- Cosmology –Particle Physics:

  Dark Matter , Baryon Asymmetry, Inflation,

  Dark Energy.

- Theoretical reasons:

  (As discussed before)

Theoretical basis

- Hierarchy (of scales).
- Unification of forces.
- Existence of particle flavors?

Present "Observational" Evidence for New Physics (NP)

- Neutrino Masses ★ ★ ★
- Dark Matter ★ ★ ★
- Matter-Antimatter Asymmetry ★ ★
- Inflation ★

Neutrinos are Massive: New Physics is there!

- Is the above a trivial statement?

→ SM built to have massless neutrinos (no RH neutrino, no isospin triplet scalar Higgs), →

find that neutrinos are massive and claim that new physics (NP) discovered!

- No, Neutrino mass is “real” NP with a new energy scale associated to it.

SM & COSMOLOGY

- The gauge theories describe condensation of nuclei (light) from quarks, with merging of Micro Particle Physics Phenomena and the Macro physics (Cosmology) in terms of Hot Big Bang Standard Model. In particular, nucleo-synthesis has been explained, resulting in Big Bang Nucleo-synthesis. .
- However there remain many issues like dark matter, dark energy, inflation and matter -anti matter asymmetry.

Energy Budget of the Universe

- Stars and galaxies ~0.5%
- Neutrinos ~0.1-1.5%
- Rest of ordinary matter ~4.4%
- Dark matter ~23%
- Dark Energy ~73%
- Anti-Matter ~0%

Dark Matter: the most impressive quantative or qualitative evidence

- Latest WMAP & LSS data provides stringent bounds on Ω_{DM} and Ω_B →

Evidence for non-baryonic DM at more than 10 standard deviations!! The SM does not provide any candidate for such non- baryonic DM.

- LSS formation requires DM to be COLD

→ New particles not included in the spectrum of the fundamental building blocks of the SM!

Rise and Fall of Neutrinos As Dark matter

- Massive neutrinos are only candidates in the SM to account for DM. From here the “prejudice” of neutrinos of a few eV to correctly account for DM.

- Neutrinos decouple at ~1 MeV; being their mass<< decoupling temperature, neutrinos remain relativistic for a long time. Being very fast, they smooth out any possible growth of density fluctuation forbidding the formation of proto-structures.

- The “weight” of neutrinos in the DM budget is severely limited by the observations disfavoring scenarios, where first super large structures arise and then galaxies originate from their fragmentation.

DARK MATTER SEARCH AT LHC

- Evidence For DM (non-luminous, non-baryonic, non-radiating) comes from:

  1. Motion of cluster of galaxies where the inward gravitational pull from within the center is insufficient to balance the outward “centrifugal force” of the matter. Without DM the cluster will pull itself apart.

  2. Similar mechanism helps to stabilize individual galaxies when their rotational motion is considered. This balancing ensures that the speed of galaxies in the cluster beyond the bright core should fall inversely *with square root of distance from the center*.

This is contrary to observation. Rotational **curves remain constant at radii** far **exceeding the luminous core** indicating a dark component of mass, i.e., the galaxies **possess a halo of DM**.

DARK MATTER SEARCH AT LHC (CONTD.)

- Measurement of Cosmic Microwave Background Radiation (CMBR) provides an opportunity to make a precise mapping of the Dark Matter.
- Various new and sensitive technologies have measured variations from uniform CMBR to the tune of $1:10^{-5}$.
- These variations in the CMBR arise from the early universe as quantum fluctuations from an inflationary epoch, in which the universe is assumed to expand exponentially for a very short fraction of a second.
- These fluctuations under gravity and with time can grow into **large scale structures to form galaxies and clusters that we see today**.
- The amount and type of material that clumps under gravity determines the structure.
- Simulations indicate that **substantial DM component is consistent with observations**.
- A universe **without the DM component** grows insufficient structure.

DM and Baryonic Density

- These fluctuations create inhomogeneities and anisotropies in the CMBR after passing through the DM background.

$$\Omega_{DM} h^2 = 0.106 \pm 0.008 \qquad (1)$$

$$\Omega_{baryon} h^2 = 0.0223^{+0.0007}_{-0.0009} \qquad (2)$$

- where Ω_{DM} is DM density parameter, and Ω_{baryon} is the matter density of protons and neutrons. (Note (1) is nearly six times (2))

- Big bang expansion resulted in cooling that gave rise to light elements ,(nucleosynthesis). The abundance parameters of hydrogen, deuterium, helium and lithium, can be easily estimated from basic thermodynamics.

- Baryonic density can also be independently measured from nucleo-synthesis.

- These measurements imply a baryon density
- $0.017<\Omega_{baryon}h^2<0.024$………(3)

Gravitational Lensing

- The phenomena of bending of light around intense gravitational sources lead to confirm Einstein general theory of relativity in 1919 by A.R. Eddington, who observed the starlight around the complete solar eclipse, confirming sun causes the bending. This phenomenon is also used these days to estimate DM component.

Section 2: What is Dark Matter

- It is now clear that DM is not made up of baryons and is non-luminous/non-radiant and is not “Jupiter-like dust”. It cannot be a light nuclear particle like the neutrino because $(m_{\nu_e} \lesssim 1\ eV)$ neutrino being so light cannot comprise so large energy density as one quarter of the energy density of universe.

- It is safe to assume that the DM would be found in weak TeV scale provided the DM was in thermal equilibrium with all the species existing after the Big-Bang.

- We can then calculate the amount remaining as a “thermal relic” by balancing the expansion rate of the universe against the tendency of DM to self annihilate into ordinary matter.

- For sufficiently large expansion rate, DM cannot find other DM particles with which it might annihilate. At this point the **amount of DM** freezes out and annihilation become unimportant.

- This determines the amount of DM in equilibrium balancing the fight between expansion and annihilation, resulting in the equation:

$$\Omega_{DM} h^2 = c \frac{T_o^3}{M_{pl}^3} \frac{1}{<\sigma v>} \qquad (4)$$

Using:

$$\Omega_{DM} h^2 = 0.106 \pm 0.008$$

$$< \sigma v > \approx 1\, pb.$$

Approx. mass scale can be calculated from identifying

$$< \sigma v > \approx \frac{\alpha^2}{m^2},$$

to be Dark matter mass

$$m \approx 100\, GeV\,!$$

Section 2.1: SUSY Dark matter

- SUSY is the leading theory of new physics (NP) for physics beyond SM(BSM). SUSY as discussed leads to doubling of spin degrees of freedom.

- However as seen such particles do not exist at low energies, which means SUSY is broken.

- LHC holds a possibility of producing some SUSY particles, especially the LSP (lightest super-symmetric particle).

- LHC will be operating at 14 TeV in the CM for pp-collisions with luminosity ~10^{34}.

LSP Scenarios

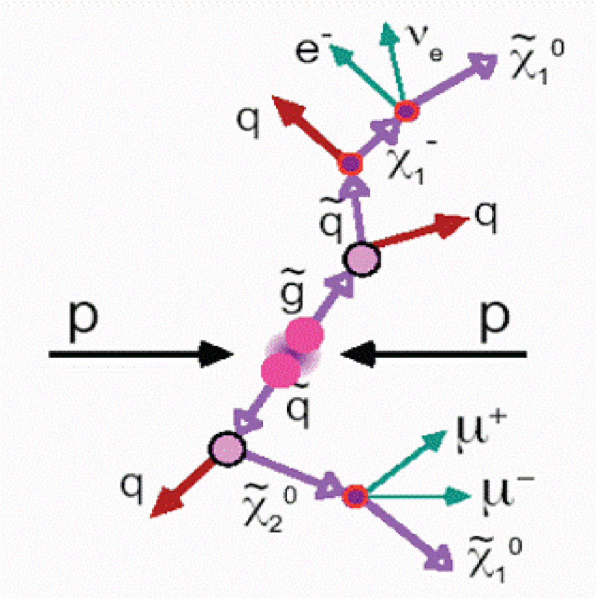
- There are three scenarios, where a neutral sparticle may be found:

1. It may be a SUSY partner of neutrino called sneutrino. This is excluded by the underground detection experiments designed to look for DM by scattering off nuclei.
2. It could be the spartner to the graviton called gravitino. This possibility is still open and under study in Supergravity theory.
3. Another viable possibility is that DM is a neutral superposition of spartners of SM gauge bosons, namely photino, zino, bino and higgsino. This admixture is known as neutralino.
4. One last possibility is that there could be a single scalar particle coupled to the Higgs boson. This will modify the properties of the Higgs boson, which could dominantly decay invisibly and its discovery could occur simultaneously to the discovery of Dark matter.

Section 2.2: What if we do not see DM at LHC

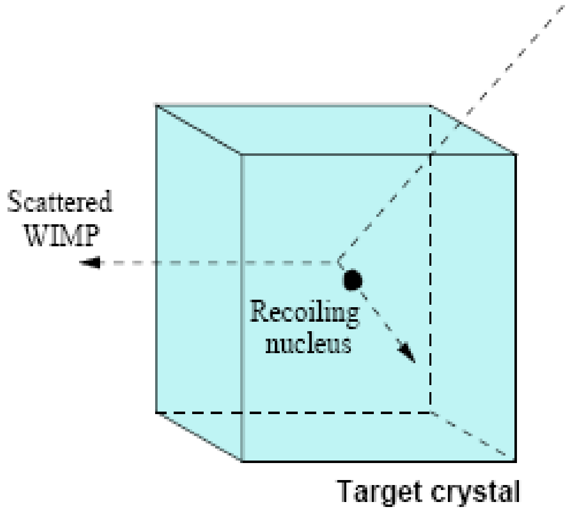
- This may imply that we do not understand DM in terms of TeV scale. One possibility is then axion for dark matter:
- Axion is a particle causing strong CP-violation through violation of Pecci-Quinn symmetry. Axion decay rate through two photons relevant to LHC energies have already been predicted

LSP as Dark Matter Candidate



Neutralino Dark Matter

If Neutralinos constitute the bulk of dark matter they would cluster with ordinary stars in galactic halos, raising the hope of their direct detection through their elastic scattering with nuclei inside a detector.

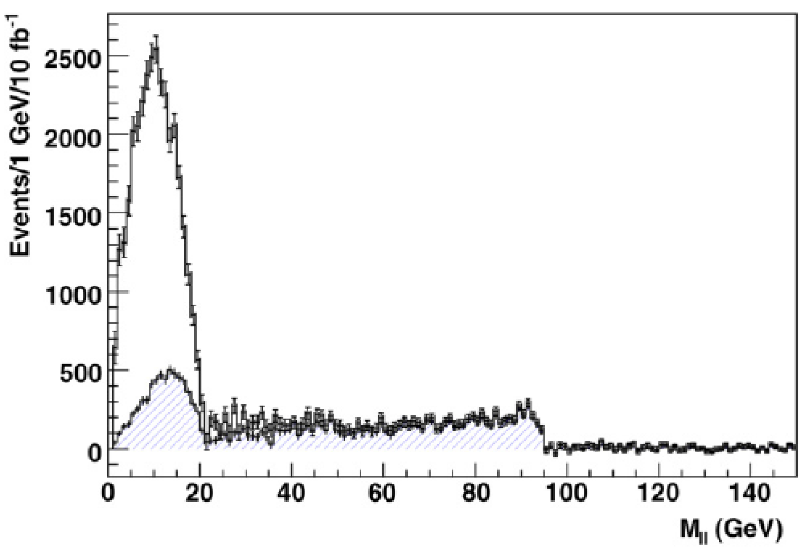


The recoiling energy can be detected by

- Ionization on solids
- Ionization in scintillators (measured by emission of photons)
- Increase in the temperature (measured by the released phonons)

Gravitino Dark Matter

- It is a spin 3/2 partner of graviton. It may be produced at LHC.



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