QGP Hydrodynamics

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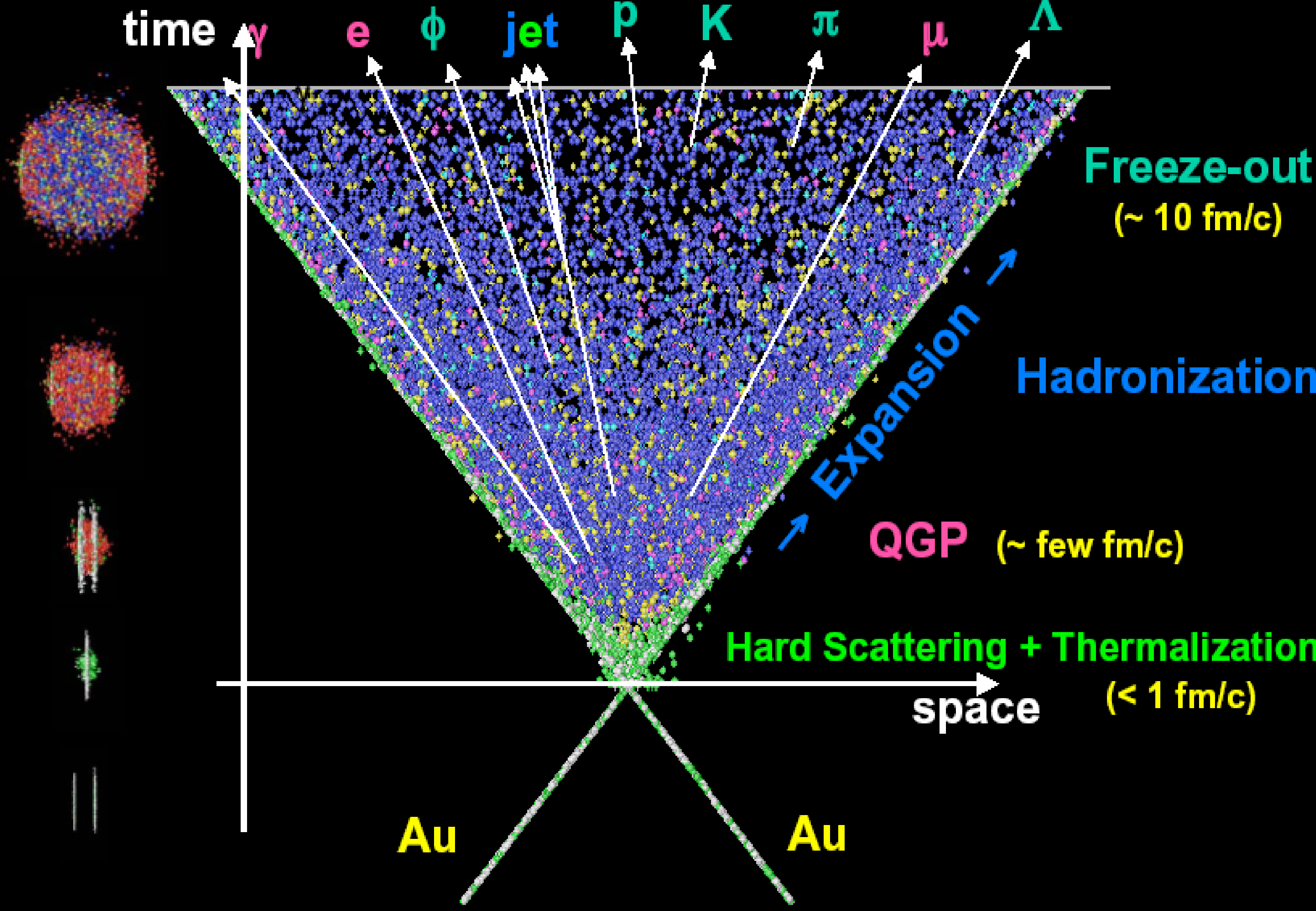
Outline

- QGP Evolution
- Centrality
- Why Hydrodynamics?
- What is a flow?
- Percolation in QGP

Study of QGP

- QGP is mainly defined theoretically by lattice QCD.
- Fascinating phenomena discovered and studied already
- Quantitative estimate of some fundamental quantities.
- Models are used to map the T, S, viscosity, size, time dependence onto observables.
- Hydrodynamics is a good start

Space-time Evolution of RHIC Collisions



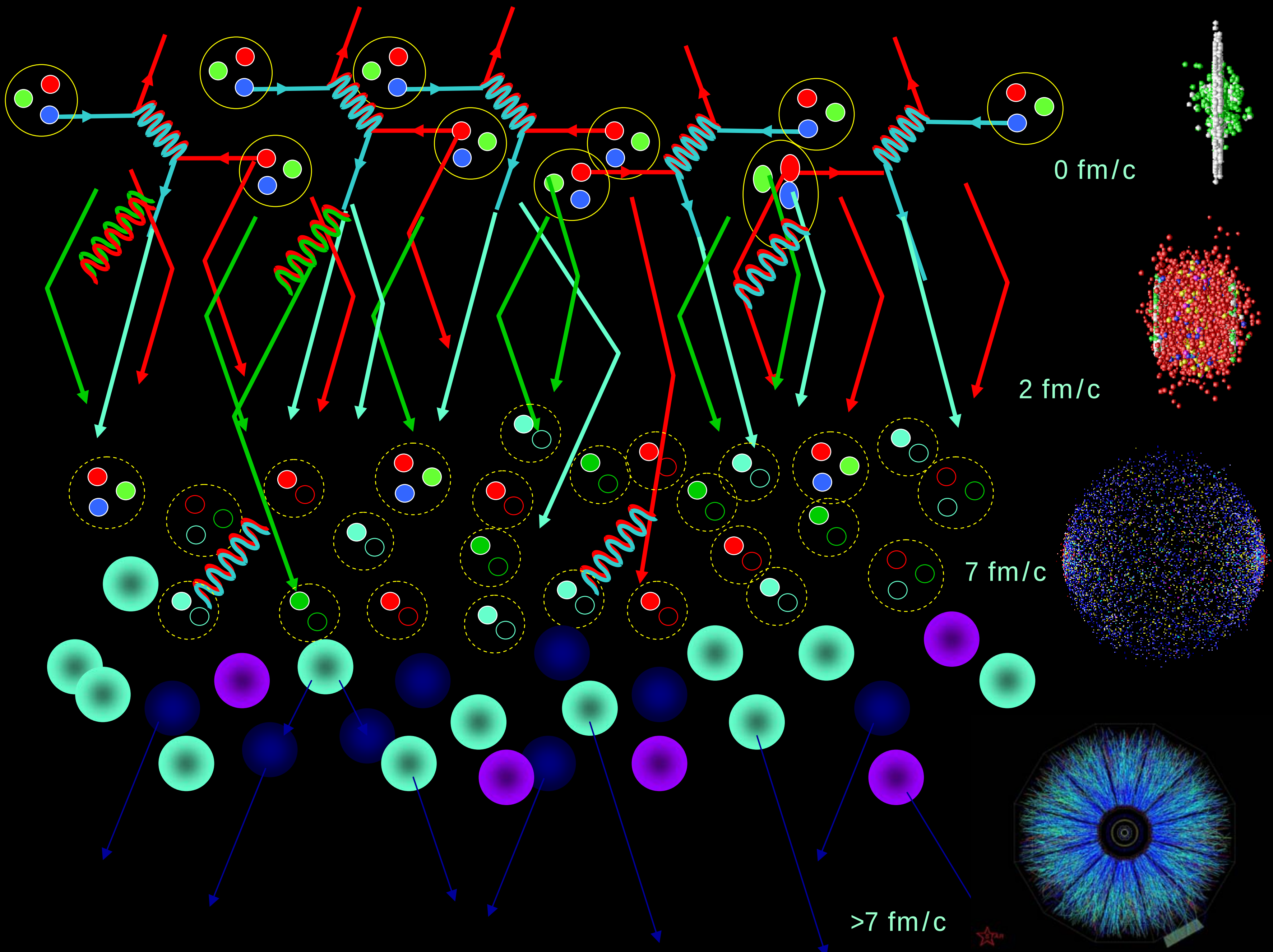
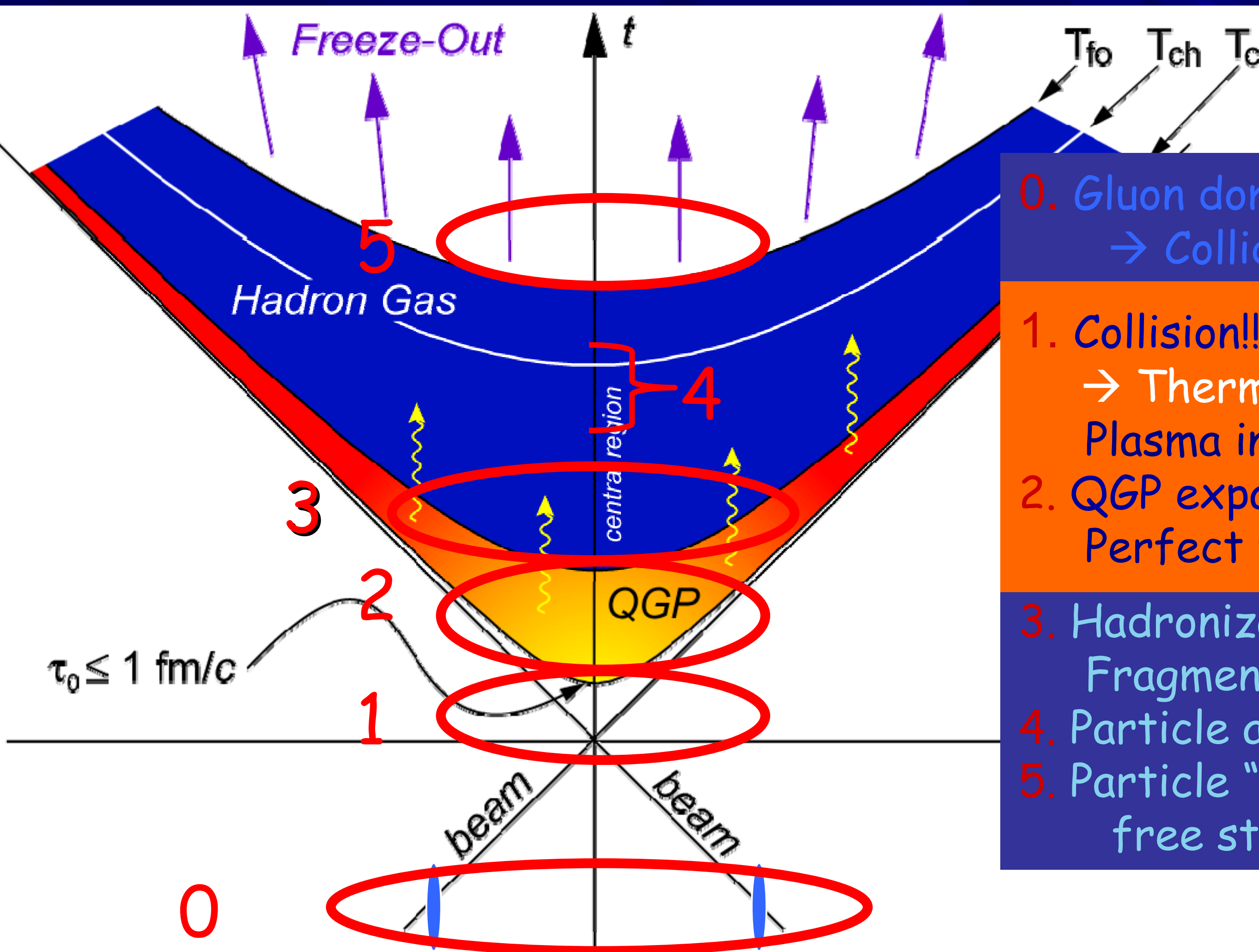
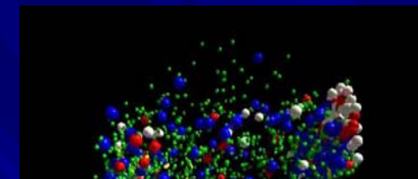
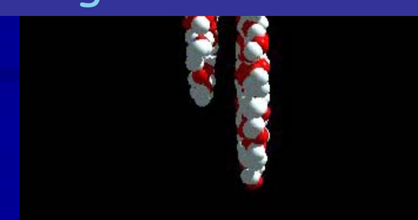


Diagram from Peter Steinberg

Details in Heavy Ion Collision



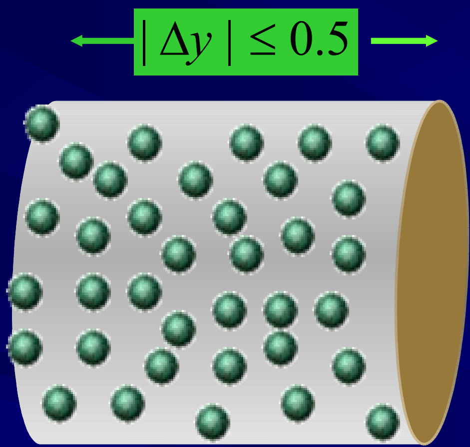




0. Gluon dominant nuclei
   → Colliding Ions
1. Collision!! Hard Collisions
   → Thermalization → QGP
   Plasma instabilities??
2. QGP expands and cools down
   Perfect fluid??
3. Hadronization
   Fragmentation vs recombination
4. Particle abundances fixed
5. Particle "freeze out"
   free streaming

Original figure by T. Chujo's, modified

Energy Density



Particle streaming from origin

$$\frac{z}{t} = v_z = \tanh y$$

$$\rightarrow dz = \tau \cosh y \, dy$$

$R = 1.18 \; A^{1/3} \approx 7 \; fm$

$\tau_{SPS} \leq 1$ fm/c

$\tau_{RHIC} \leq 0.4 - 1$ fm/c

Energy density (Bjorken):

$$\varepsilon = \frac{dE_T}{A_T \, dz} = \frac{1}{\pi R^2 \tau} \frac{dE_T}{dy}$$

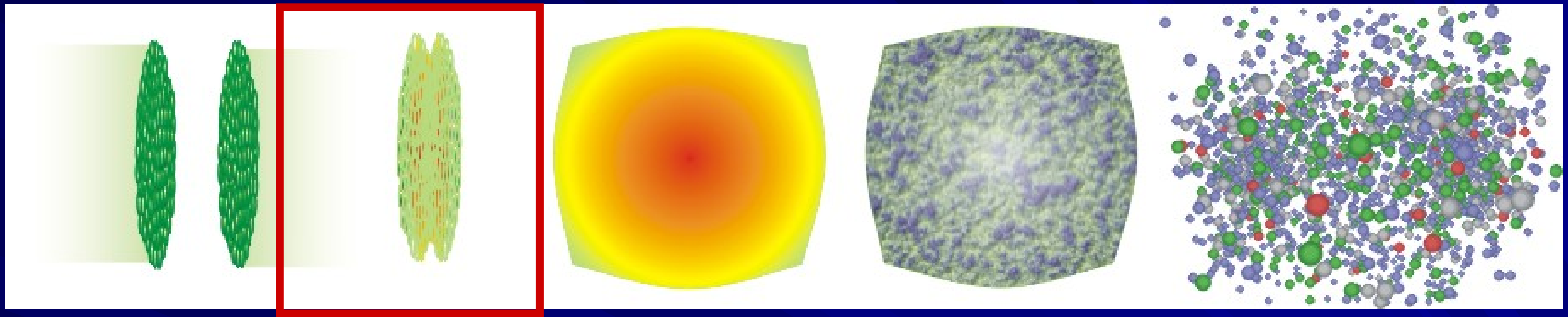
Estimate ε for RHIC:

$dE_T/dy \sim 720$ GeV

Time estimate from hydro:

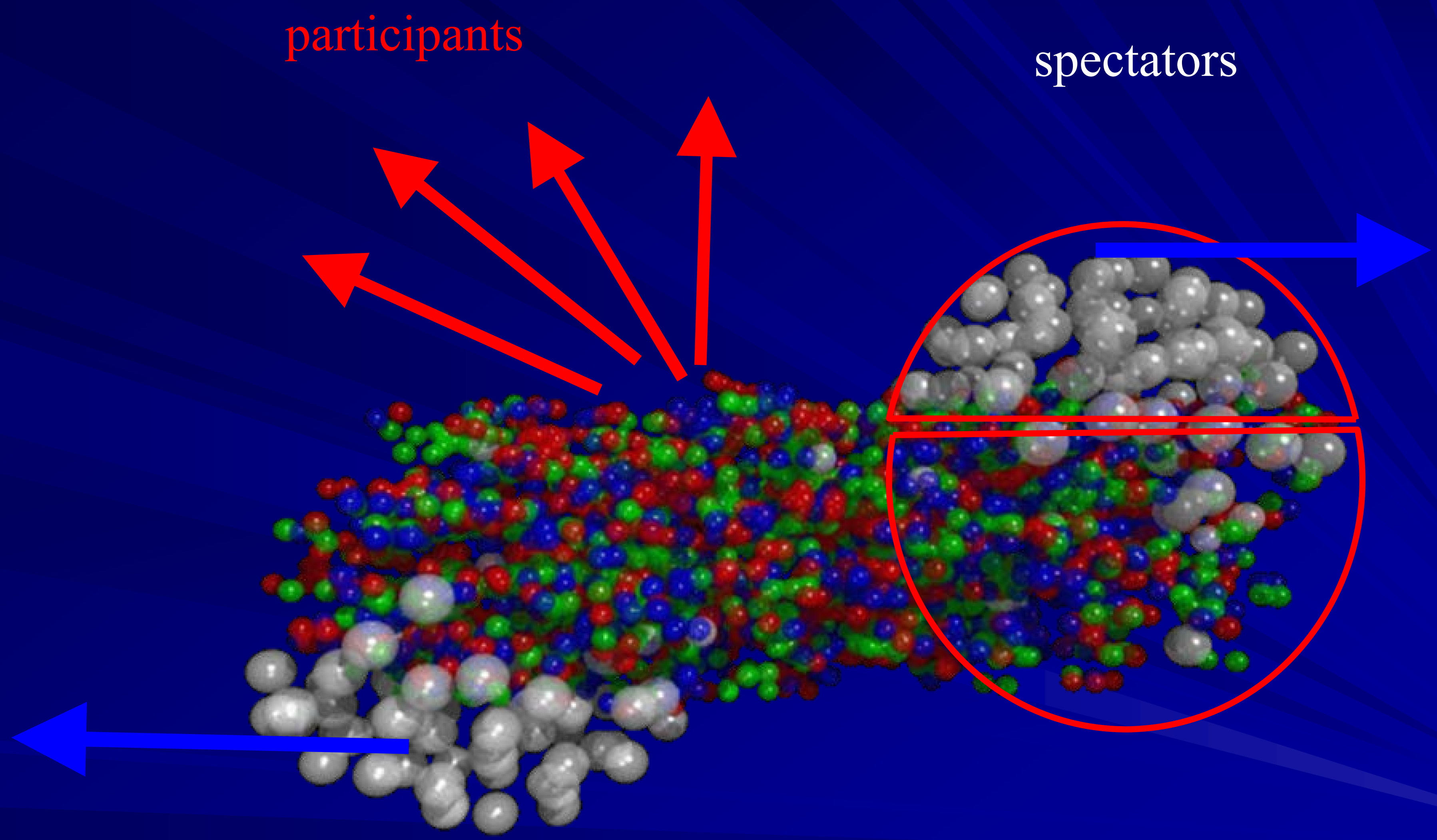
$$\tau = 0.6 \; fm/c \Rightarrow \varepsilon \sim 8 \; GeV/fm^3$$

→ $T_{initial} \sim$ 300-350 MeV



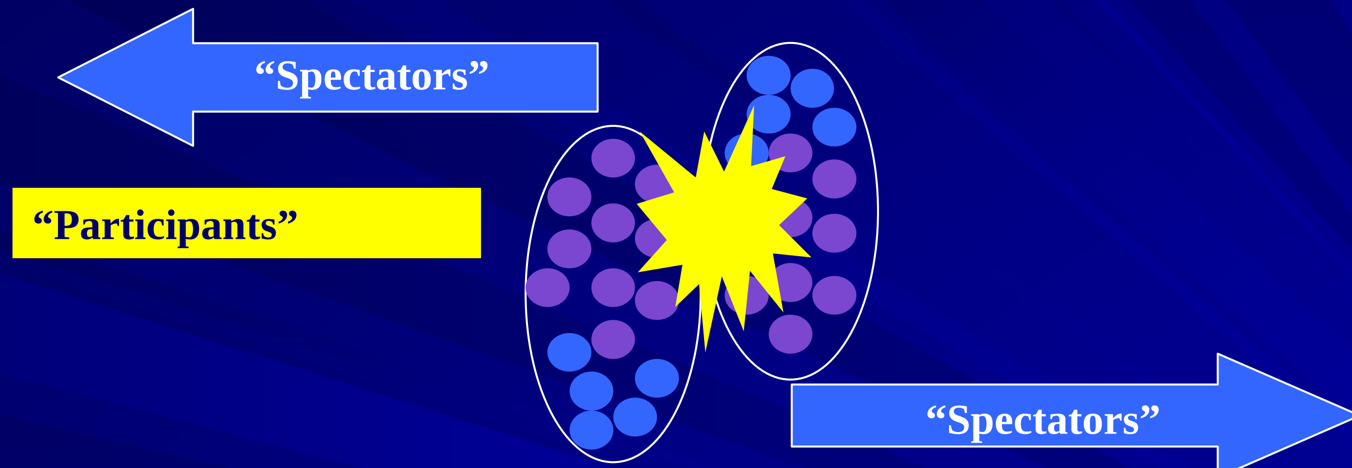
high-p_t and early times: manifestations of pre-equilibrium

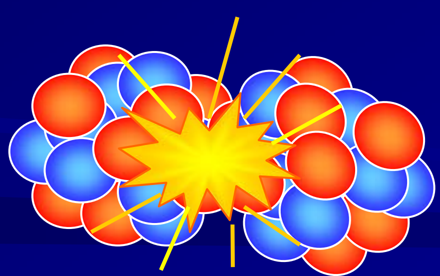
- jet production and quenching
- [photons & leptons]


participants
spectators

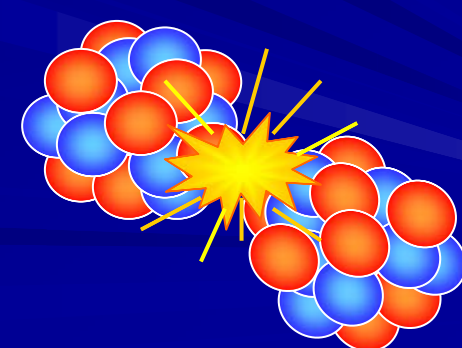
Centrality

The most central collision, the most dense matter





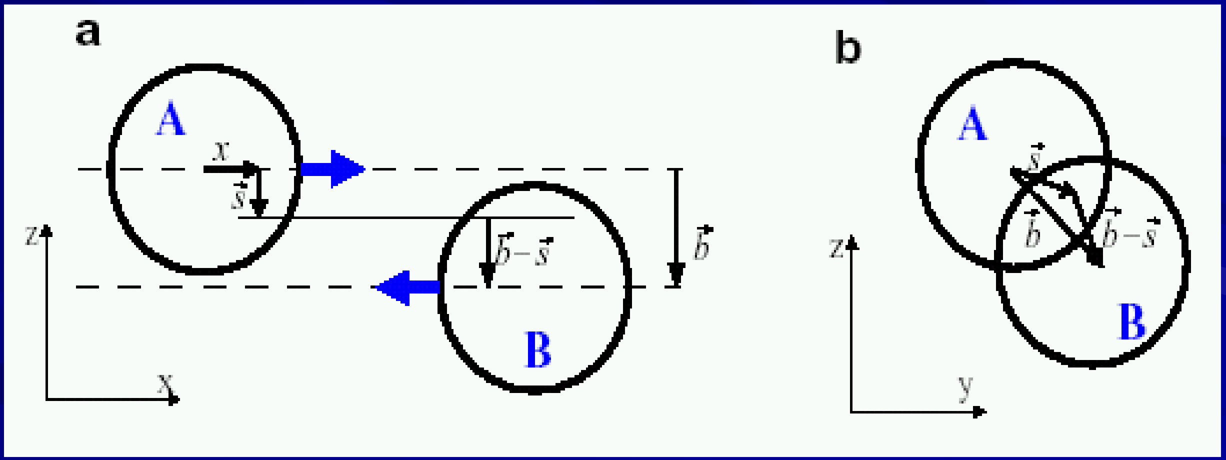
Very central collisions



Very peripheral collisions

Centrality: impact parameter

- In heavy ion collisions the volume and energy of the "fireball" is determined (at given beam energy) mostly by the number of participating nucleons N_{part}, which in turn depends on the impact parameter **b**



x = beam axis

(y,z) = transverse plane

Centrality: number of collisions

The average number of N-N collisions at impact parameter **b** is:

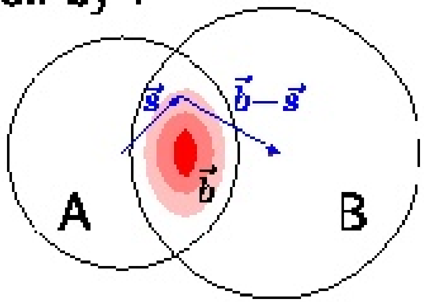
$$<\nu(\mathbf{b})> = \sum_{k=1}^{AB} kP(k,\mathbf{b}) = AB\sigma_0 T_{AB}(\mathbf{b})$$

Under the assumption of an inelastic A-B collision, the average number $N_{coll}(\mathbf{b})$ of N-N collisions is the same as $<\nu(\mathbf{b})>$ except for very large **b**:

$$N_{coll}(\mathbf{b}) = \sum_{k=1}^{AB} kP(k,\mathbf{b}) \,/\, \sum_{k=1}^{AB} P(k,\mathbf{b})$$
$$= AB\sigma_0 T_{AB}(\mathbf{b}) \,/\, \sigma_{AB}(\mathbf{b})$$

The local collision density $n_c(\vec{b},\vec{s})$ is given by :

$$n_c(\vec{b},\vec{s}) = \frac{\sigma_0\, AB}{\sigma_{AB}(b)}\, T_A(\vec{s})\, T_B(\vec{b}-\vec{s})$$

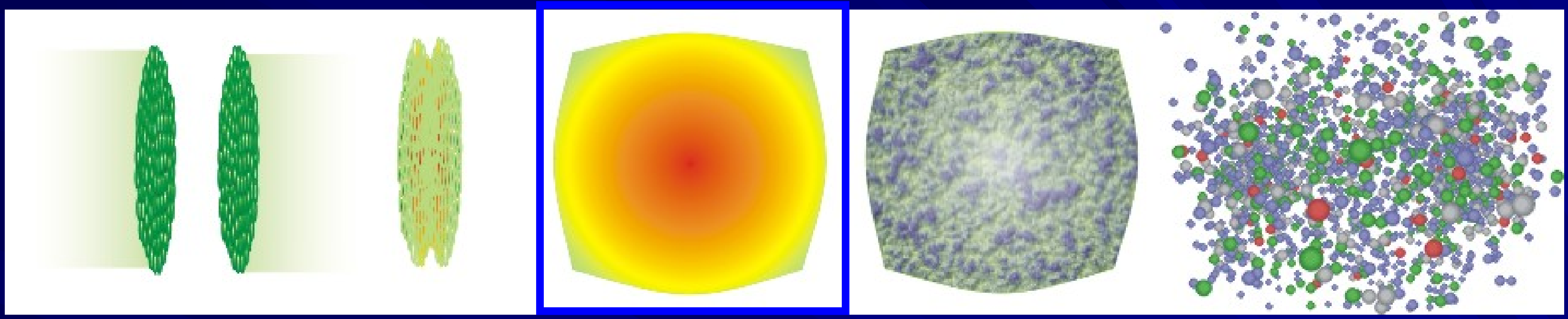


Centrality: number of participants

- The number of participants (wounded) nucleons from both nuclei A and B is on average:

$$N_W(\mathbf{b}) = N_A(\mathbf{b}) + N_B(\mathbf{b})$$
$$= [A/\sigma_{AB}(b)]\int T_A(\mathbf{s})\,\sigma_B(\mathbf{b-s})\,d^2s + [B/\sigma_{AB}(b)]\int T_B(\mathbf{b-s})\,\sigma_A(\mathbf{s})\,d^2s$$
$$\approx A\int T_A(\mathbf{s})\{1-[1-T_B(\mathbf{b-s})\sigma_0]^B\}d^2s$$
$$+ B\int T_B(\mathbf{b-s})\{1-[1-T_A(\mathbf{s})\sigma_0]^A\}d^2s$$

- N_W is the number of nucleons having suffered at least one inelastic collision; there are other ways to count participants, which can lead to different numbers:
  - $N_{part} = A + B - N$ (spectators),
    e.g.: $N_{part}^{pro} = A(1-E_F/E_{beam})$
  - N_{part} from a dynamical simulation may (or may not) include rescattering with produced particles



low-p_t and intermediate times: creation and evolution of the QGP

- Hydrodynamics and anisotropic flow
- Thermalization

Why Hydrodynamics?

Static
- EoS from Lattice QCD
- Finite T, μ field theory
- Critical phenomena
- Chiral property of hadron

Energy-momentum: $\partial_\mu T^{\mu\nu} = 0,$

Conserved number: $\partial_\mu n_i^\mu = 0$

Dynamic Phenomena in HIC
- Expansion, Flow
- Space-time evolution of thermodynamic variables

Hydrodynamic quark model

- Hydrodynamics provides a direct link between the equation of state (EOS) of the expanding fluid and the flow pattern manifested in the emitted hadron spectra.

- A quantitative determination of the EOS requires both precision flow data and systematic theoretical studies of

  the influence of the initial conditions

  - equation of state, non-ideal transport effects and the final decoupling kinetics on the observed hadron spectra.

- Theoretically limited by the difficulty of computations in *viscous relativistic hydrodynamics*.

- Determination of the equation of state also a big issue.

Note that the hydrodynamics model also breaks down for more peripheral collisions, lower energy collisions etc.

Landau Hydrodynamics

Landau, Izv. Akad. Nauk SSSR 17, 51 (1953)
Nuovo Ciment, Suppl. 3, 11115 (1956)

pp collision
Initial condition – initial entropy of the system
adiabatic hydrodynamic motion
constant total entropy - constant number of particles
longitudinal expansion followed by transverse expansion

has successfully explained

1. total number of produced charged particles
2. rapidity distribution dN / dy

Hydrodynamic equations

Energy momentum tensor

$$\partial_\mu T^{\mu\nu} = J^\nu$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

Longitudinal expansion

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{01}}{\partial z} = 0$$

$$\frac{\partial T^{01}}{\partial t} + \frac{\partial T^{11}}{\partial z} = 0$$

Transverse expansion

$$\frac{\partial T^{02}}{\partial t} + \frac{\partial T^{22}}{\partial x} = 0$$

<u>constraints</u>:

$$\begin{cases} \partial_\mu (n u^\mu) = 0 & \leftrightarrow \text{ baryon number conservation} \\ \partial_\mu (s u^\mu) = 0 & \leftrightarrow \text{ entropy conservation} \\ u_\mu u^\mu = 1 & \leftrightarrow \text{ flow velocity normalization} \end{cases}$$

Relativistic (Ideal) Hydrodynamics

conservation of energy and momentum $\partial_\mu T^{\mu\nu} = 0$ **4**

and conserved currents (baryon-number) $\partial_\mu j^\mu = 0$ **1**

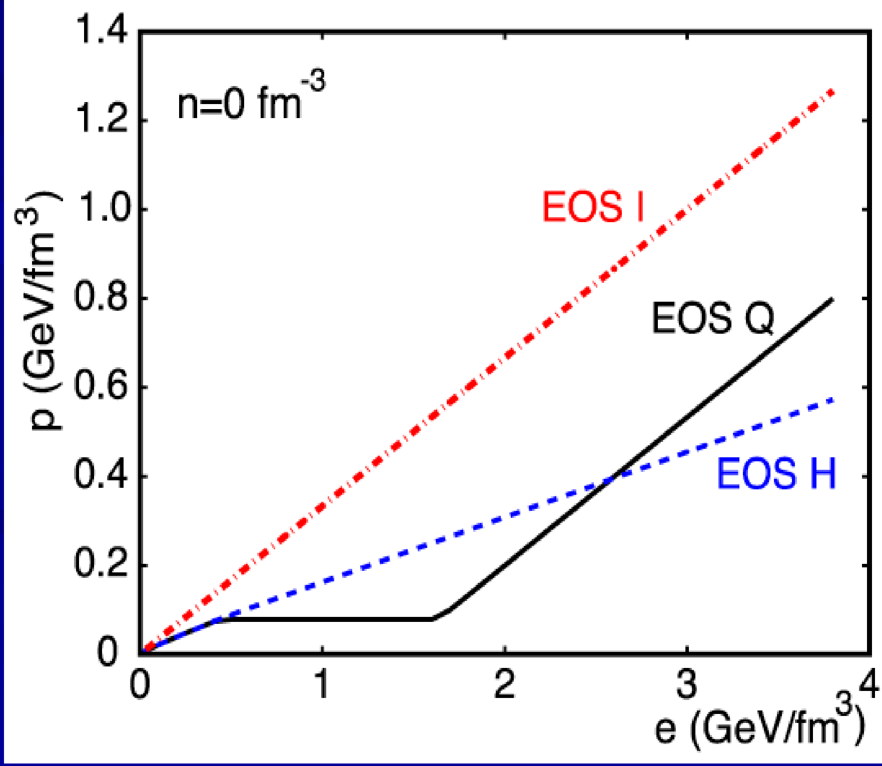
With baryon current $j^\mu(x) = n(x) u^\mu(x)$

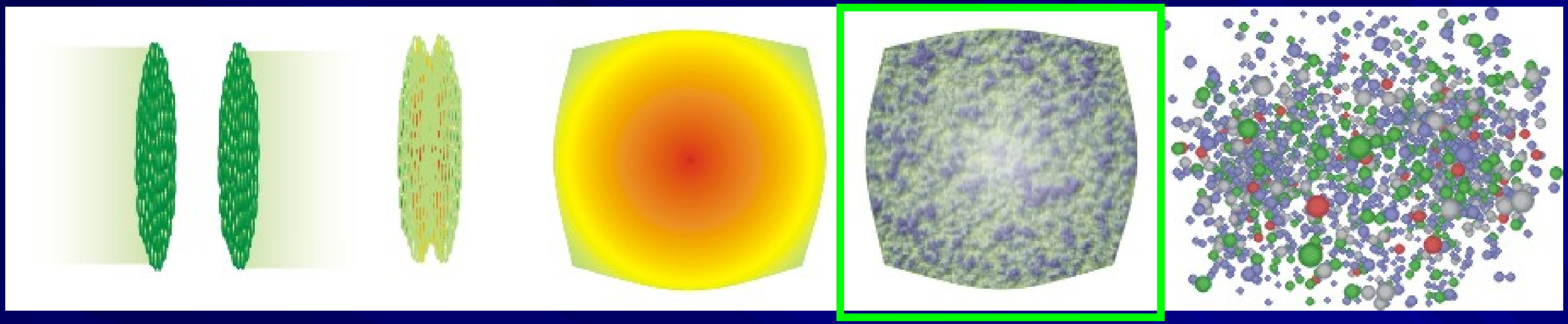
u^x, u^y, u^z

5 equations for 6 fields

close the system by supplying an equation of state, e.g.

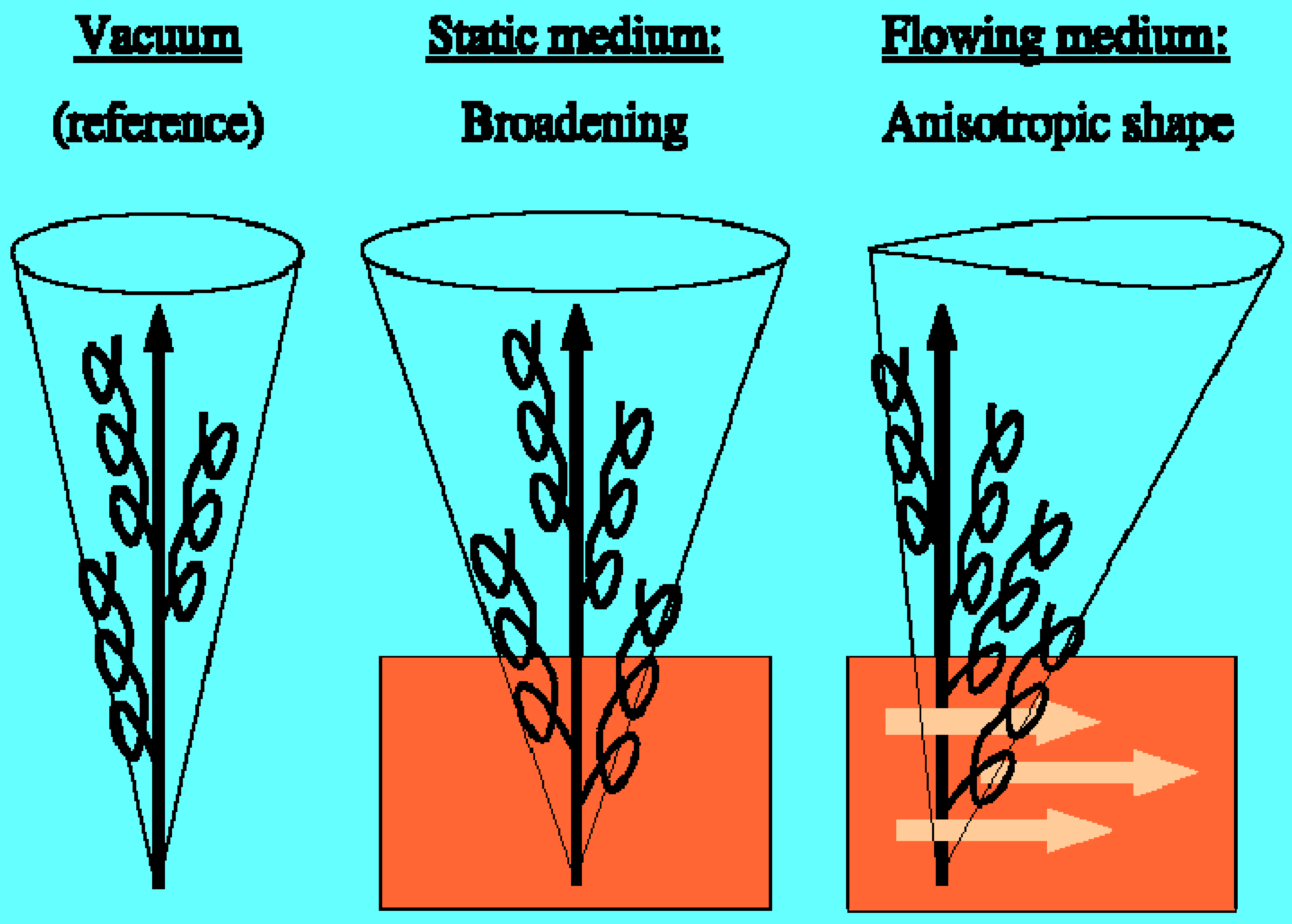
- **EOS I** : ultrarelativistic, ideal gas, $P = \varepsilon/3$
- **EOS H**: interacting resonance gas, $P \sim 0.15\,\varepsilon$
- **EOS Q**: Maxwell construction of those two:
  critical temperature T_{crit}= 0.165 MeV
  bag constant $B^{1/4}$ = 0.23 GeV
  latent heat ε_{lat}=1.15 GeV/fm^3





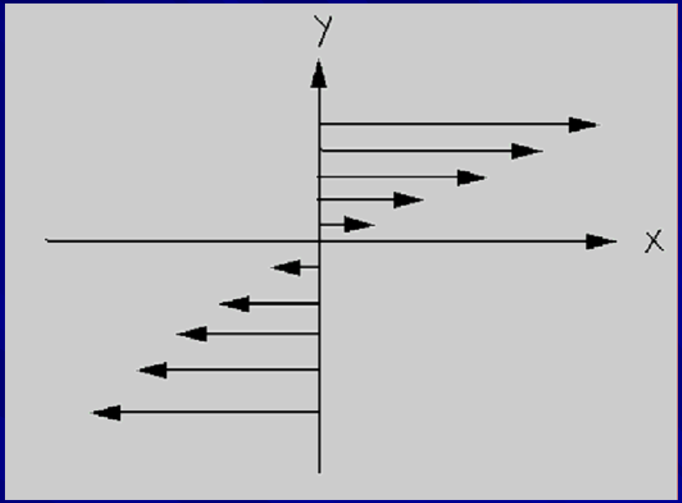
Intermediate-p_t and late(r) times: dynamics of hadronization

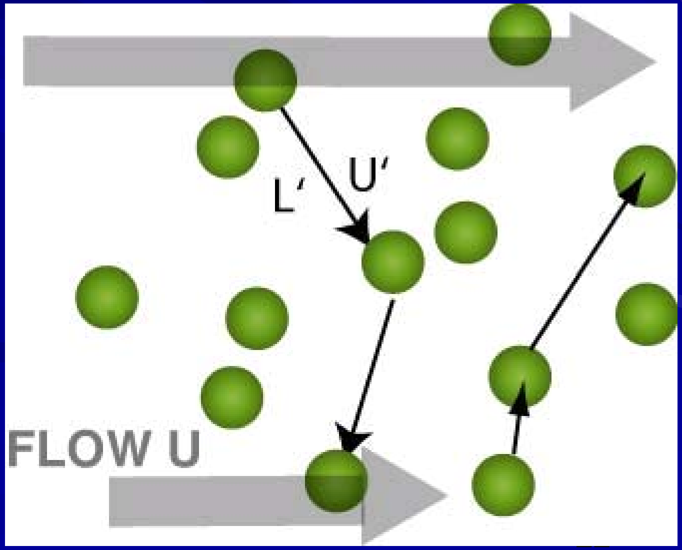
- Recombination & Fragmentation
  - Recombination + Fragmentation Model
  - Results: spectra, ratios and elliptic flow
  - Challenges: correlations, entropy balance & gluons



Armesto et al, nucl-ex/0405301

Viscosity





Think of a not-quite-ideal fluid:

"not-quite-ideal"≡"supports a shear stress"

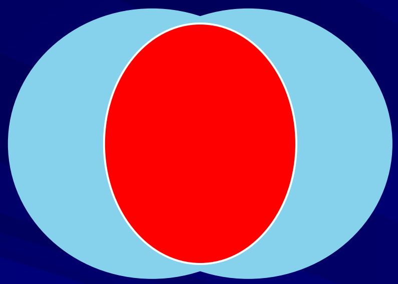
Viscosity η is defined as

$$\frac{F_x}{A} = -\eta \frac{\partial v_x}{\partial y}$$

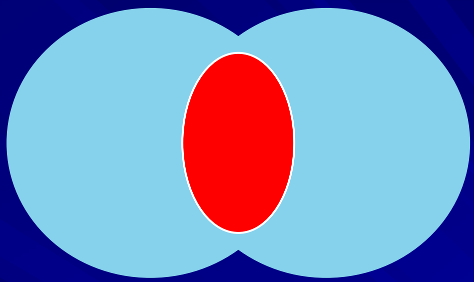
$$\eta \approx (\textit{momentum density}) \times (\textit{mean free path})$$

$$\approx n\,\bar{p}\,mfp = n\,\bar{p}\,\frac{1}{n\sigma} = \frac{\bar{p}}{\sigma}$$

- The event geometry in complicated events
  - Degree of overlap

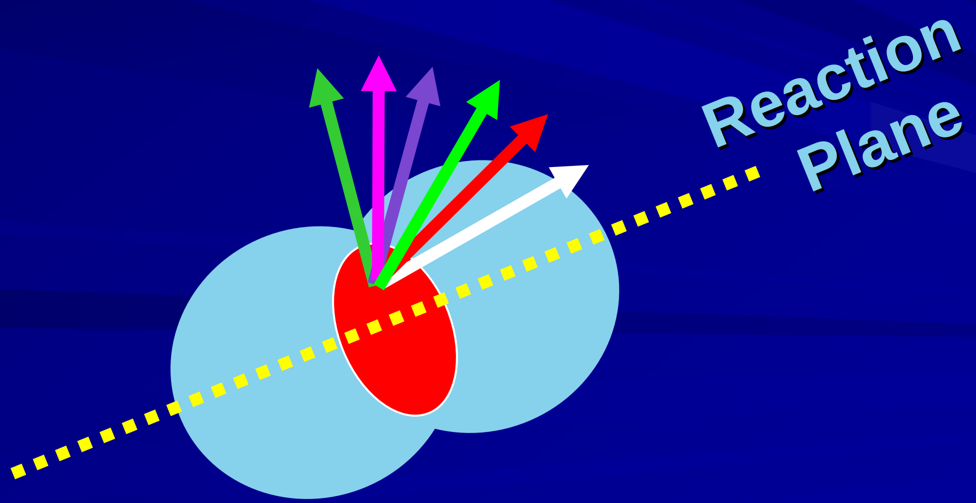


“Central”

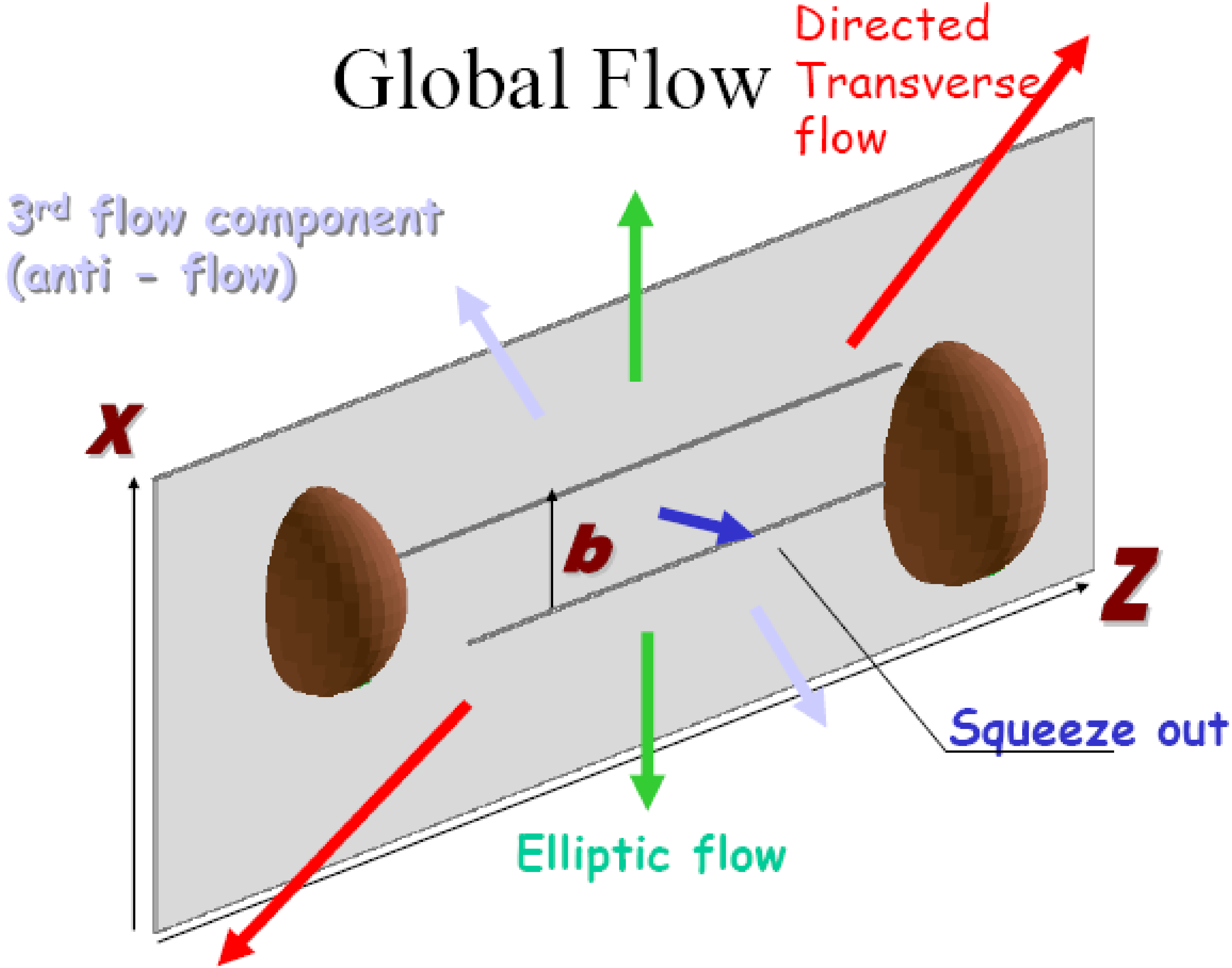


“Peripheral”

  - Orientation with respect to overlap



Global Flow

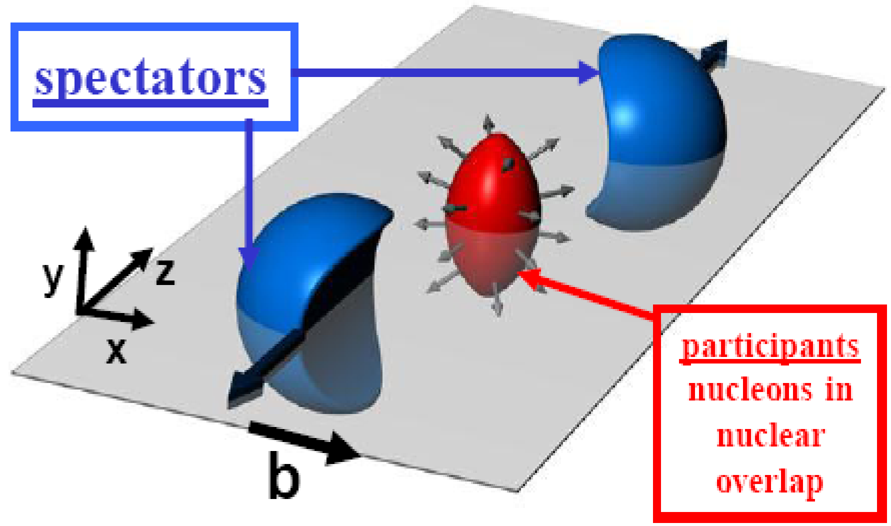

Directed
Transverse
flow
3rd flow component
(anti - flow)
x
b
Z
Squeeze out
Elliptic flow

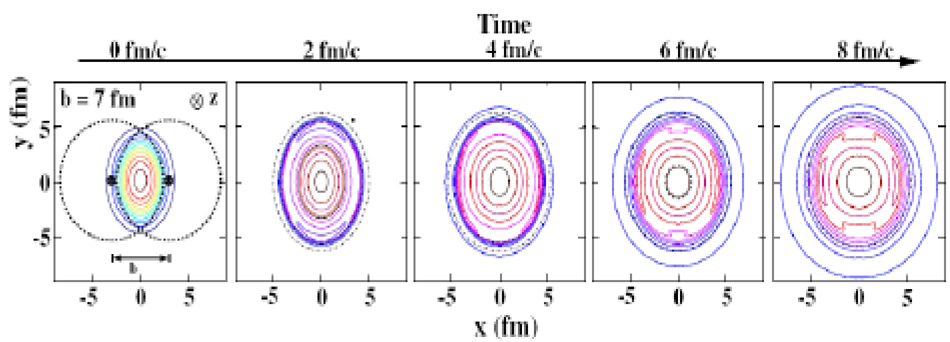
Types of flow in nuclear collisions

- radial flow
  - driven by **pressure gradient**
    - increases for central collisions
  - acts over **long time**
    - until freeze-out

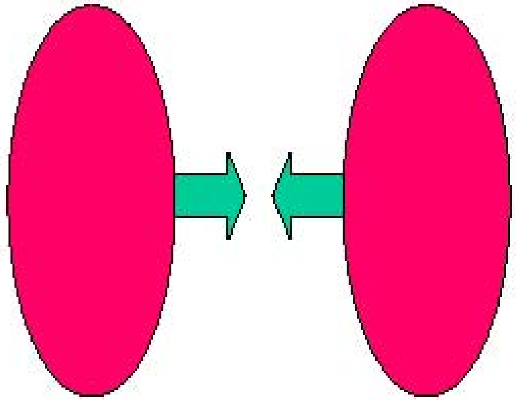
not so interesting

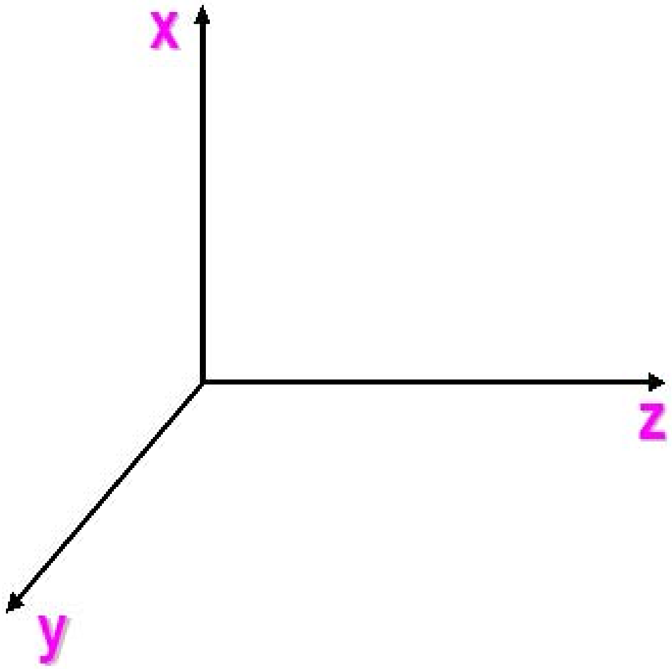
- elliptic flow **very interesting !**
  - **spatial anisotropy** => pressure anisotropy
    - azimuthal dependence of flow
    - strong for peripheral, zero for central
  - acts at **early times**
    - until anisotropy disappears





Definitions. Central Collisions (b=0)



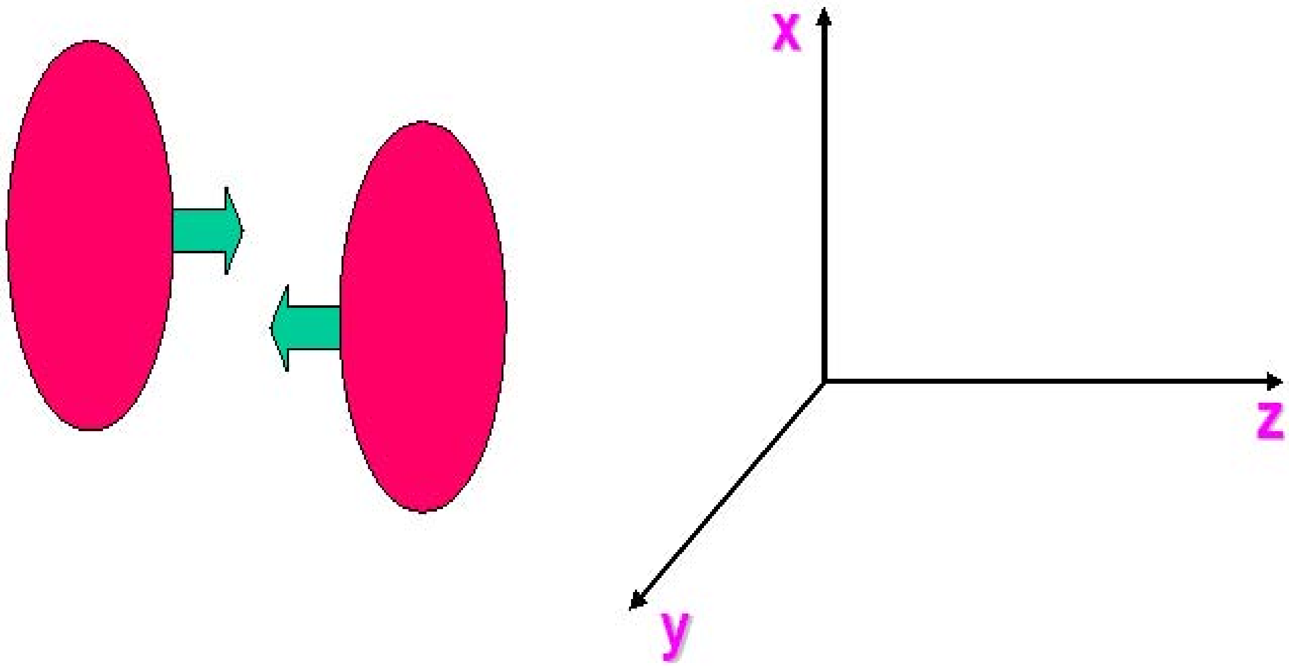


Collective Flow =

Longitudinal Flow + Transverse Radial Flow

(isotropic)

Definitions. Non-central Collisions (b>0)



Collective Flow = Longitudinal Flow

+ Transverse Radial Flow (isotropic)

+ Transverse Anisotropic Flow

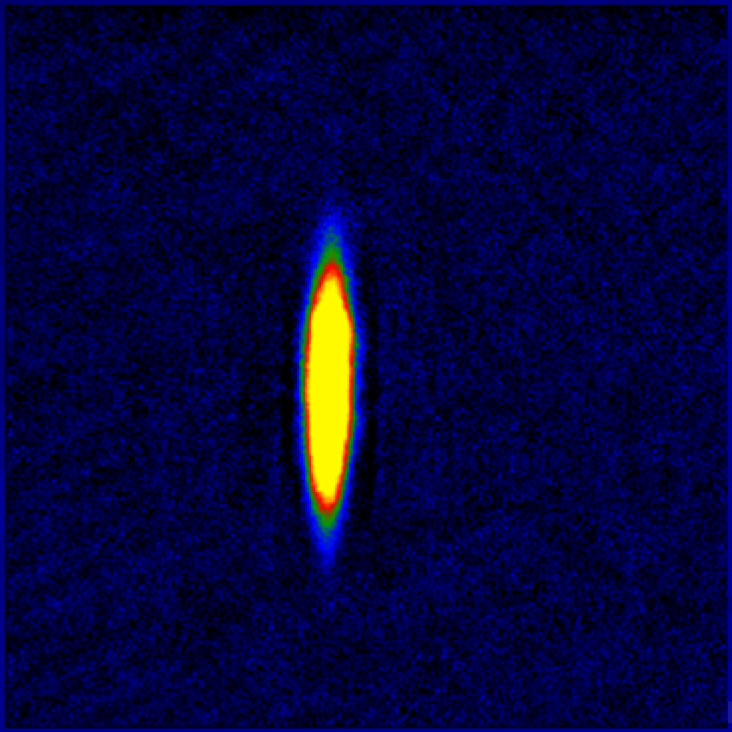
Elliptic flow

- Due to rapid expansion along the beam axis, an anisotropy in momentum space develops
- The elliptic flow is a measure of the anisotropy for the number of particles produced with respect to φ.
- It arises from the elliptical shape of the overlapping region in colliding nuclei and is usually parameterized with dependencies on parameters $v_2(p_T)$ and φ.
- The angular dependence is well known so elliptical flow is often used to mean the elliptic flow coefficient $v_2(p_T)$,

  - a measure of "the small differences between the p_t spectra with momenta pointing into and perpendicular to the reaction plane".

Isotropic expansion

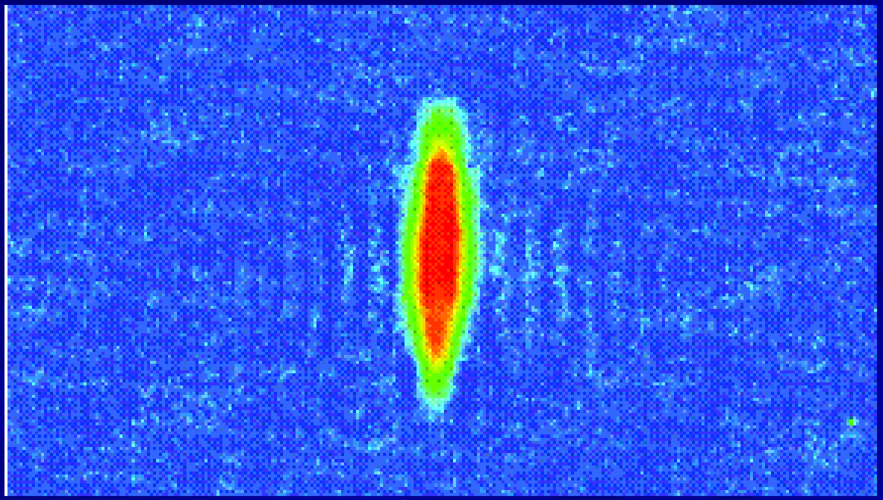
- nano-Kelvin gas of ^{6}Li atoms
- magnetic trap
- small scattering length leads to viscous hydrodynamics
- isotropic expansion when trapping field dropped



Ken O'Hara (Penn. St.)

Anisotropic expansion

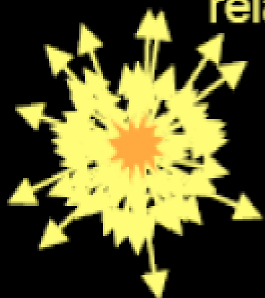
- resonance tuned for large scattering length
- nearly ideal hydrodynamics
- anisotropic expansion when trapping field dropped



Azimuthal Angular Distributions

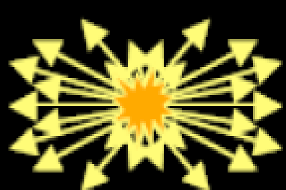
1) Superposition of independent p+p:

momenta random
relative to reaction plane

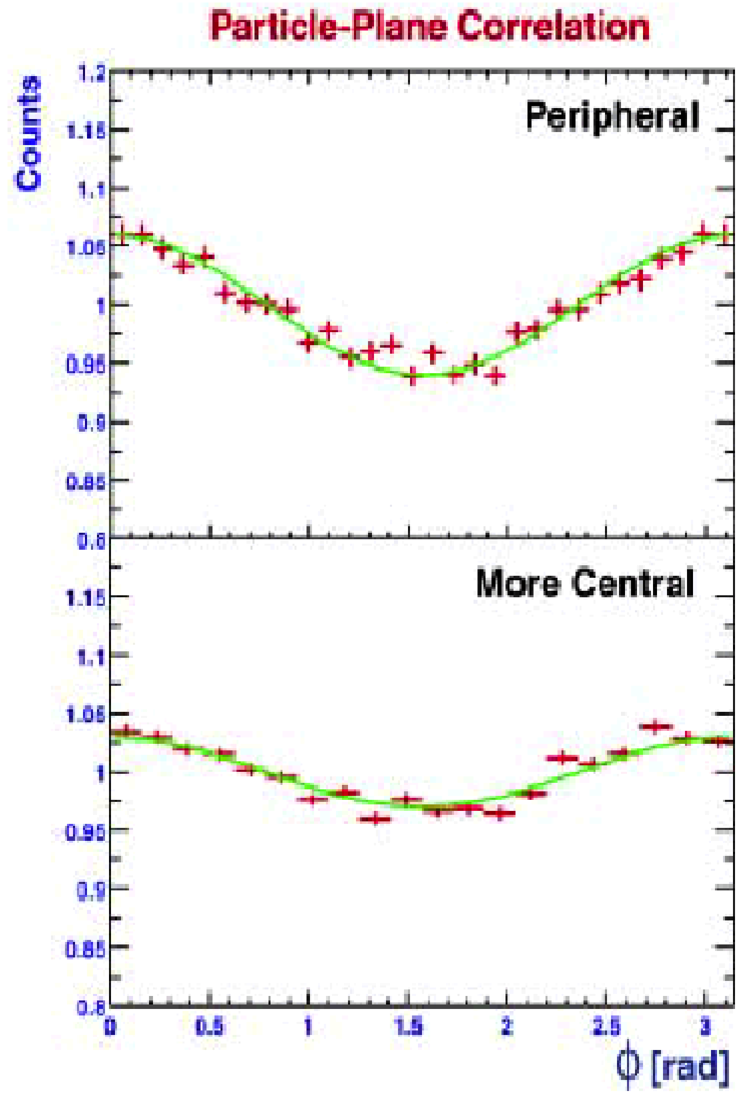


2) Evolution as a **bulk system**

Pressure gradients (larger in-plane)
push bulk “out” → flow



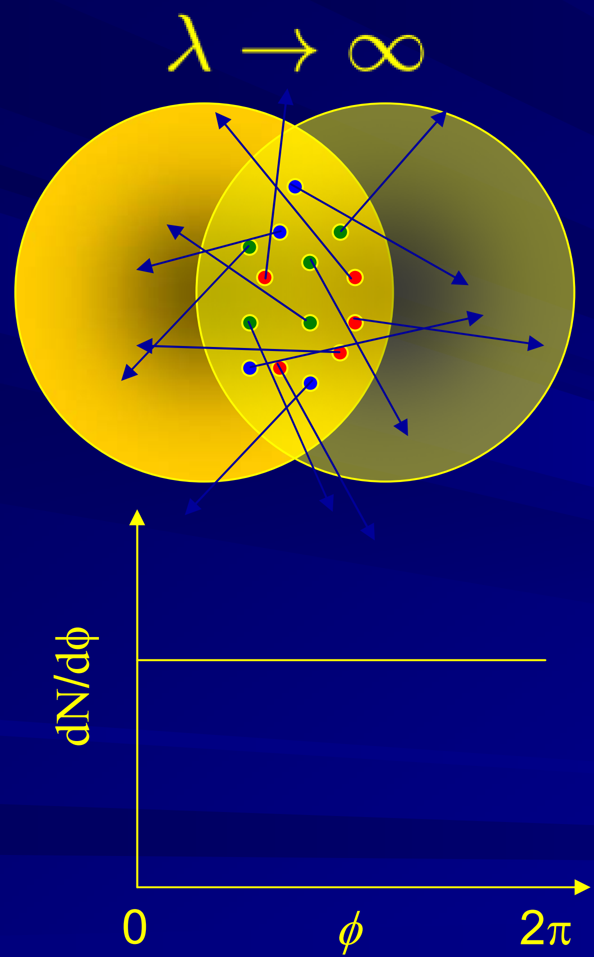
more, faster particles
seen in-plane

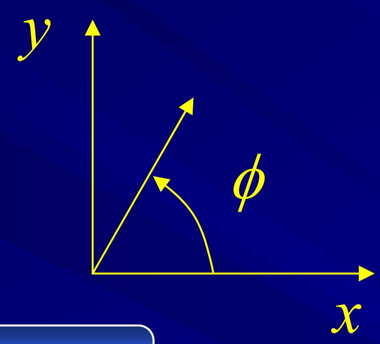


Ollitrault ('92)

How does a system respond to spatial anisotropy?

No secondary interaction





INPUT

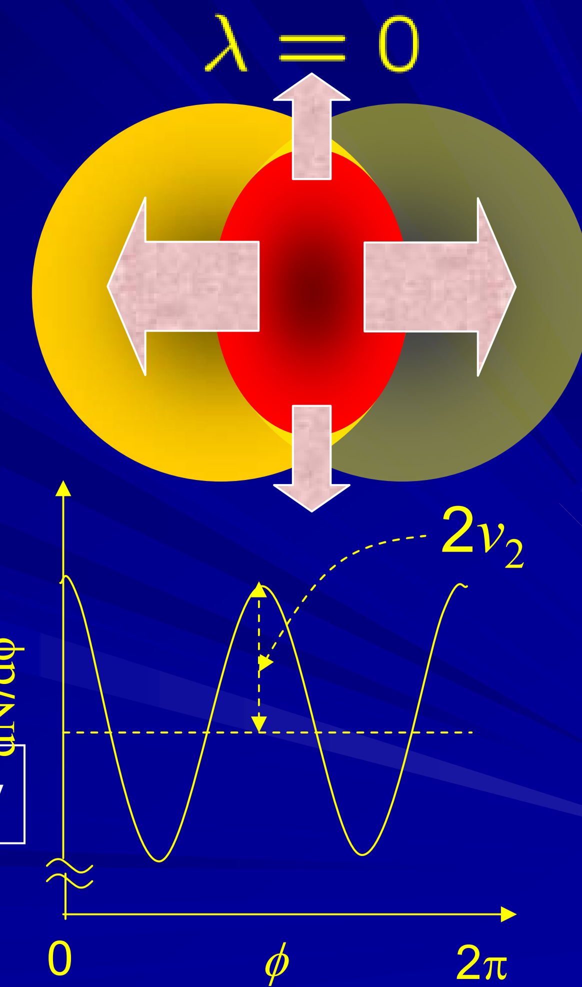
Spatial Anisotropy

Interaction among produced particles

OUTPUT

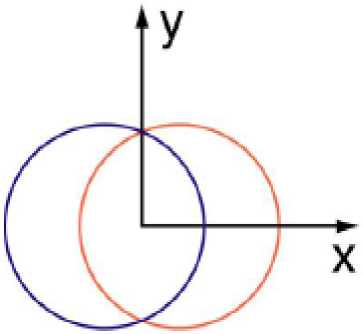
Momentum Anisotropy

Hydro behavior

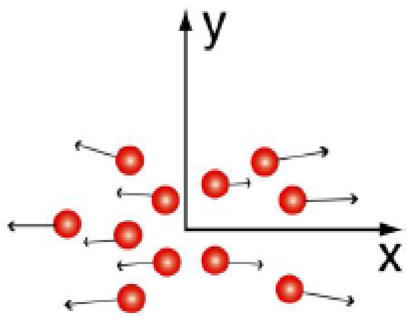


Particle Production in the Transverse Plane

Coordinate space:
initial asymmetry



Momentum space:
final asymmetry



$$\frac{dN}{dp_T d\varphi} = \frac{dN}{dp_T}\left[1 + 2\sum_n v_n \cos(n\varphi)\right]$$

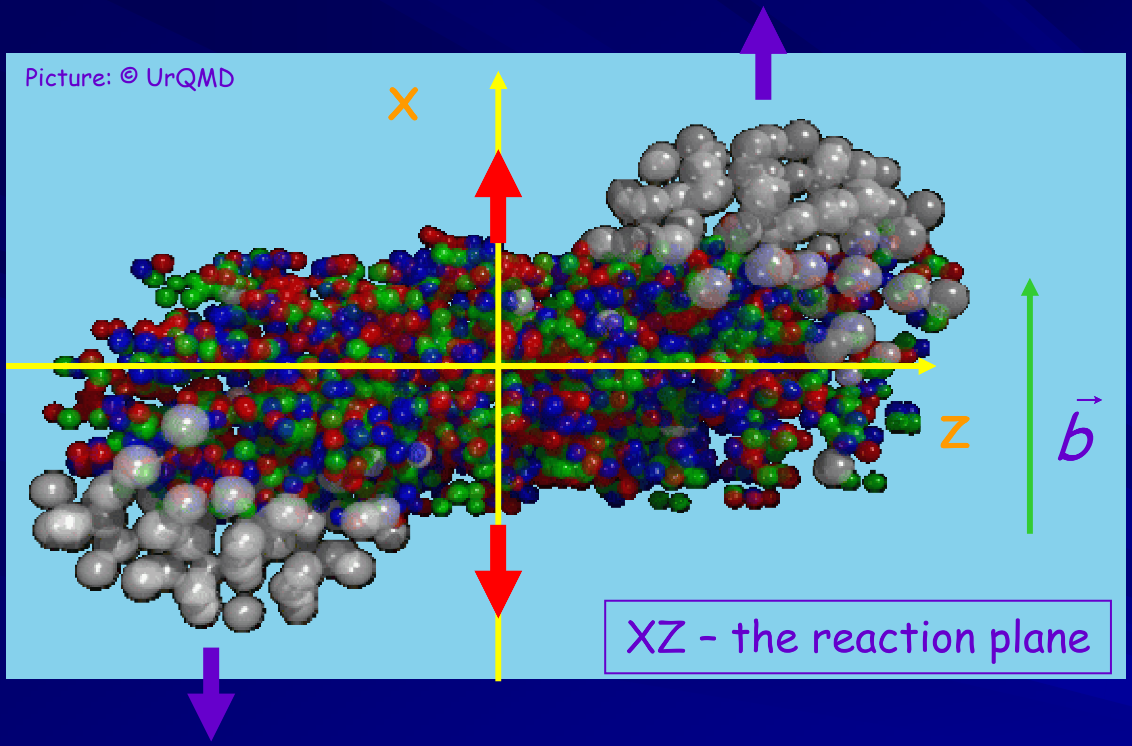
v_2 is the 2nd harmonic Fourier coefficient of the particle distribution in the x-y plane

$$v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Collective motion → asymmetric pressure gradients are more effective at pushing particles out along the "reaction plane" direction rather than perpendicular to it, as measured by the elliptic flow v_2

Large v_2 is an indication of early thermalization

The coefficients v_1 and v_2



$$v_1 = \left\langle \frac{p_x}{p_t} \right\rangle = \left\langle \cos(\phi - \Phi_R) \right\rangle$$

$$v_2 = \left\langle \left(\frac{p_x^2}{p_t^2} - \frac{p_y^2}{p_t^2} \right) \right\rangle = \left\langle \cos 2(\phi - \Phi_R) \right\rangle$$

$$p_t = \sqrt{p_x^2 + p_y^2}$$

$$\phi = \tan^{-1} \frac{p_y}{p_x}$$

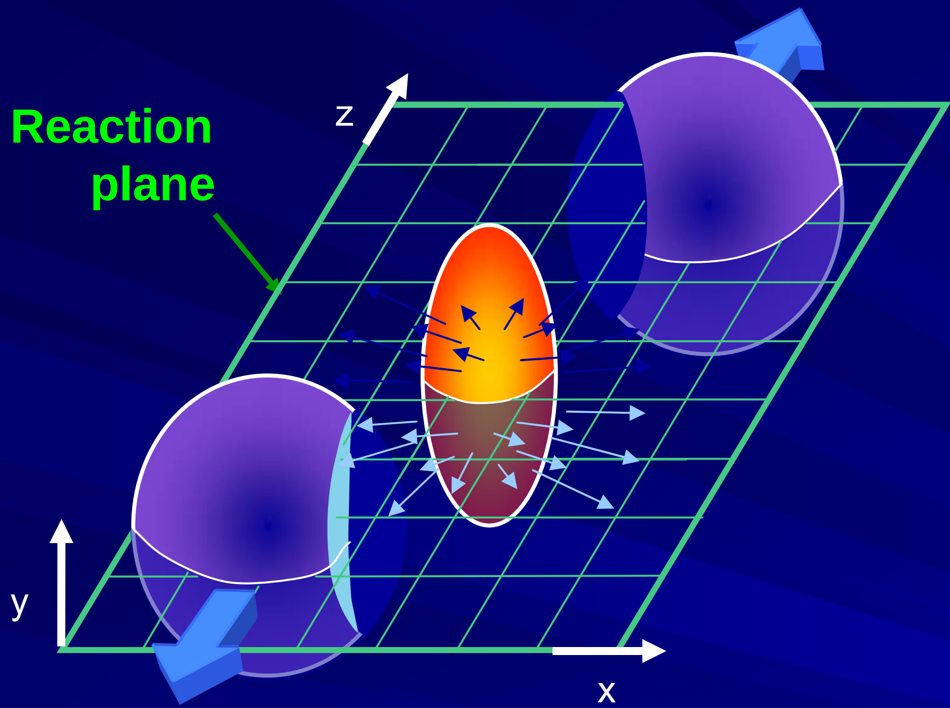
Anisotropic flow ≡ correlations with respect to the reaction plane

$$\frac{d^3N}{dp_t\, dy\, d\varphi} = \frac{d^2N}{dp_t\, dy} \frac{1}{2\pi} (1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \ldots)$$

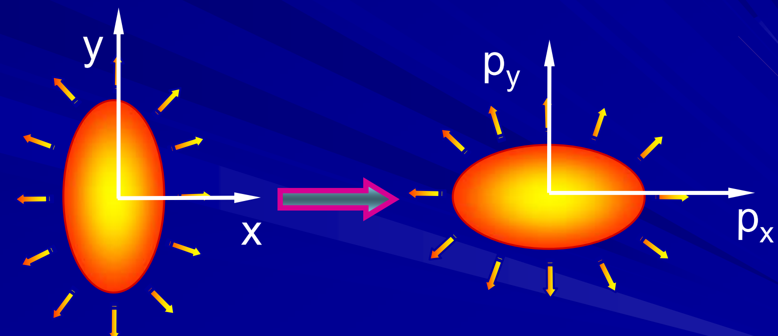
Directed flow

Elliptic flow

Collision Geometry: Elliptic Flow



- The application of fluid-dynamics implies that the medium is in local thermal equilibrium!
- Note that fluid-dynamics cannot make any statements how the medium reached the equilibrium stage...



elliptic flow (v_2):

- gradients of almond-shape surface will lead to preferential emission in the reaction plane
- asymmetry out- vs. in-plane emission is quantified by 2nd Fourier coefficient of angular distribution: v_2

➢ calculable with fluid-dynamics

Hydrodynamics
(for further reading)

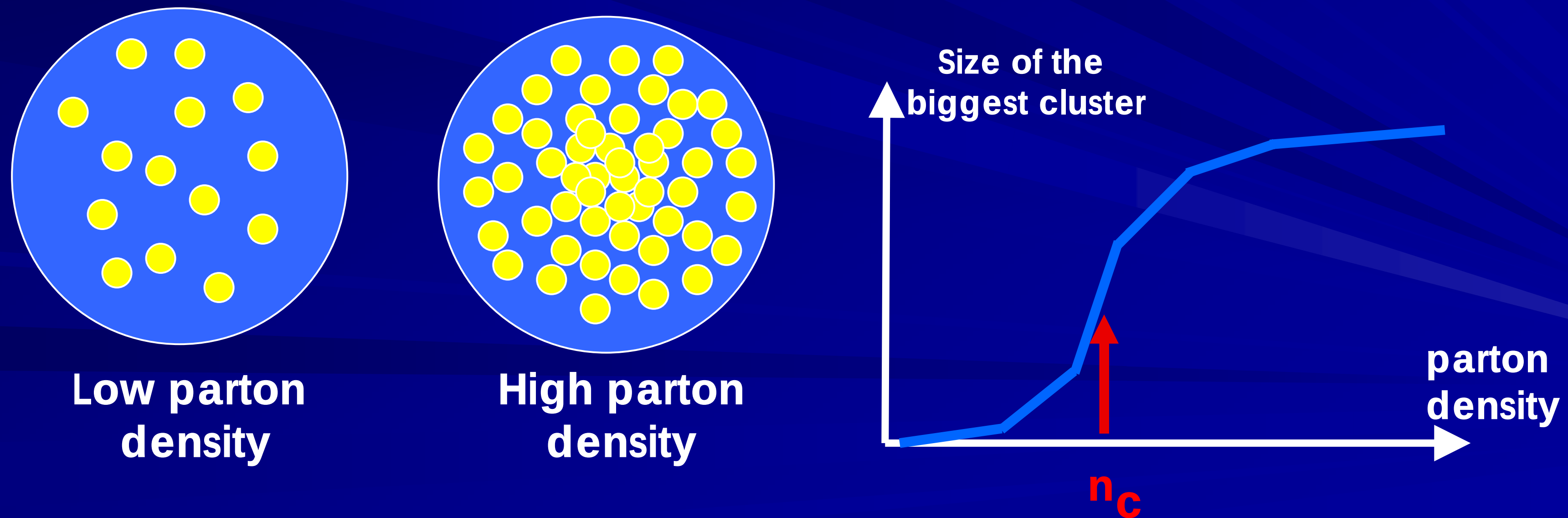
Ultrarelativistic Heavy Ion Collisions
Author: Ramona Vogt
Elsevier (2007)

Hydrodynamic Models for Heavy Ion Collisions
Authors: P. Huovien, P.V. Ruuskanen
An invited review for Nov. 2006 edition of Annual Review of Nuclear and Particle Physics; nucl-th/0605008

Hydrodynamic Approaches to Relativistic Heavy Ion Collisions
Author: Tetsufumi Hirano, invited talk given at XXXIV International Symposium on Multiparticle Dynamics, Sonoma, USA, July 26 - August 1, 2004
Journal-ref: Acta Phys.Polon. B36 (2005) 187-194; nucl-th/0410017

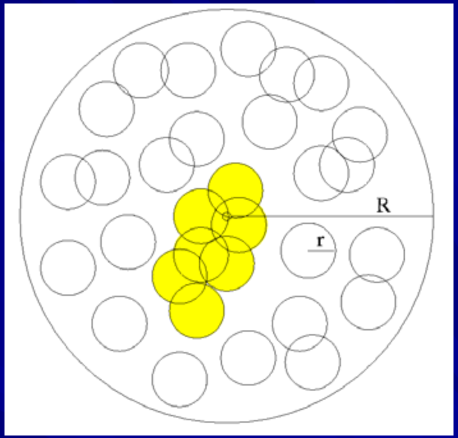
Percolation

- Parton percolation is a geometric, pre-equilibrium form of deconfinement
- an essential prerequisite for QGP production is cross-talk between the partons from different nucleons



Percolation Model: geometrical transition

- In Central collisions nucleons undergo several interactions and, since each collision establishes a string, we will obtain a spaghetti like of intertwined overlapping QCD strings.



- Deconfinement is expected when there is enough internetting between nucleons.

- Deconfinement is a function of string size (QCD) and deconfinement string density

H. Satz, M. Nardi

Parton Percolation in Nuclear Collisions

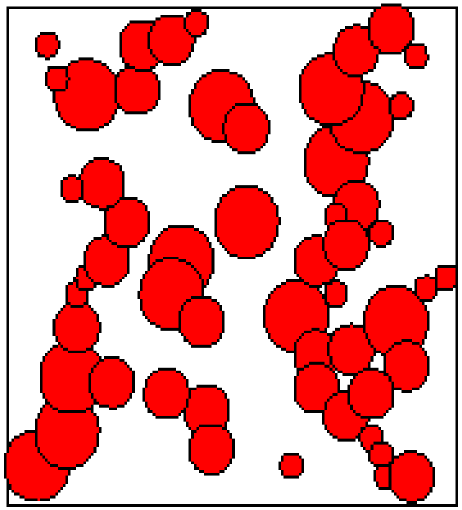
- large-scale interconnected system
- partons lose independent existences, knowledge of origin
- onset of color deconfinement
- prerequisite for later thermalization, QGP formation

Deconfinement and Hadron Percolation

Consider hadrons as spheres of radius $r_h \simeq 0.8$ fm

percolation occurs for density $n_c = \frac{0.34}{(4\pi/3)\, r_h^3} \simeq 0.16 \text{ fm}^{-3}$

$\Rightarrow$ formation of hadronic matter



for $n \leq n_c$: only isolated hadrons, clusters

for $n = n_c$: connected hadronic medium

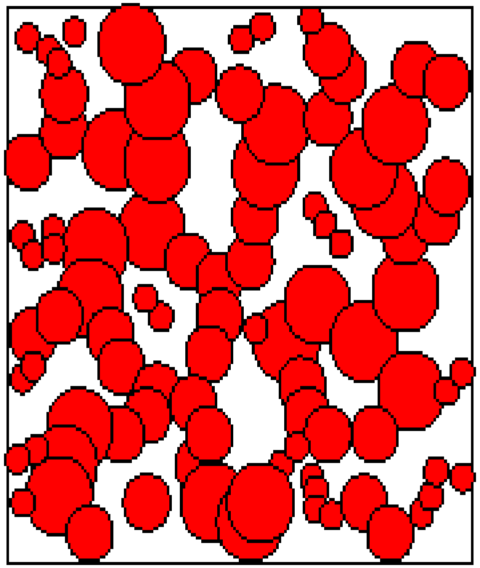
31 % hadronic clusters
69 % empty space

for $n \geq n_c$: both "media" percolate

When does the percolating vacuum disappear? Or, starting from high density side, when does vacuum first percolate?

Percolation condition for

"hadronic size" vacuum bubbles $\bar{n}_c = \frac{1.24}{(4\pi/3)\, r_h^3} \simeq 0.58 \text{ fm}^{-3}$



for $n = \bar{n}_c$: end of connected vacuum

69 % hadronic clusters

31 % empty space

for $n \geq \bar{n}_c$: only isolated vacuum bubbles

in dense interacting matter

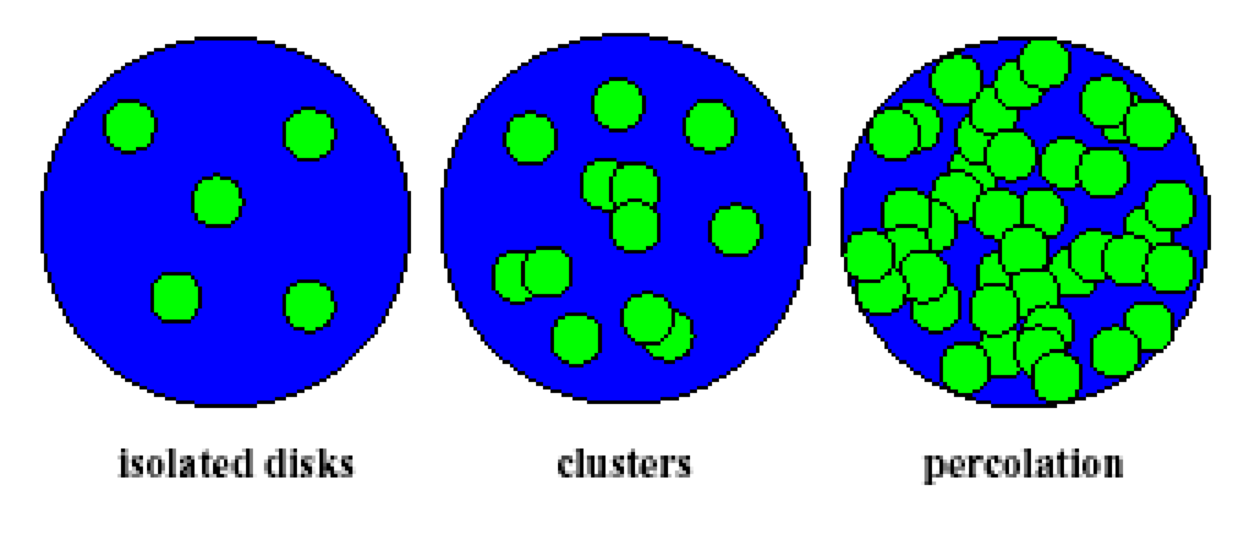
Deconfinement as percolation:

when a hadronic medium becomes so dense that only isolated vacuum bubbles survive, then it becomes a quark-gluon plasma

Deconfinement and coalescence

Believe that

- there is a very good chance that the effect of the light nuclei emission in heavy ion collisions may be one of the accompanying effects of percolation cluster formation and decay.
- that light nuclei could be formed as a result of coalescence mechanism.

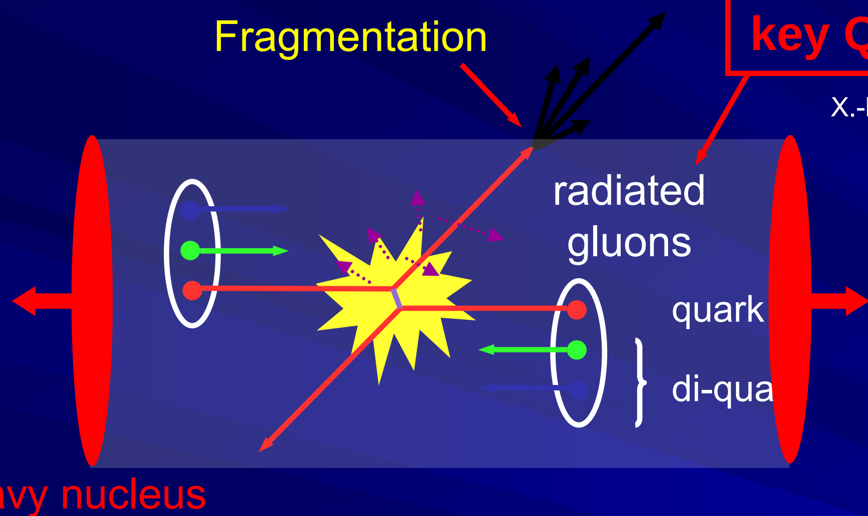


Jets in heavy ion collisions

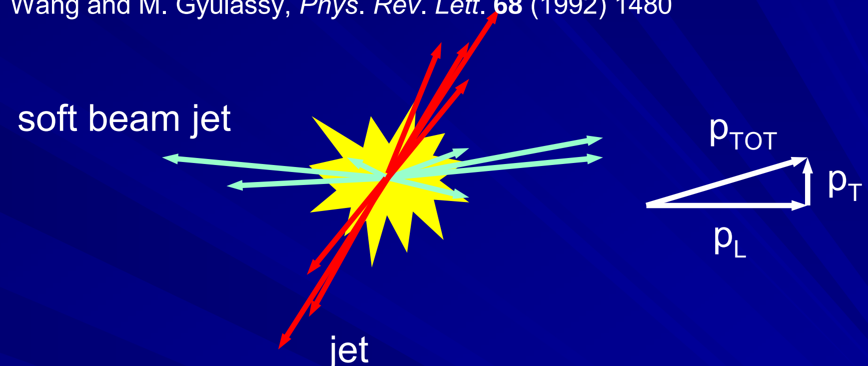
- Studying deconfinement with jets

key QCD prediction: jets are quenched

X.-N. Wang and M. Gyulassy, *Phys. Rev. Lett.* **68** (1992) 1480



Interaction at the quark (parton) level



The same interaction at the hadron level

- **Models of jet suppression**

  Various approaches; main points:

  ΔE_{med} is independent of parton energy.

  ΔE_{med} depends on length of medium, L.

  ΔE_{med} gives access to gluon density dN_g/dy or transport coefficient $\hat{q} = \frac{\langle k_T^2 \rangle}{\lambda}$

  Leads to a deficit of high p_t hadrons compared to p+p collisions (no medium).

Multiple soft scattering: Weidemann et al.
Opacity expansion: Gyulassy et al.
Twist expansion: Wang et al.

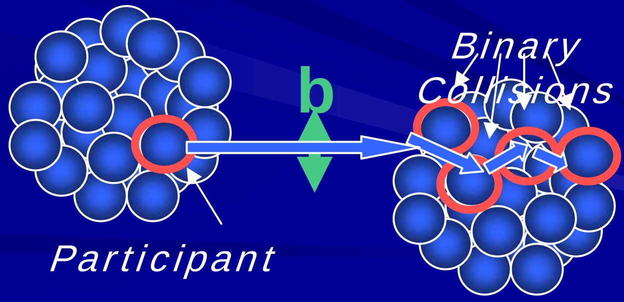
Jet Suppr. - Nuclear Modification Factor

- We can study jet suppression using leading hadrons
- We define a nuclear modification factor, R_{AA}, in terms of the ratio of the p_t spectra in nucleus-nucleus collisions divided by the p_t spectra in p+p collisions

$$R_{AA} = \frac{1}{T_{AB}} \frac{dN_{AA} / d\eta d^2 p_t}{dN_{pp} / d\eta d^2 p_t} \qquad T_{AA} = \langle N_{bin} \rangle / \sigma^{pp}_{inelastic}$$

- We also define a nuclear modification factor, R_{CP}, in terms of the ratio of the p_t spectra in central nucleus-nucleus collisions divided by the p_t spectra in peripheral nucleus-nucleus collisions

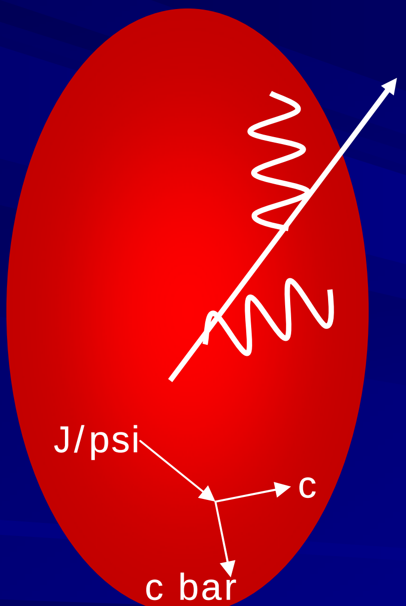
$$R_{CP} = \frac{\left(d^2N / d\eta dp_t / N_{bin}\right)_{central}}{\left(d^2N / d\eta dp_t / N_{bin}\right)_{peripheral}}$$



- With binary scaling, these factors as a function of p_t *are =1*

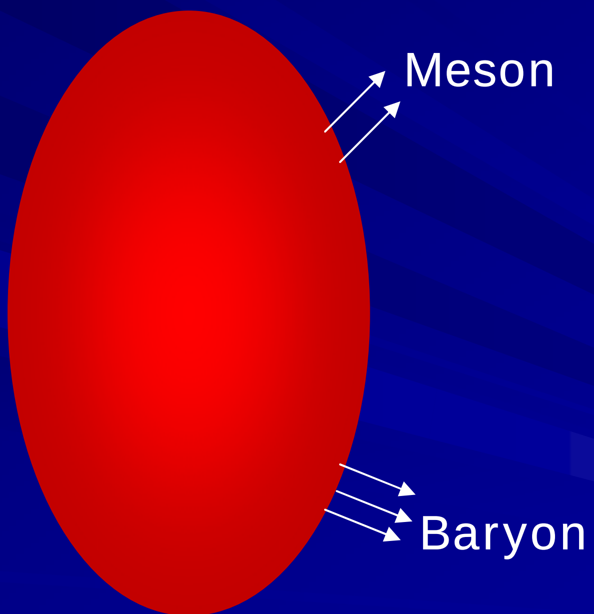
Utilization of Hydro Results

Jet quenching
J/psi suppression
Heavy quark diffusion



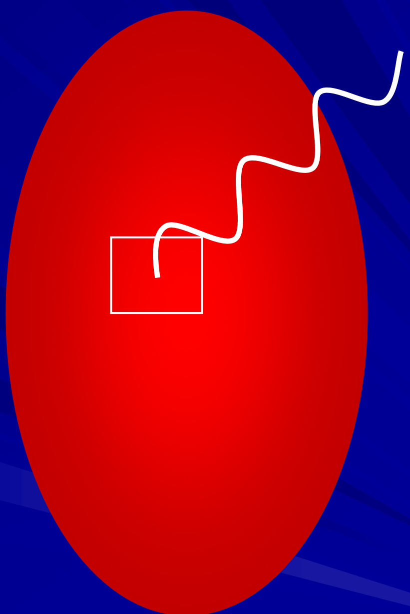
Information
along a path

Recombination
Coalescence



Information
on surface

Thermal
radiation
(photon/dilepton)



Information
inside medium

- Using transparency function the rate of yields can be calculated

$$R = \frac{n_1}{n_2}$$

(here e.g. n_1 and n_2 could be heavy flavor particles yields with fixed values of and) as a function of centrality, the masses and energy, it is expected to get the necessary information on the properties of the nuclear matter.

- With percolation model and experimental data on the behaviour of the nuclear modification factors it is possible to get information on the appearance of the anomalous nuclear transparency as a signal of formation of the percolation cluster.

@ RHIC

- Deconfined phase is showing unexpected properties
- New phenomena, new probes:
  - jet tomography
  - collective motion
  - b-quarkonia could be a useful probe
- Initial quanta: Color Glass Condensate?
- A very dense, fluid phase: strongly interacting Quarks and Gluons?
- Which excitations populate the QG Liquid?

High energy limit of QCD

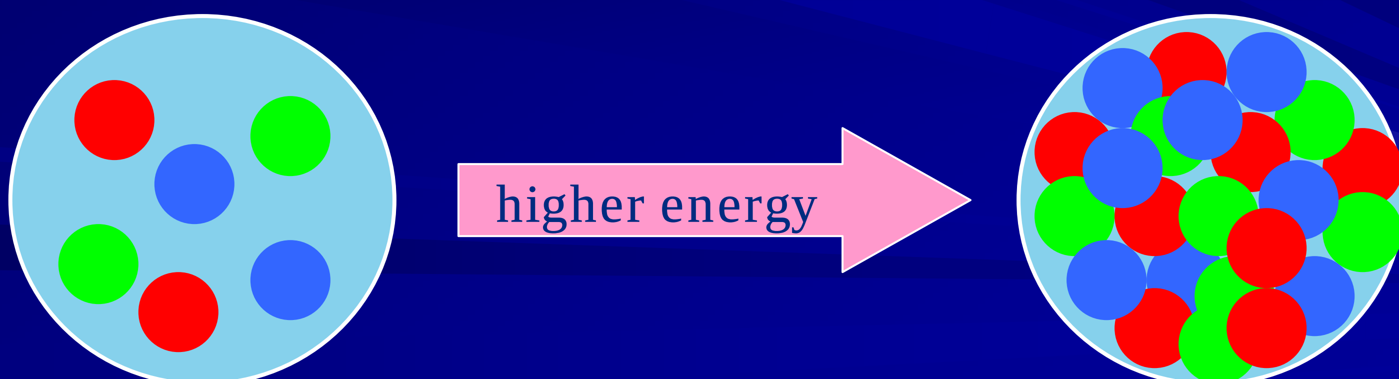
A universal form of matter at high energy

→ *Color Glass Condensate (CGC) !!*

Gluons have "color"

created from "frozen" random color source, that evolves slowly compared to natural time scale

High density ! occupation number ~ $1/\alpha_s$ at saturation



Dilute gas

CGC: high density gluons

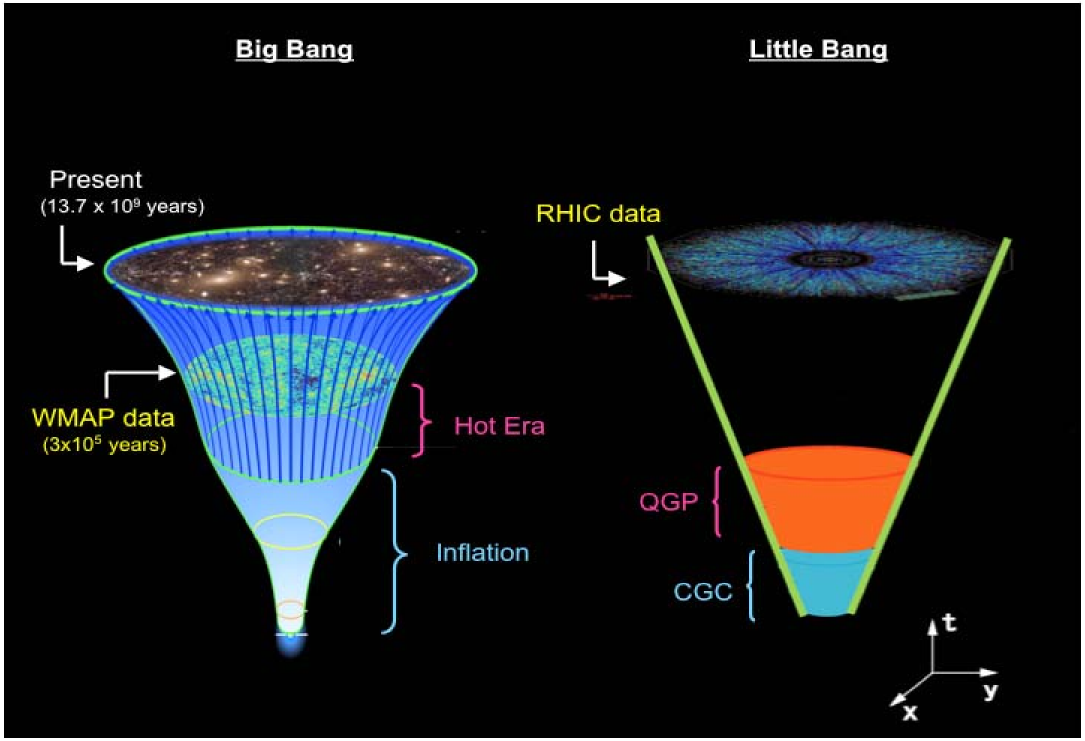
The Color Glass Condensate and Glasma

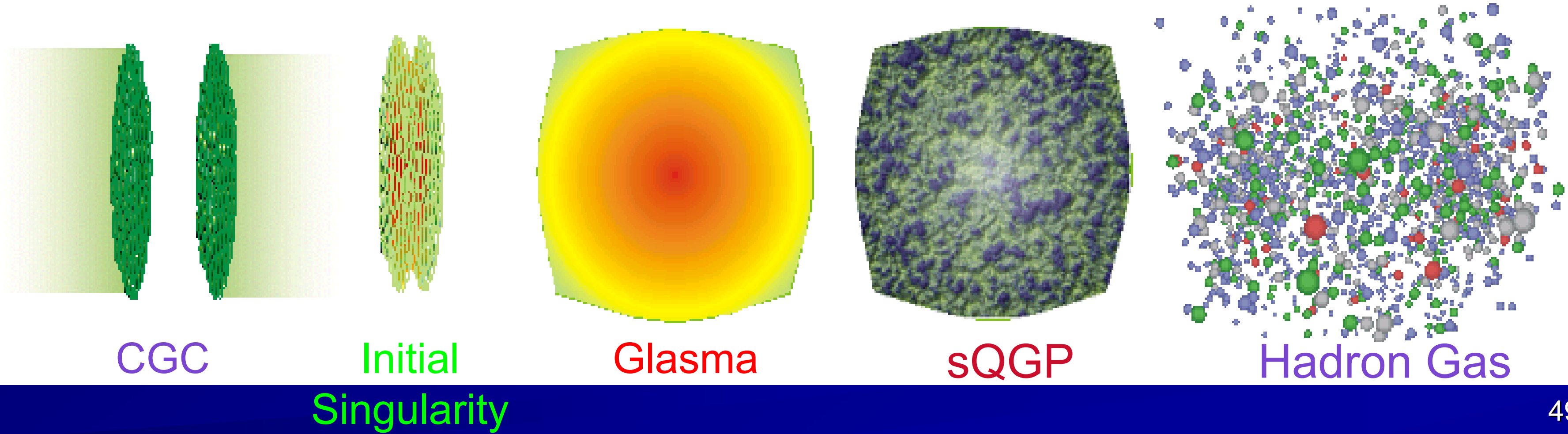
What is the high energy limit of QCD?

What are the possible form of high energy density matter?

How do quarks and gluons originate in strongly interacting particles?

Art due to Hatsuda and S. Bass





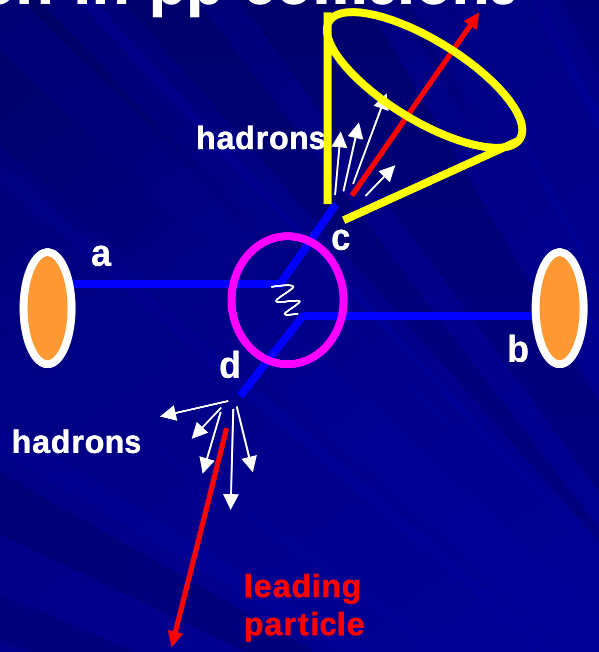
Thank You

Extra Slides

High p_T Particle Production

High p_T (≳ 2.0 GeV/c) hadron production in pp collisions

Jet: A localized collection of hadrons which come from a fragmenting parton



Parton Distribution Functions
Hard-scattering cross-section
Fragmentation Function

$p_{had} = z\,p_c$, $z<1$ energy needed to create quarks from vacuum

$$\frac{d\sigma^h_{pp}}{dy\,d^2p_T} = K \sum_{abcd} \int dx_a dx_b f_a(x_a,Q^2) f_b(x_b,Q^2) \frac{d\sigma}{d\hat{t}}(ab \rightarrow cd) \frac{D^0_{h/c}}{\pi z_c}$$

"Collinear factorization"

High p_T Particle Production in A+A

$$\frac{dN^h_{AB}}{dy\,d^2p_T} = ABK\sum_{abcd}\int dx_a dx_b \int d^2\mathbf{k}_a d^2\mathbf{k}_b$$

$$p_c^* = p_c(1-\varepsilon)$$

$$z_c^* = z_c/(1-\varepsilon)$$

$$\otimes f_{a/A}(x_a,Q^2)\, f_{b/B}(x_b,Q^2)$$ Parton Distribution Functions

$$\otimes g(\mathbf{k}_a)\, g(\mathbf{k}_b)$$ Intrinsic k_T , Cronin Effect

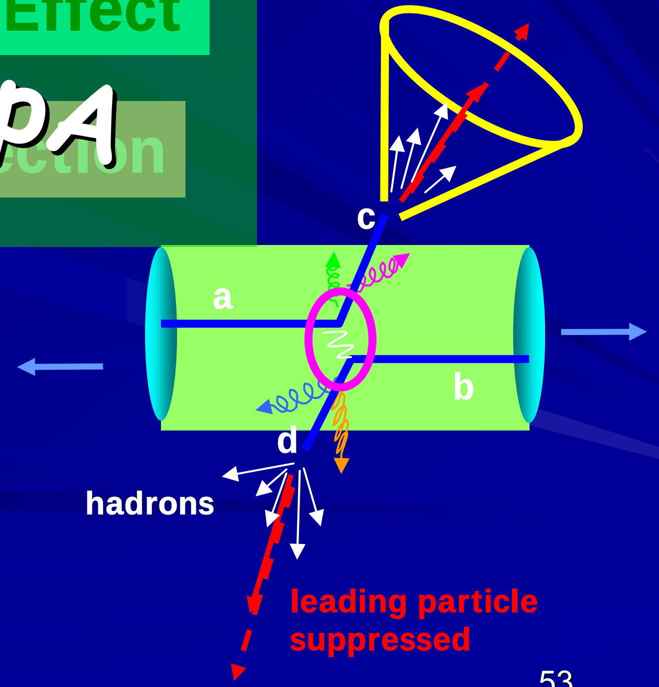
$$\otimes S_A(x_a,Q_a^2)\, S_B(x_b,Q_b^2)$$ Shadowing, EMC Effect

$$\otimes \frac{d\sigma}{d\hat{t}}(ab\rightarrow cd)$$ Hard-scattering cross-section

Known from pp and pA

$$\otimes \int_0^1 d\varepsilon P(\varepsilon)\frac{z_c^*}{z_c}$$ Partonic Energy Loss

$$\otimes \frac{D^0_{h/c}(z_c^*,Q_c^2)}{\pi z_c}$$ Fragmentation Function



- Expect to get a result which would demonstrate the changing of absorption properties of medium depending on the kinematical characteristics of heavy particles.
- A comparison of yields in different ion systems by using nuclear modification factors such as R_{CP} (involving Central and Peripheral collisions) should provide information on hadronization.
- R_{CP} highlights the particle type dependence at intermediate p_T as suggested by coalescence models --- hadrons result from the coalescence of quarks in the dense medium.
- At high p_T, jet fragmentation becomes the dominant process to explain the hadron formation.
- Thus, the quark constituents may be the relevant degrees of freedom for the description of the collision.

p_t limit for hydrodynamics

- Particles with very large transverse momenta (jets) are never expected to suffer sufficiently many interactions with the fireball medium to fully thermalize before escaping; hence a hydrodynamic approach can never work at very high p_t .
- However, we can turn this inescapable failure of hydrodynamics in small collision systems and at high p to our favour :

  since ideal fluid dynamics appears to work well in near-central collisions,

  and at low $p_t \leq 1.5\text{-}2$ GeV/c (...),

  we can study its gradual breakdown at larger impact parameters,

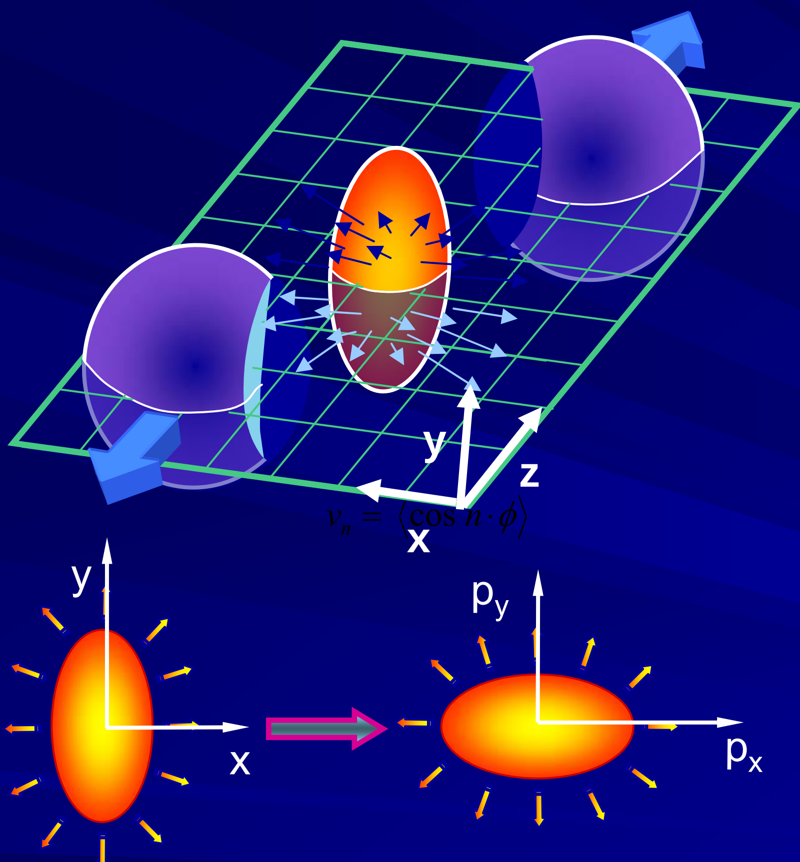
  rapidities and transverse momenta

  in order to learn something about the mechanisms for the approach to thermal equilibrium at the beginning of the collision

  and the decay of thermal equilibrium near the end of the expansion stage, Hence about the transport properties of the early quark–gluon plasma and

  the late hadron resonance gas created in these collisions.
- Breakdown also consistent with the expectated behavior of v_2 with varying shear viscosity.

Elliptic flow



- Look at non-central collisions
- Overlap region is not symmetric in coordinate space
- Almond shaped overlap region
  - Larger pressure gradient in *x-z* plane than in *y* direction
- Spatial anisotropy -> momentum anistropy
  - Process quenches itself -> sensitive to early time in the evolution of the system
  - Sensitive to the equation of state
- Perform a Fourier decomposition of the momentum space particle distributions in the *x-y* plane
  - v_n is the n^{th} harmonic Fourier coefficient of the distribution of particles with respect to the reaction plane
    - v_1: directed flow
    - v_2: elliptic flow