COMSATS Institute of Information Technology

QGP Thermodynamics and Phase Transitions

Mahnaz Q. Haseeb
Department of Physics
CIIT, Islamabad

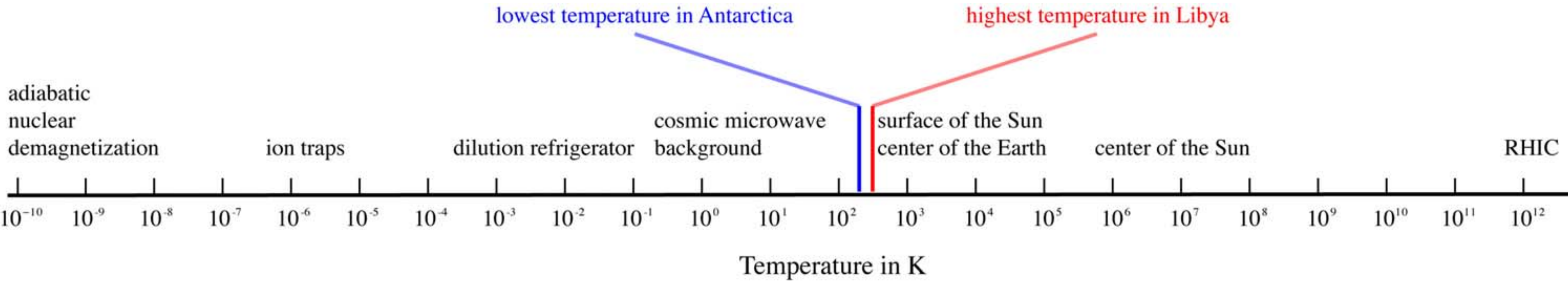
Outline

- Thermodynamics in QGP
- What are the phase transitions?
- Order of the phase transition
- Where to study QCD phase transitions?
- Phenomenological Models
- Statistical models

Temperature and Energy Density

- Very high temperatures are created in HIC collisions

$$T_{ch} = 170 \pm 10 \text{ MeV}$$



- Very high energy densities are created in HIC collisions

$$\varepsilon_{Bj} = \frac{dE_T}{dy}\frac{1}{\tau_0 \pi R^2} = 4.9 \pm 0.3 \text{ GeV/fm}^3$$

Grand Canonical Ensemble

Start out with a system completely out of equilibrium and lots of kinetic energy.

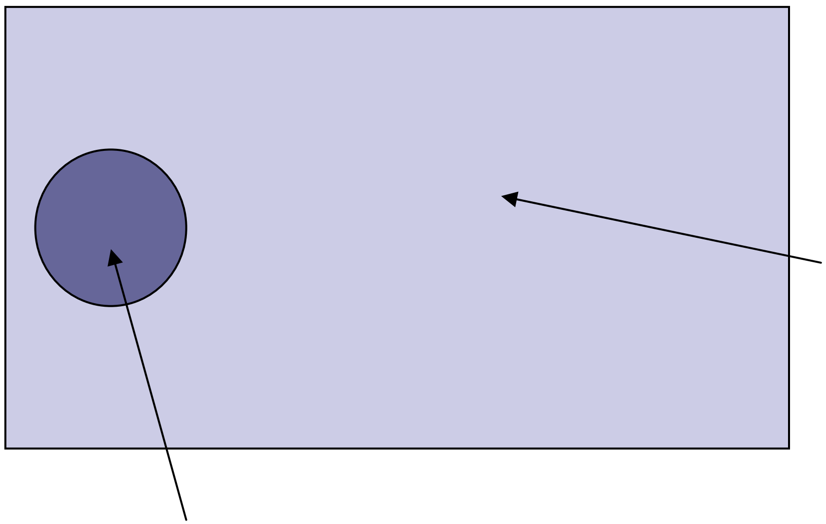
Use the Grand Canonical Ensemble to calculate the abundances of all the final measured particles.

$$\overline{n}_s = \frac{1}{e^{(\varepsilon_s - \mu)/kT} \pm 1}$$

Fermions or Bosons

Depends on Temperature and Chemical Potential.

Grand Canonical Ensemble



Infinite heat bath with which a system can exchange energy and particles, hence having a temperature and chemical potential.

$$N_i \propto g_i V \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{(\sqrt{p^2+m^2}-\mu_B)/T} \pm 1}$$

Canonical and Micro Canonical Ensembles

If the volume of the system is large, GCE is appropriate. For small volumes, one must conserve quantum numbers in every event !

In Canonical Ensembles, the total particle number is constant while the system interchanges heat energy e.g., the strangeness is suppressed for very small volumes and reaches the GCE limit for large volumes.

In a Microcanonical Ensemble there is constant Temperature, Volume and Particle number

Thermodynamical Variables

- The Gibbs free energy for GCE is

$$E = TS + \mu N - PV$$

  - taken per unit volume

$$\varepsilon = Ts + \mu n - P$$

- So $dE = T\,dS + \mu\,dN - P\,dV$
- When *N* or *n* is fixed (CE)

$$\epsilon = T\frac{\partial P}{\partial T} - P$$

$$s = \frac{S}{V} = \frac{1}{V}\frac{\partial P}{\partial T}\ .$$

Partition function

$$Z = \mathrm{Tr}\,\exp\left[-\left(\hat{H} - \mu\hat{N}\right)/T\right] = \int D\bar{q}\,Dq\,D\varphi\,\exp\left[\int_0^{1/T} d(it)\int_V d^3x\left(L + \mu_q\,\bar{q}\gamma^0 q\right)\right]$$

$$(\mu_q = \mu/3)$$

Effective potential

$$U(T,\mu) = -\frac{T}{V}\ln Z \quad \Rightarrow \quad U = \Omega/V = -P$$

- mean field approximation: $\varphi \leftrightarrow \langle\varphi\rangle$

$$\boxed{U(T,\mu,\varphi) = U_0(\varphi) - \nu_q T\int\frac{d^3p}{(2\pi)^3}\ln\left\{1 + \exp\left[(E-\mu_q)/T\right]\right\} - (\mu_q \to -\mu_q)}$$

$$E = \sqrt{p^2 + m_q^2} \qquad \boxed{m_q^2 = g^2\varphi^2 = g^2\left(\sigma^2 + \pi^2\right)}$$

Partition function

In terms of the partition function

$$Z = \sum_a <a|\, exp[-(1/T)(H-\mu N)]|a>$$

So that

$$n = (T/V)(\partial \ln Z/\partial\mu)$$

$$\varepsilon = (T^2/V)(\partial \ln Z/\partial T) + \mu n$$

Melting the Hadrons

Can we melt the hadrons and liberate these quark and gluon degrees of freedom?

$$\varepsilon = g\frac{\pi^2}{30}T^4$$

Energy density for "g" massless d.o.f.

$$\varepsilon = 3\cdot\frac{\pi^2}{30}T^4$$

Hadronic Matter: quarks and gluons confined
For T ~ 200 MeV, 3 pions with spin=0

$$\varepsilon = \left\{2\cdot 8_g + \frac{7}{8}\cdot 2_s\cdot 2_a\cdot 2_f\cdot 3_c\right\}\frac{\pi^2}{30}T^4$$

Quark Gluon Matter:
8 gluons;
2 quark flavors, antiquarks,
2 spins, 3 colors

$$\varepsilon = 37\cdot\frac{\pi^2}{30}T^4$$

37 !!!

What are the phase transitions?

Transformation of a system from phase to Another

Let’s consider various physical situations where we can view them

Cosmological phase transitions

- QCD phase transition T=175 MeV
- Electroweak phase transition T=150 GeV

  baryogenesis ?
- GUT phase transition(s) ? $T=10^{16}$ GeV

  monopoles,cosmic strings ?
- "inflation" $T=10^{15}$ GeV

  primordial density fluctuations !

  primordial magnetic fields ?

Phase transition in early universe…

…when the universe cools below 175 MeV

10^{-5} seconds after the big bang

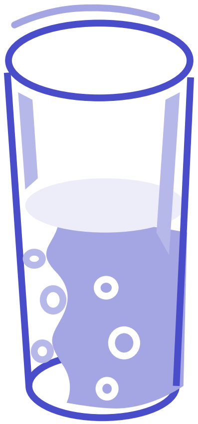
Quarks and gluons form baryons and mesons

before: simply not enough volume per particle available

Phase Transitions of Water

ICE

WATER



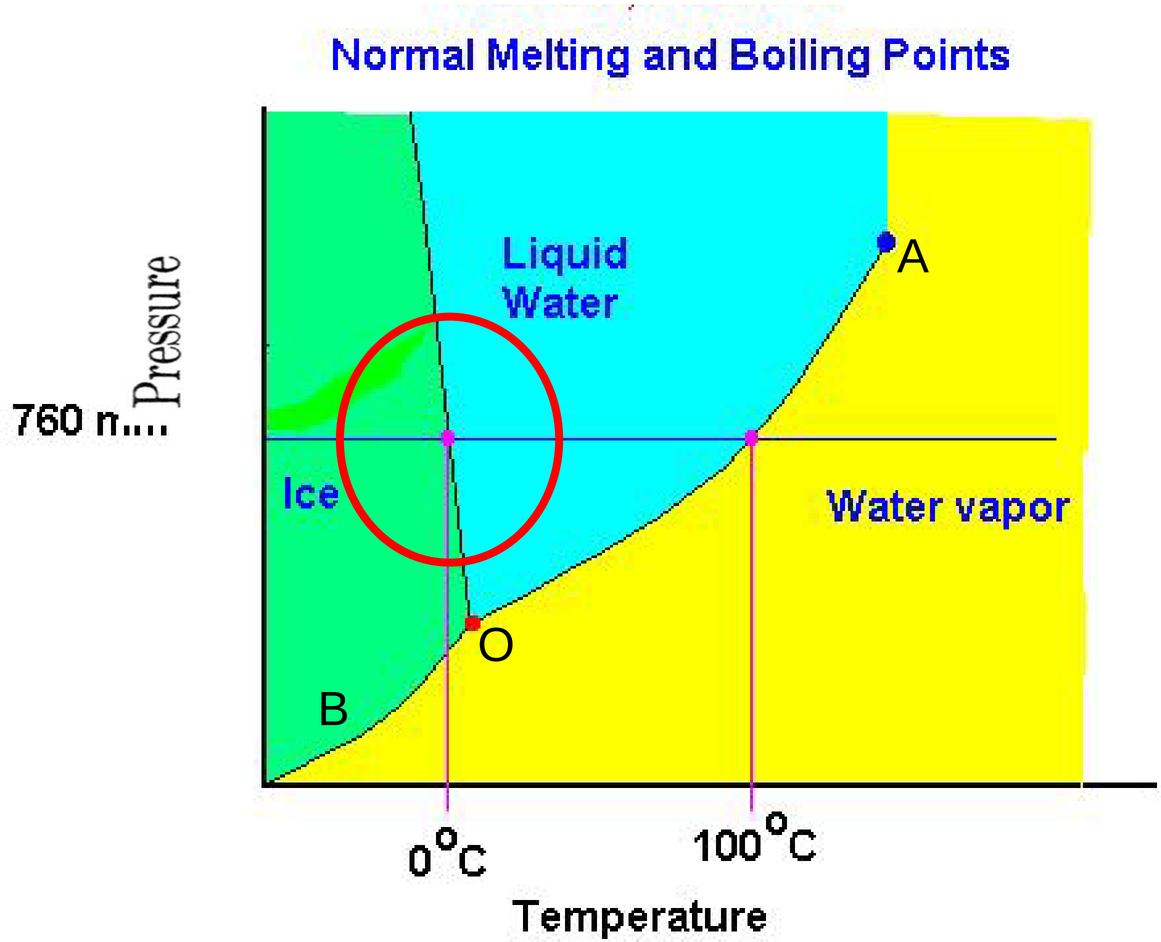
Add heat

Symmetric phase

Broken symmetry phase

Phase Diagram of Water

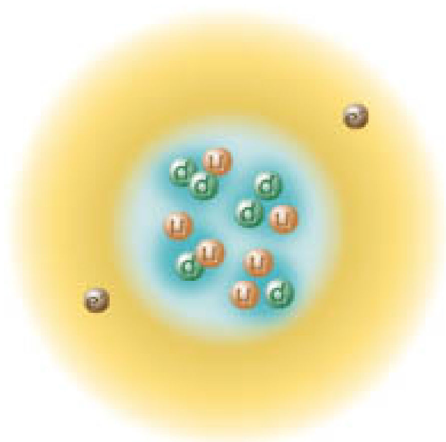
No end to the line:
(B) always a phase transition
Triple point: (O)
2 lines meet,
3 phases coexist
Critical point: (A)
2 phases no longer distinct

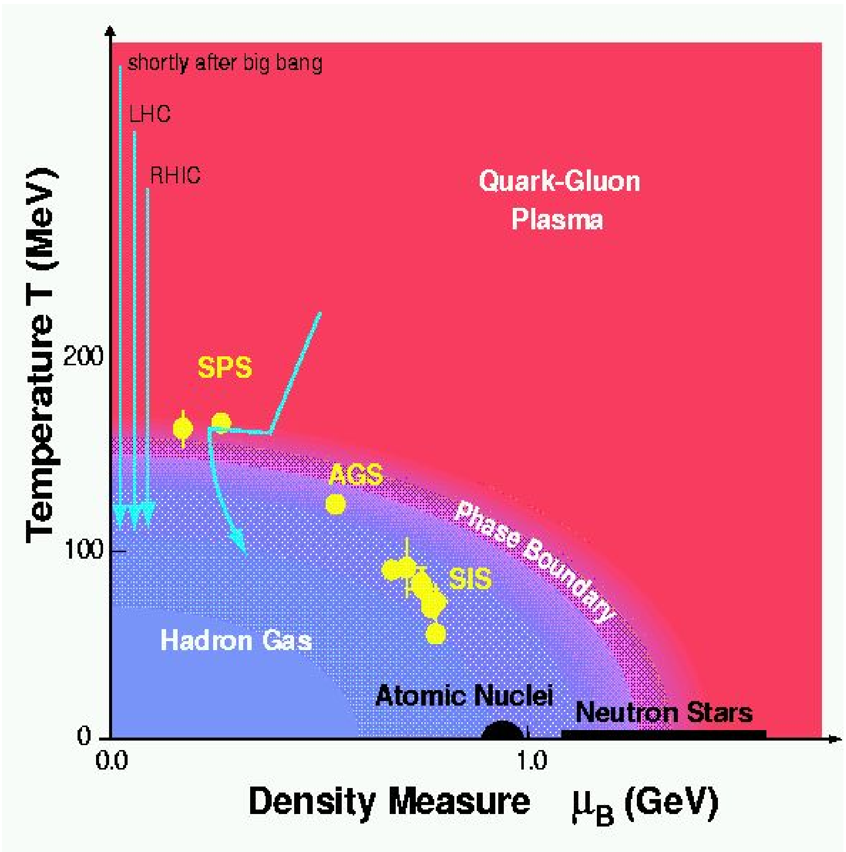


We heat up the system

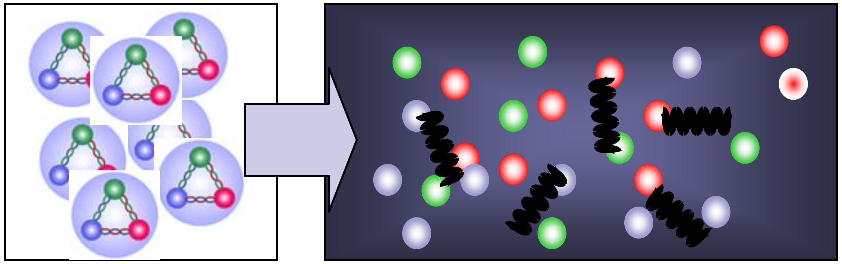
Phases of QCD Matter

- strong interaction analogues of the familiar phases:



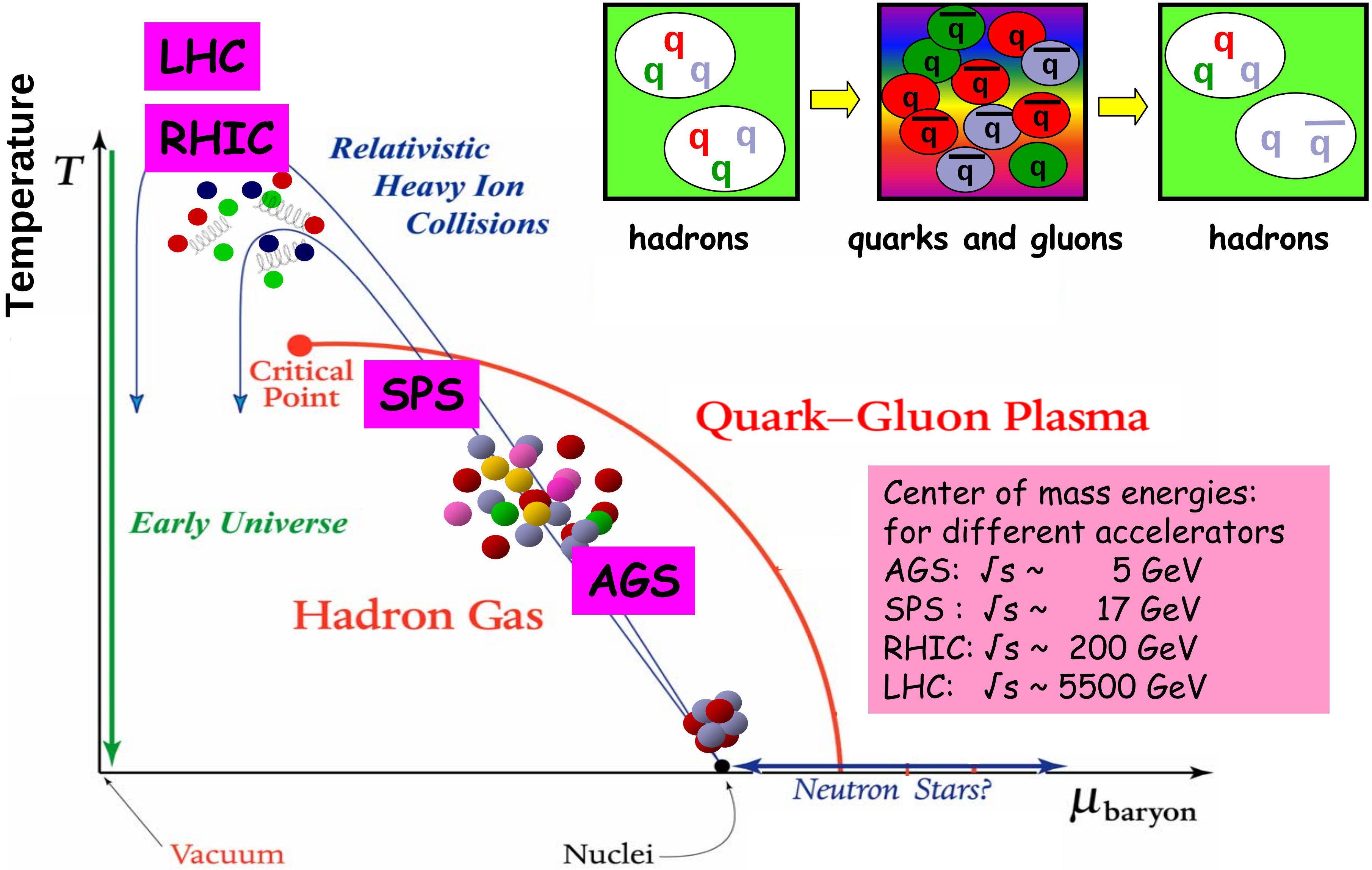


- Nuclei behave like a liquid
  - Nucleons are like molecules
- Quark Gluon Plasma:
  - "ionize" nucleons with heat
  - "compress" them with density



new state of matter!

Phase Diagram of Nuclear Matter

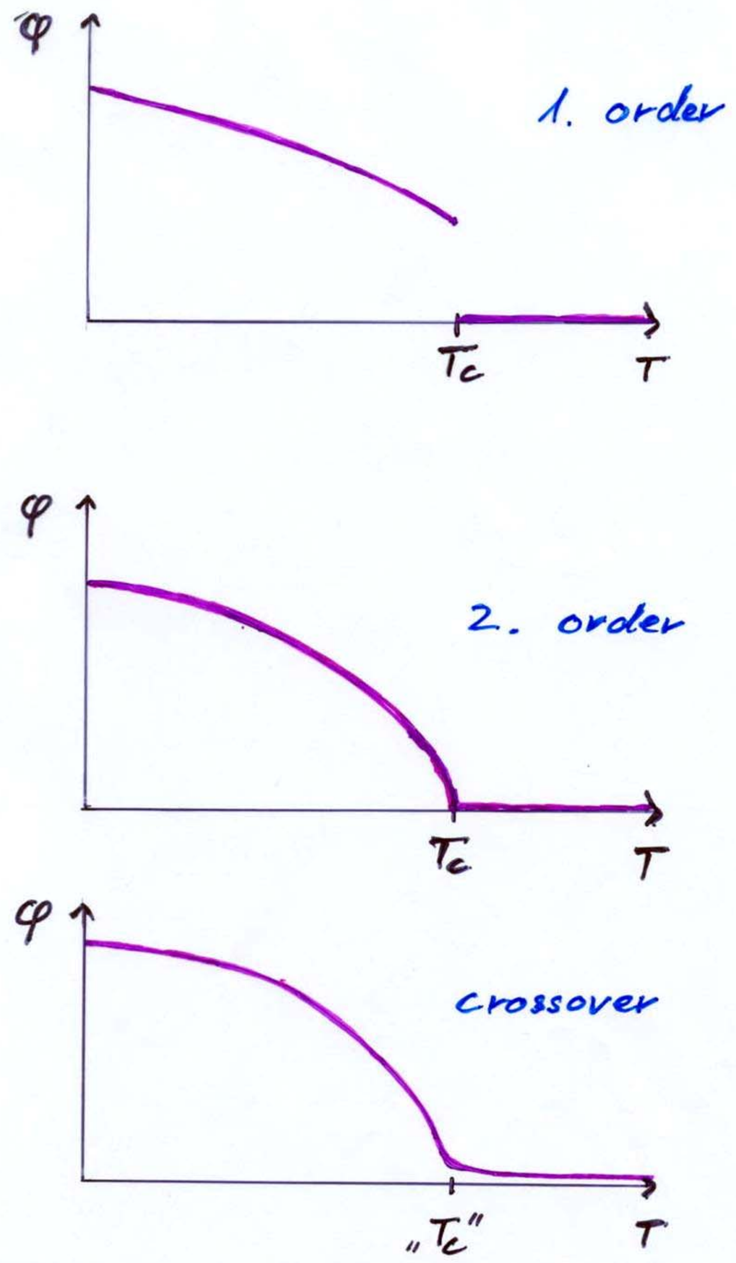


Center of mass energies:
for different accelerators
AGS: $\sqrt{s}$ ~ 5 GeV
SPS : $\sqrt{s}$ ~ 17 GeV
RHIC: $\sqrt{s}$ ~ 200 GeV
LHC: $\sqrt{s}$ ~ 5500 GeV

Order of the phase transition is crucial ingredient for phase transition in experiments (heavy ion collisions)

Order of the phase transition

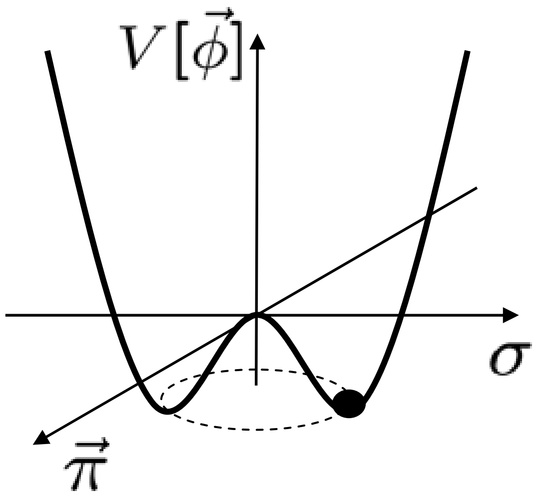
temperature dependence of order parameter
$\xi = \xi_0(T)$,
- the minimum for *Landau free energy*
$F_L(\xi,T) = E(\xi,T) - T\,S(\xi,T)$



Spontaneous Breaking of Chiral symmetry

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$$

⇒ pion is the massless Nambu-Goldstone particle



$$\vec{\phi} = (\pi_1, \pi_2, \pi_3, \sigma)$$

$$V[\phi] = r_0 \phi_i^2 + u \phi_i^4$$

$$\sigma = \langle \bar{q} q \rangle \text{ (Chiral condensate)}$$

⇔ Ferromagnet

O(3) symmetry is spontaneously broken

Chiral symmetry restoration at high temperature

Low T

SSB

$\langle\varphi\rangle = \varphi_0 \neq 0$

High T

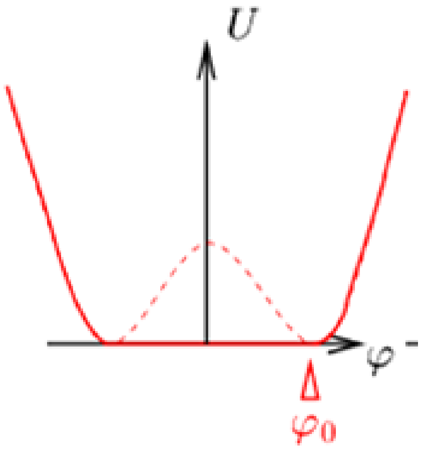
Symmetric

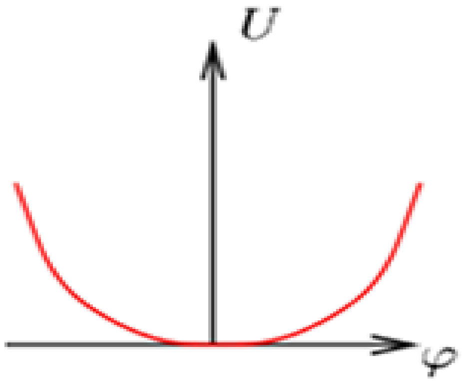
$\langle\varphi\rangle = 0$

at high T :
less order
more symmetry

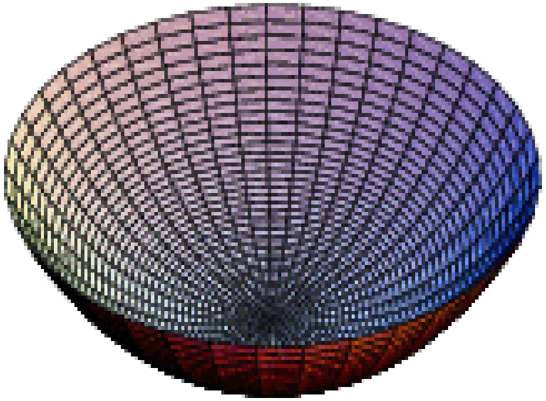
examples:

magnets, crystals

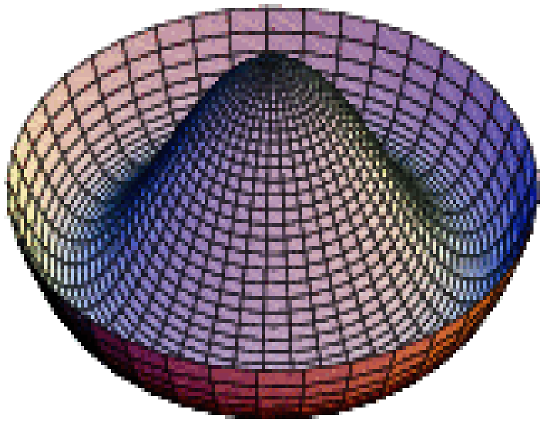




Chiral Symmetry



Chiral symmetry restored phase



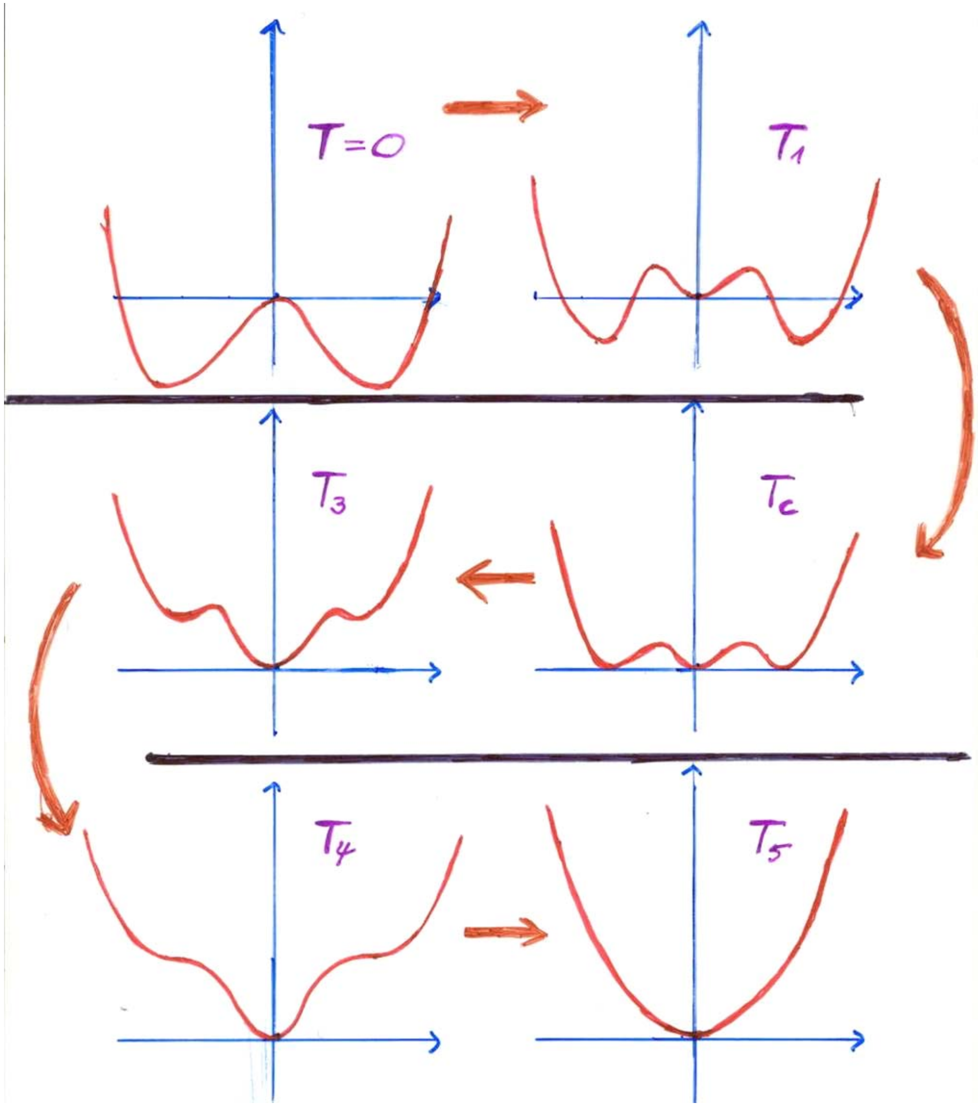
Chiral symmetry broken phase

- Spontaneously broken chiral symmetry

$$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$$

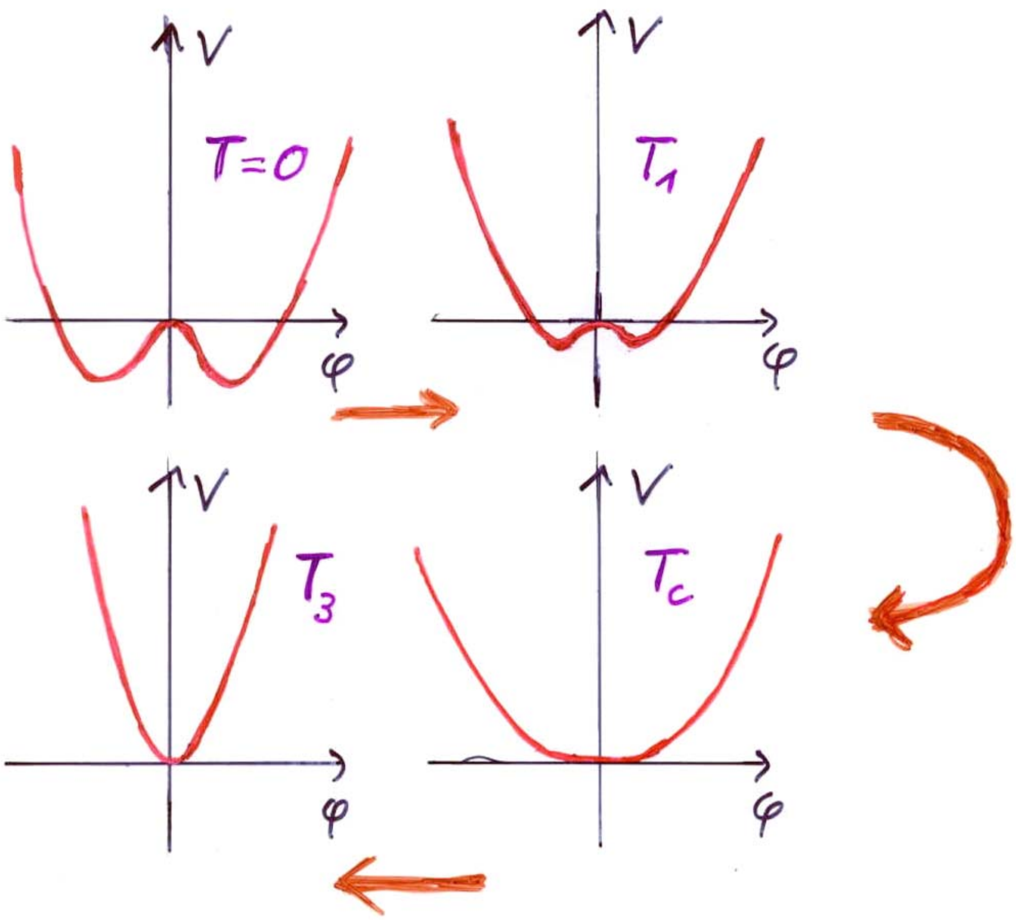
- If QGP phase transition corresponds to the chiral phase transition, it would be the second order phase transition.
- Chiral transitions ~ 150 MeV !!!

First order phase transition



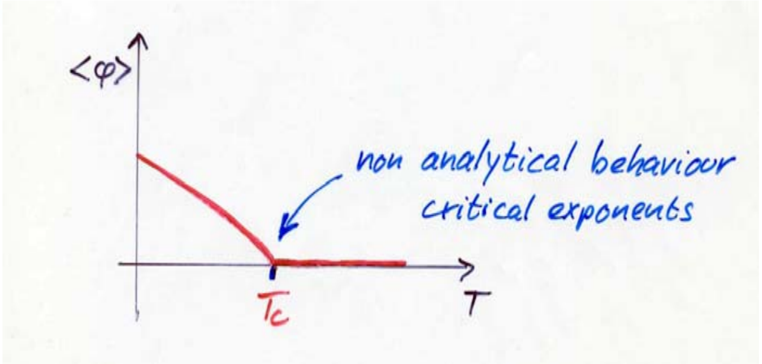
ξ = 0

Second order phase transition



$\xi^2 = \xi_0^2$

$V = -\mu_0^2 \varphi^\dagger\varphi + cT^2 \varphi^\dagger\varphi + \frac{\lambda}{2}(\varphi^\dagger\varphi)$



Phase transition

- The quark-gluon and hadron equations of state
  - The energy density of (massless) quarks and gluons is derived from Fermi-Dirac statistics and Bose-Einstein statistics.

$$\varepsilon_g = \frac{1}{2\pi^2}\int \frac{p^3 dp}{e^{\beta p} - 1} \qquad \Rightarrow \varepsilon_g = \frac{\pi^2 T^4}{30}$$

$$\varepsilon_q = \frac{1}{2\pi^2}\int \frac{p^3 dp}{e^{\beta(p+\mu)} + 1} \qquad \Rightarrow \varepsilon_q + \varepsilon_{\bar{q}} = \frac{7\pi^2 T^4}{120} + \frac{\mu^2 T^2}{4} + \frac{\mu^4}{8\pi^2}$$

where μ is the quark chemical potential, μ_q = - μ_q and $\beta = 1/T$.

  - Taking into account the number of degrees of freedom

$$\varepsilon_{TOT} = 16\varepsilon_g + 12\left(\varepsilon_q + \varepsilon_{\bar{q}}\right)$$

  - Consider two extremes:
    1. High temperature, low net baryon density ($T > 0$, $\mu_B = 0$).
    2. Low temperature, high net baryon density ($T = 0$, $\mu_B > 0$).

$\mu_B = 3\ \mu_q$

Critical parameters

- High temperature, low density limit - the early universe
  - Two terms contribute to the total energy density
  - For a relativistic gas:
  - For stability:

$$\varepsilon_{qg} = 37\frac{\pi^2}{30}T^4$$

$$P_{qg} = \frac{1}{3}\varepsilon_{qg}$$

$$P_{net} = P_{qg} - B \geq 0$$

$$T_c = \left[\frac{90}{37\pi^2}B\right]^{1/4} = 100-170\ \text{MeV}$$

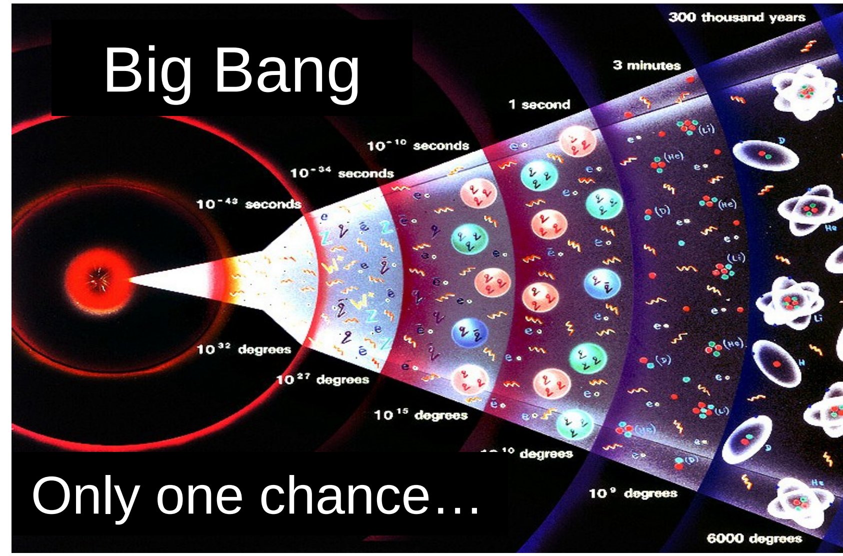
- Low temperature, high density limit - neutron stars
  - Only one term contributes to the total energy density
  - By a similar argument:

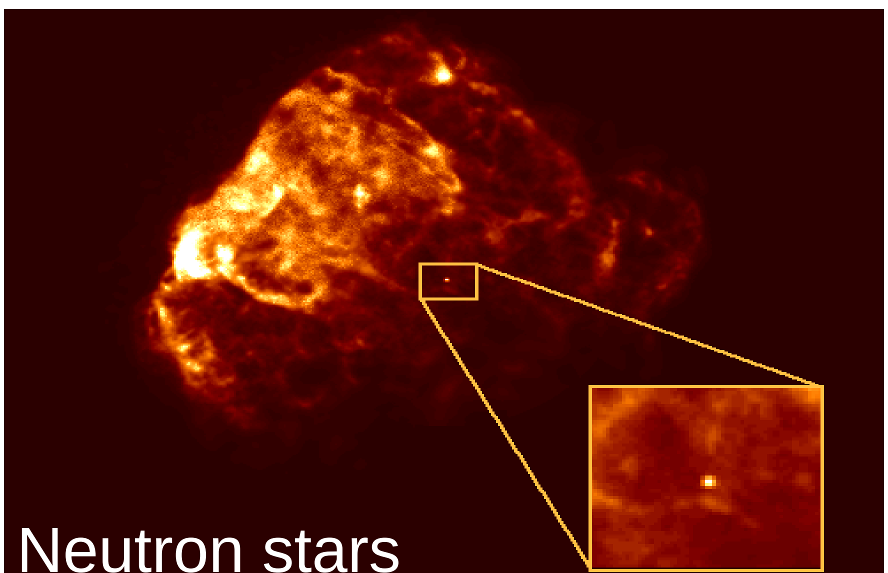
$$\varepsilon_q = \frac{3}{2\pi^2}\mu_q^4$$

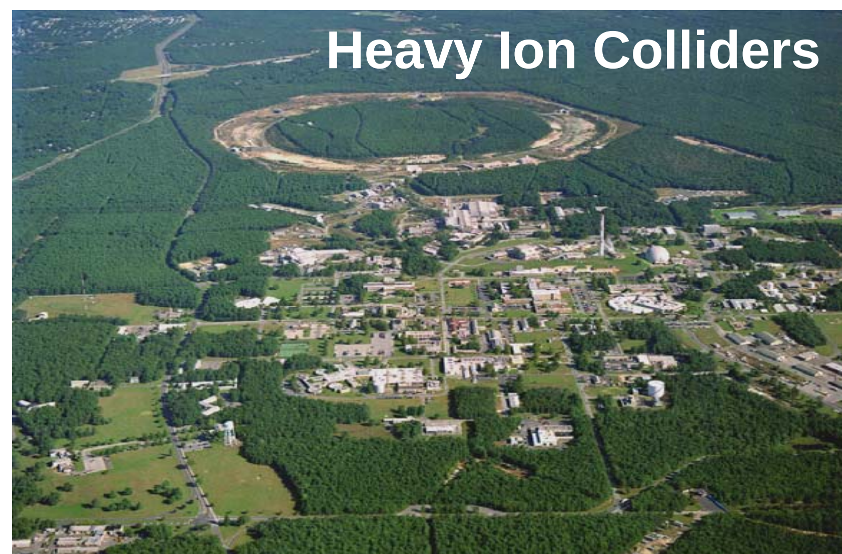
$$\mu_c = \left(2\pi^2 B\right)^{1/4} = 300-500\ \text{MeV}$$

~ 2-8 times normal nuclear matter density given p_{Fermi} ~ 250 MeV and $\rho \sim 2\mu^3/3\pi^2$

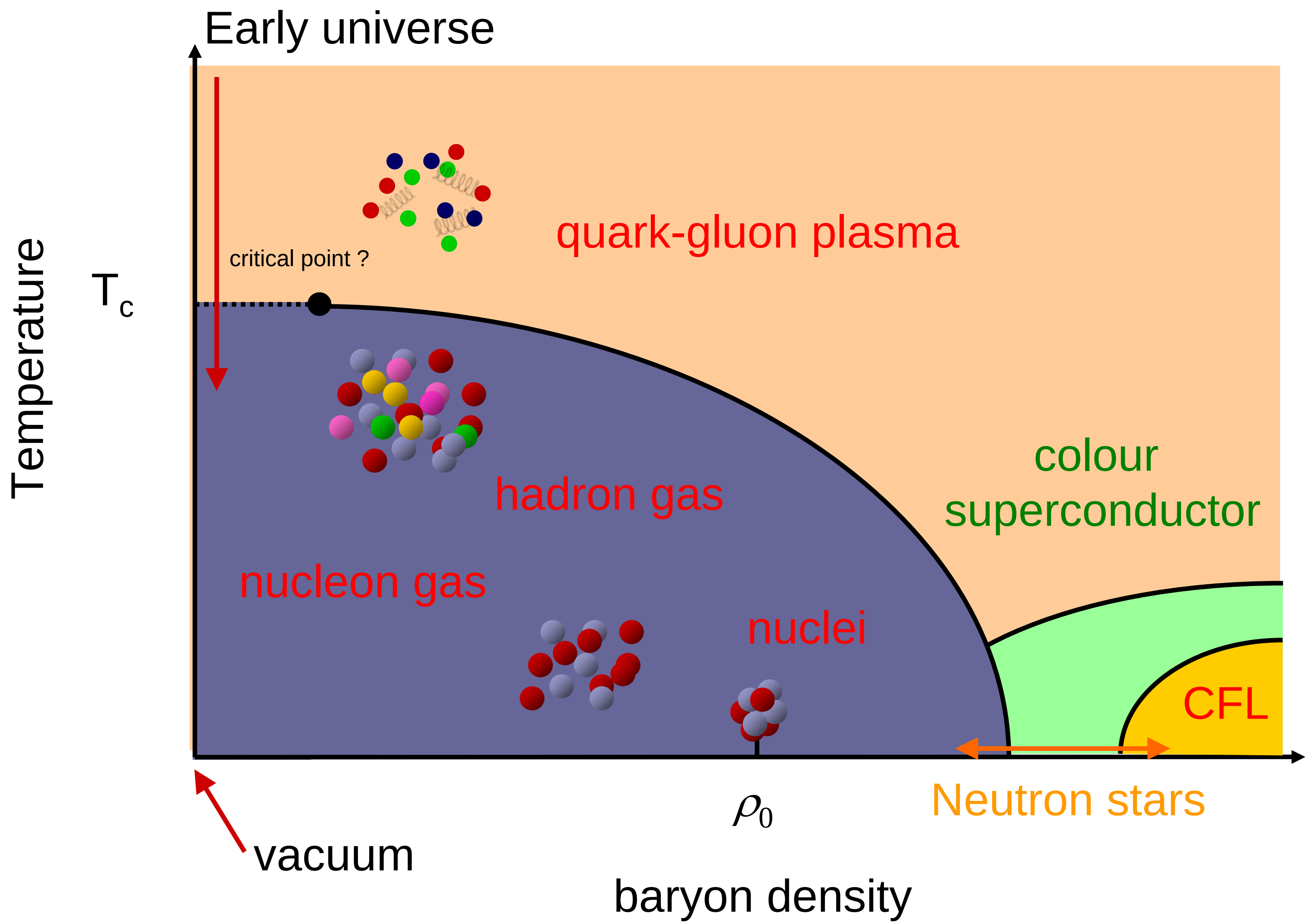
Where to study QCD Phase Transition?







The phase diagram of QCD



Estimating the critical parameters, T_c and ε_c

- Mapping out the Nuclear Matter Phase Diagram
  - Perturbation theory highly successful in applications of QED.
  - In QCD, perturbation theory is only applicable for very hard processes.
  - Two solutions:

    1. Phenomenological models (MIT Bag model)
    2. Lattice QCD calculations

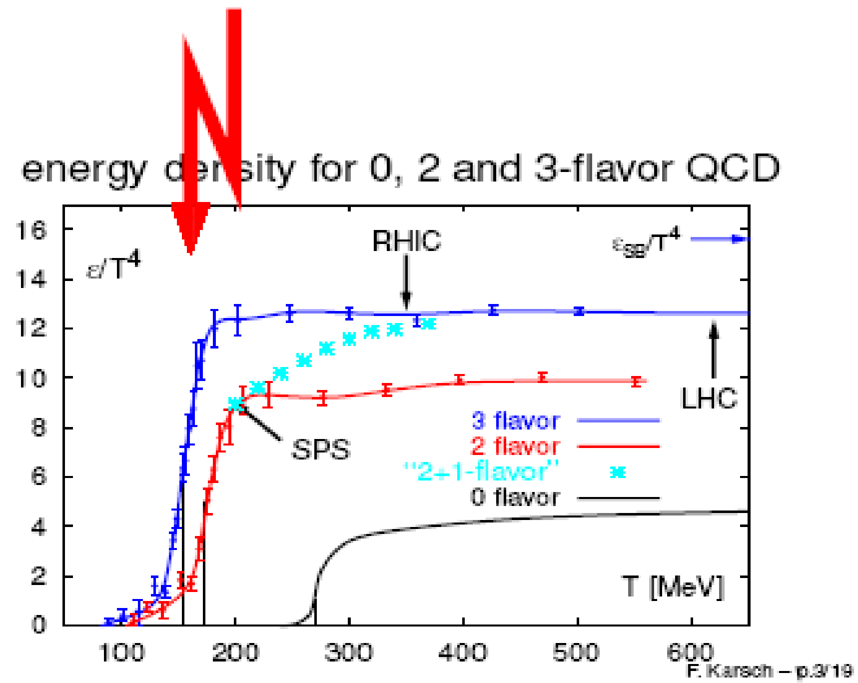
Lattice QCD – QGP phase transition

$\varepsilon_{SB} = n_f\, \pi^2 /30\ T^4$

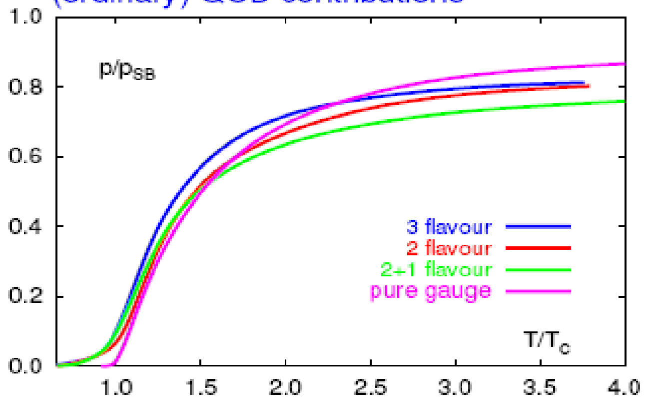
T_C ~ 155-175 MeV ε_C ~ 0.3-1.0 GeV/fm^3

n_f in hadron gas: 3 (π^+, π^-, π^0)

$$\varepsilon_{SB} = \begin{array}{l} \{2_f \cdot 2_s \cdot 2_q \cdot 3_c \frac{7}{8} + 2_s \cdot 8_c\} \frac{\pi^2}{30} T^4 = 37\ \frac{\pi^2}{30} T^4 \\ \{3_f \cdot 2_s \cdot 2_q \cdot 3_c \frac{7}{8} + 2_s \cdot 8_c\} \frac{\pi^2}{30} T^4 = 47.5\ \frac{\pi^2}{30} T^4 \end{array}$$



$T \gtrsim (2\text{-}3)T_c$: deviations from ideal gas understood in terms of non-perturbative (ordinary) QCD contributions



$T \lesssim 2T_c$: strong deviations from ideal gas
large screening masses,
remnants of confinement

F. Karsch – p.4/19

References

- **Ultrarelativistic Heavy Ion Collisions**

  Author: Ramona Vogt

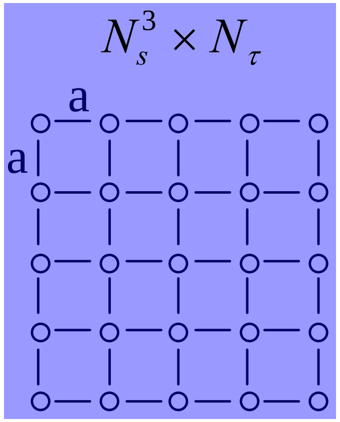
  Elsevier (2007)

Prediction according to Lattice QCD?

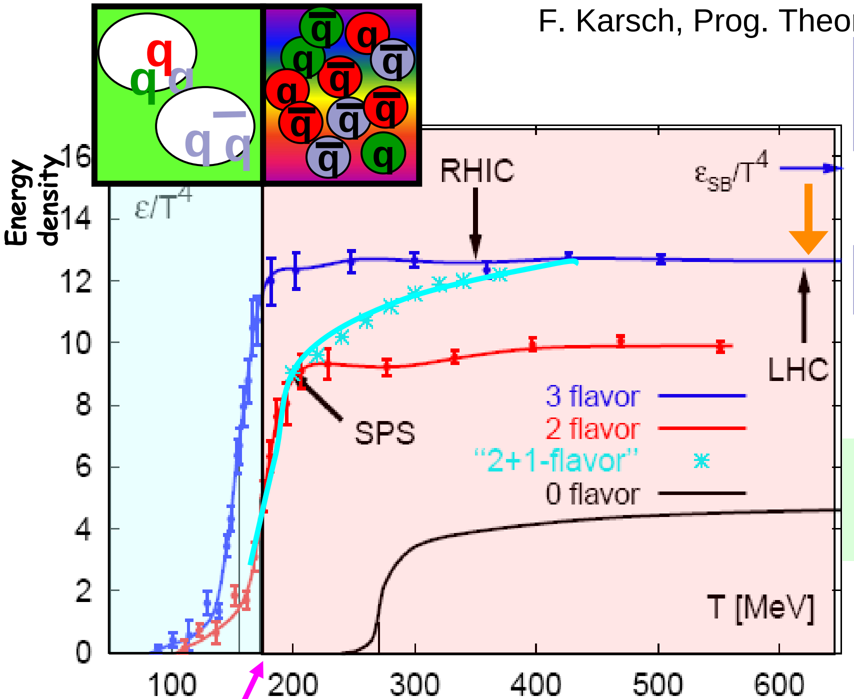
Essay on lattice QCD

F. Karsch, Prog. Theor. Phys. Suppl. 153, 106 (2004)

- Quarks and gluons are studied on a discrete space-time lattice
- Solves the problem of divergences in pQCD calculations (which arise due to loop diagrams)



1. The Polyakov Loop
$\langle L \rangle \sim F_q(free-entropy)$
2. The Chiral Condensate
$\langle \overline{\psi}\psi \rangle \sim m_q \to 0$



Stefan-Boltzman limit for ideal plasma

No ideal plasma

RHIC

Lattice QCD assumes thermal equilibration

Transition point:
T ~ 170 MeV
ε~ 1.0 GeV/fm3

Temperature

Lattice QCD shows a rapid increase in the entropy associated with the deconfinement of quarks and gluons.
→Critical temperature (phase transition) Tc = 170 MeV
Ideal plasma limit not reached
→Strong coupling between partonic degrees of freedom

The ‘bag’ model

- **The quarks are placed in a bag where the perturbative QCD vacuum dominates:**

  a vacuum really ‘empty’,
  i.e. where the quark condensate is zero
  - a vacuum where the quarks do not interact.

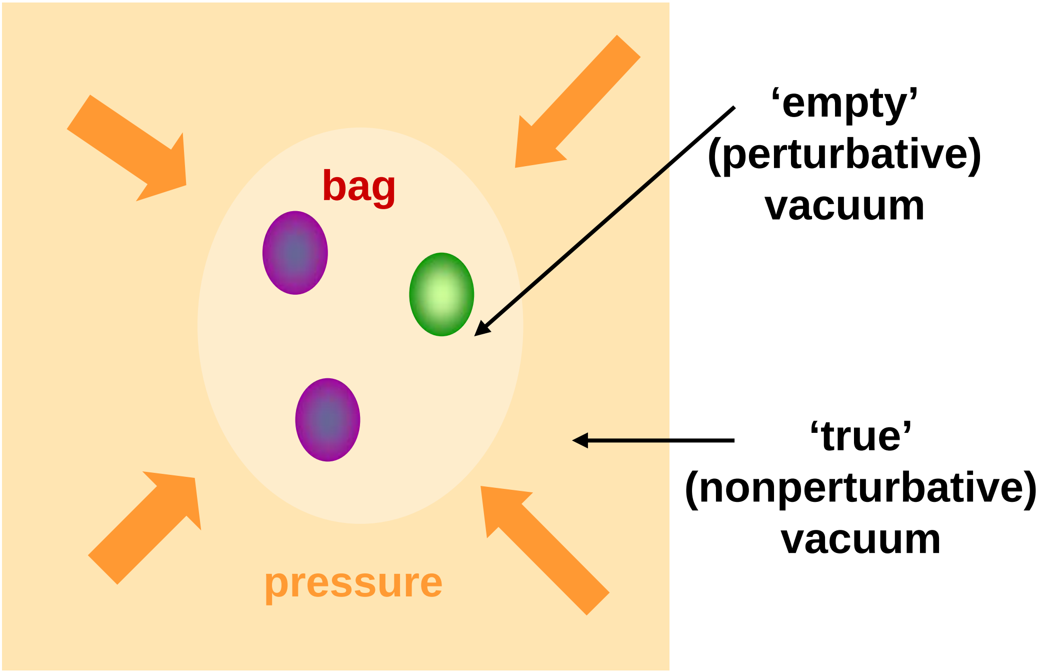
They interact only between themselves, and then have weak masses (only few MeV for u and d flavors). The quarks are maintained in the bag due to the outside pressure which represents the ‘true’ vacuum.

As a consequence, for a nucleon, this is the action of this non perturbative vacuum that confers to the quarks an effective mass of about 300 MeV.

When the system reaches T_C, the internal pressure becomes strong enough to compensate the pressure due to the non perturbative vacuum and become a stable plasma.

$P_{PQG}=P\pi \rightarrow T_C = (90/34\pi^2)^{1/4}\, B^{1/4}$

The T_C values which are obtained via this naïve approach are close to the ones predicted by the lattice QCD calculations.



B: energy density

QCD: Quantum Chromo Dynamics

Statistical Models

A) Chemical equilibration

(Braun-Munzinger, Stachel, Redlich, Tounsi, Rafelski)

B) Thermal equilibration

(Schnedermann, Heinz)

C) Hydrodynamics

(Heinz, Eskola, Ruuskanen, Teaney,Hirano)

Statistical Hadronic Models

- Assume thermally (constant T_{ch}) and chemically (constant n_i) equilibrated system
- Given T_{ch} and μ 's (+ system size), n_i's can be calculated in a grand canonical ensemble

Chemical freeze-out

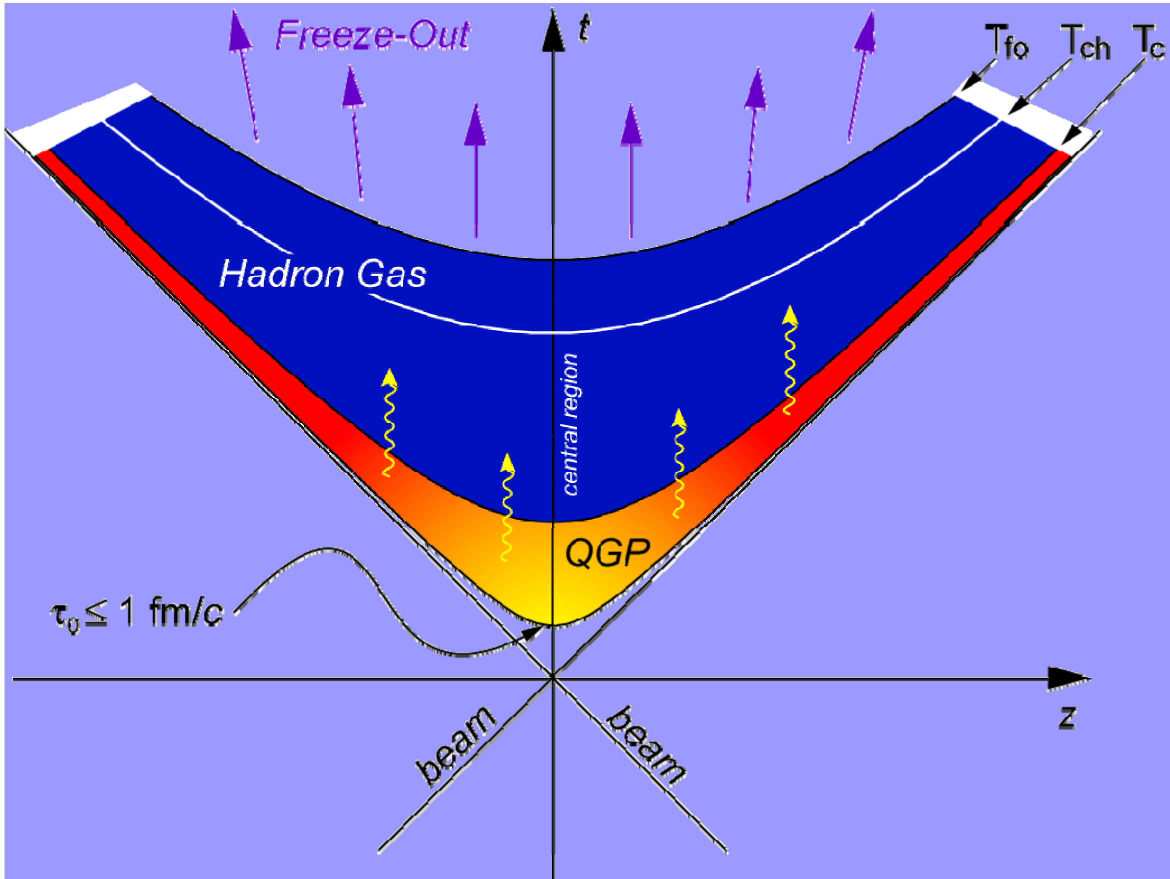
(yields & ratios)

- inelastic interactions stops
- particle abundances fixed (except maybe resonances)

Thermal freeze-out

(*shapes* of p_T, m_T spectra):

- elastic interactions stops
- particle dynamics fixed

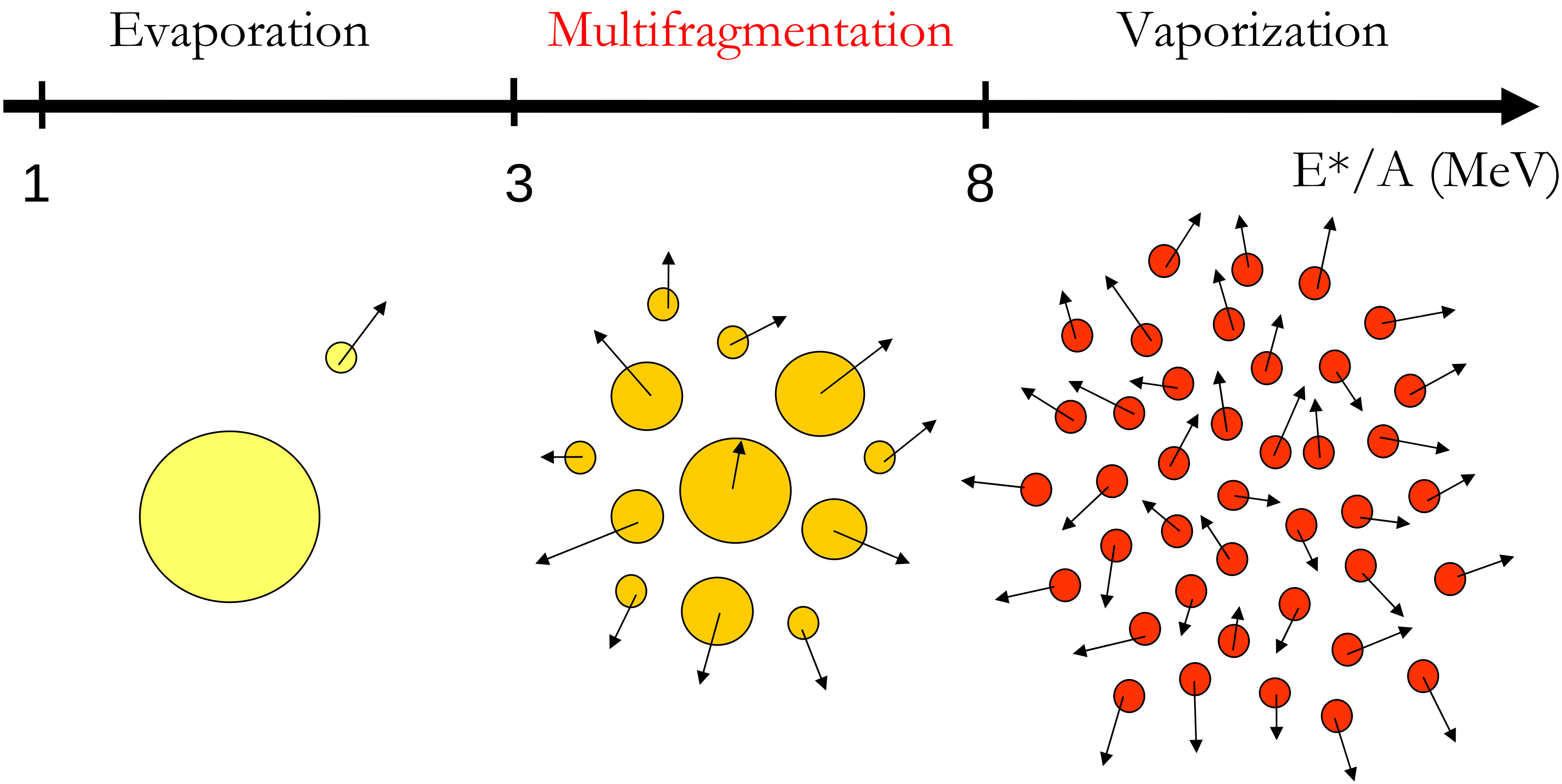


Statistical Hadronic Models

- shortcomings

- Model says <u>nothing</u> about how system reaches chemical equilibrium
- Model says <u>nothing</u> about when system reaches chemical equilibrium
- Model makes no predictions of dynamical quantities
- Some models use a strangeness suppression factor, others not
- Model does not make assumptions about a partonic phase;
- However the model findings can complement other studies of the phase diagram (e.g. Lattice-QCD)

Hot nuclei and de-excitation

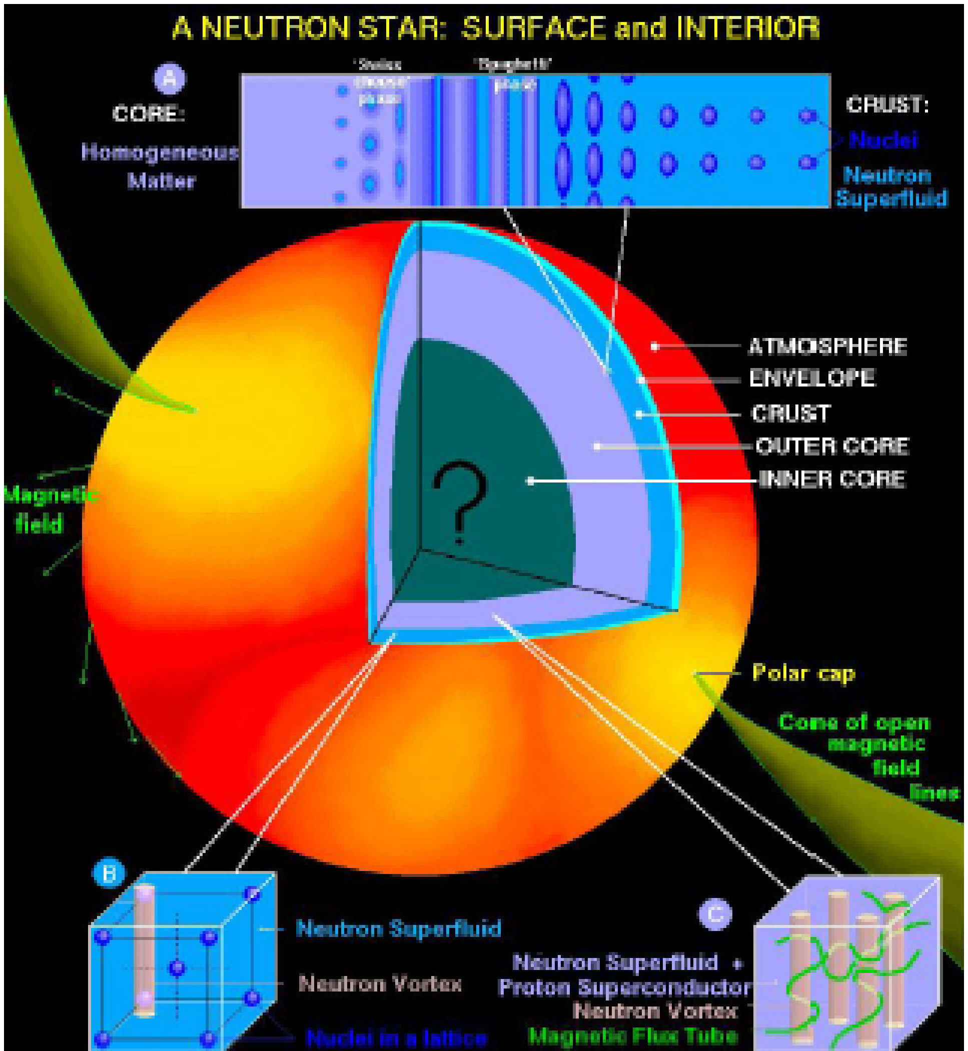


$\rho \sim \rho_0$
T < 5 MeV

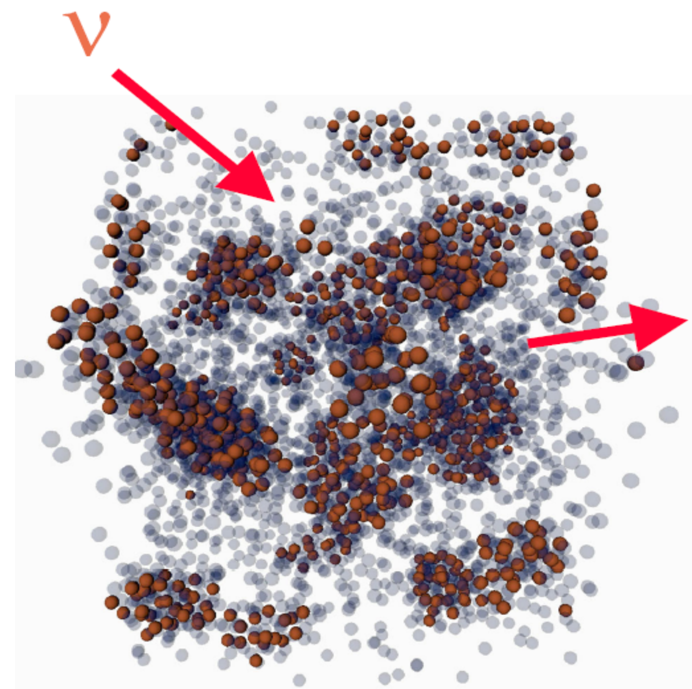
$\rho < \rho_0$
T= 5-15 MeV

$\rho << \rho_0$
T>15 MeV

Phase transition and Neutron stars



(Extended) MF theories with a density functional constraint in a large density domain are a unique tool to understand the structure of neutron stars.



Multifragmentation and Phase transition

Thank You

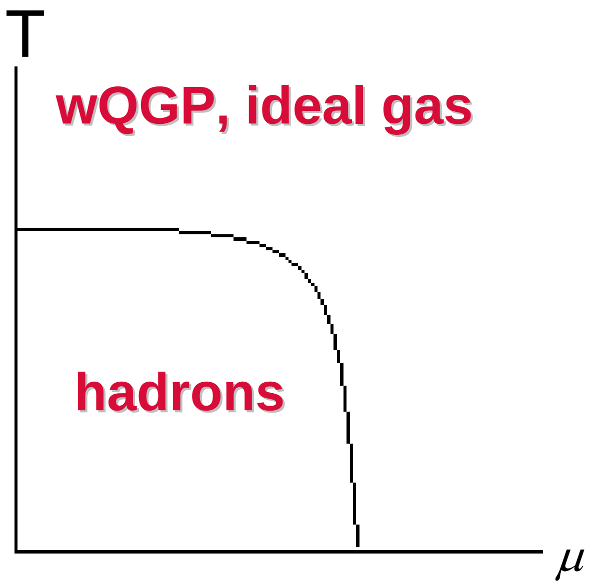
Order parameters

- Nuclear matter and quark matter are separated from other phases by true critical lines
- Different realizations of global symmetries
- Quark matter: SSB of baryon number B
- Nuclear matter: SSB of combination of B and isospin I_3

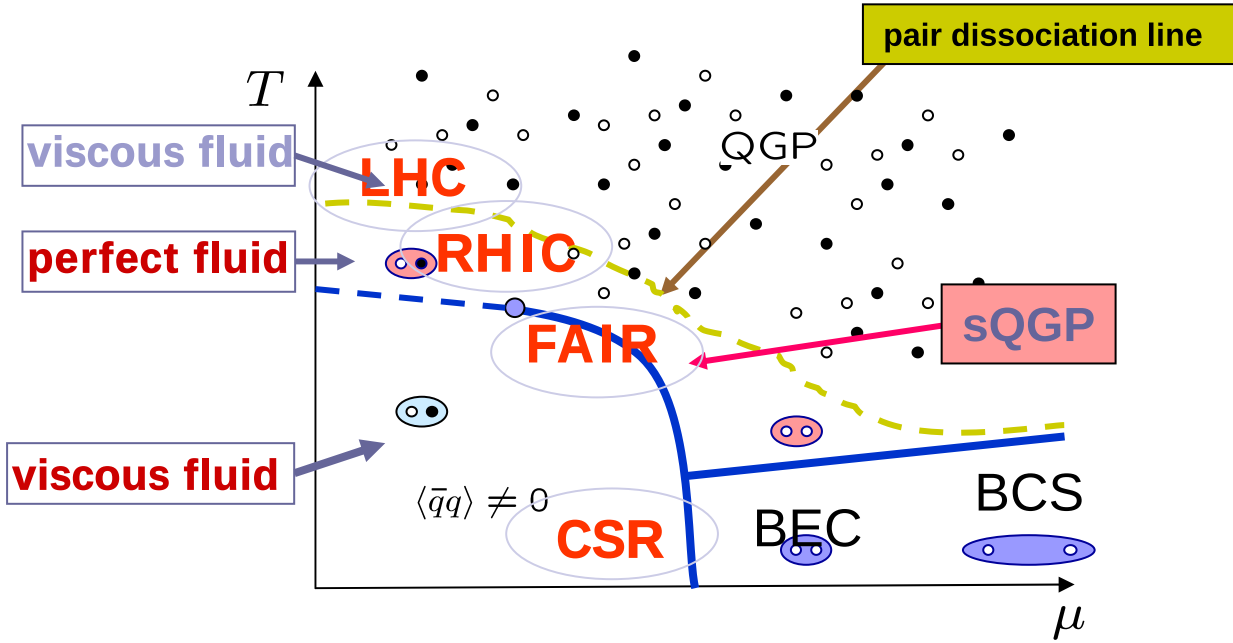
  neutron-neutron condensate

where we are?

old phase diagram



new phase diagram

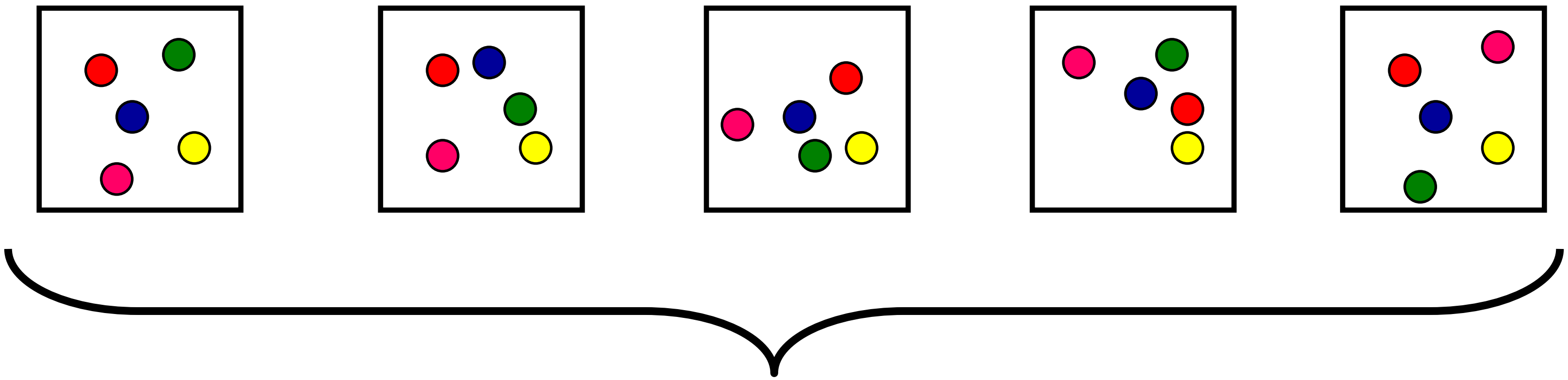


Problems

- a strongly coupled quark-gluon plasma, a quasi-parton state or a state with both partons and their bound states?
- a critical point of QCD?
- viscous fluid
- QCD phases at high density, BCS-BEC crossover (related to cold atom gas,…….)

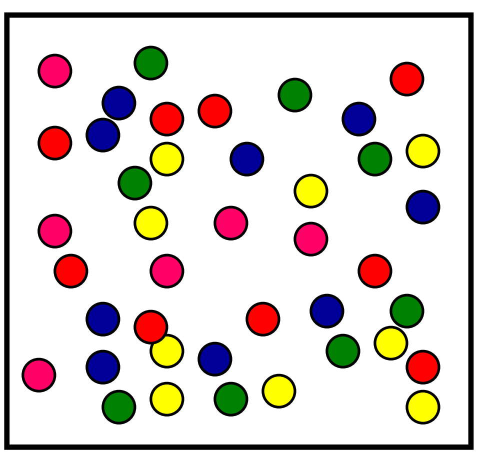
Statistics ≠ Thermodynamics

p+p



Ensemble of events constitutes a statistical ensemble
T and μ are simply Lagrange multipliers
“Phase Space Dominance”

A+A



We can talk about pressure

- T and μ are more than Lagrange multipliers

QCD at high temperature

- Quark – gluon plasma
- Chiral symmetry restored
- Deconfinement
- Lattice simulations : both effects happen at the same temperature

QCD Phase Transition

Quark –gluon plasma

Gluons :   8 x 2 = 16

- Quarks : 9 x 7/2 =12.5
- Dof :   28.5

Chiral symmetry

Hadron gas

Light mesons : 8

- pions :   3
- Dof :   8

Chiral symm. broken

Large difference in number of degrees of freedom !

Strong increase of density and energy density at T_c !