First LHC School
Lecture#2

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Strangeness and Hypercharge

$$\pi^- + p \to \Lambda^0 + K^0$$

$$p + \pi^-$$

$$\pi^+ + \pi^-$$

- Production cross-section of typical of Hadronic interaction.
- Decays have life-times of the order 10^{-10}, Characteristics of weak interaction.

Moreover

$$\pi^- + p \to K^- + p$$ was not seen

But

$$\pi^- + p \to n + K^+ + K^-$$ was seen

A new quantum number for hadrons, called **Strangeness**.
Strangeness is conserved in hadronic & EM interactions but violated in Weak interaction.

Strangeness of some particles

$$S(K^+) = 1, \quad S(K^-) = -1$$
$$S(K^0) = 1, \quad S(\bar{K}^0) = -1$$
$$S(\Lambda^0) = -1, \quad S(\Sigma^0) = -1$$
$$S(\Xi) = -2, \quad S(\Omega^-) = -3$$

Examples

$$\pi^- + p \rightarrow \Lambda^0 + K^0$$
$$0 \quad 0 \quad -1 \quad +1$$

$$\Delta \rightarrow n + \pi$$
$$0 \quad 0 \quad 0$$

They are allowed in strong process.

$$\Lambda^0 \rightarrow p + \pi^-$$
$$-1 \quad 0 \quad 0$$

$$K^0 \rightarrow p + \pi^-$$
$$1 \quad 0 \quad 0$$

They are forbidden in strong interaction but allowed in weak interaction as 'S' is not conserved in these decays.

Hypercharge

$Y = B + S$ is conserved by the Hadronic & EM interaction but not conserved in weak decays.

$$Y = 2\left\langle {}^{q}/_{e} \right\rangle$$

Average value of charge number

Isospin

Hadrons occur in the mass multiplets e.g. *p, n* are mass degenerate in mass behave in exactly the same way for hadronic interaction. So they are considered as the states of same particle N in some internal space or "charge space".

In general hadrons appear in

Singlets | Λ; η;…etc.

I=0 for singlet.

Doublets $\begin{pmatrix} p \\ n \end{pmatrix}, \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \ldots .etc$

$I=\frac{1}{2}$ for doublet
$I_3=\frac{1}{2}, -\frac{1}{2}$

Triplets $\begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}, \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}, \ldots .etc$

$I=1$ for triplet
$I_3=1, 0, -1.$

Isospin is a symmetry of hadronic interaction i.e. it is conserved in hadronic interactions but is violated by EM & Weak interactions.

The charge of a state is given by the relation

$$Q = \left(\frac{q}{e}\right) = I_3 + \langle Q \rangle = I_3 + \frac{1}{2}Y.$$

This is called the Gell-Mann-Nishijima relation.

Quantum Numbers of the Quarks ($Y = B + S$, $Q = I_3 + Y/2$)[a]

Quark	Spin	B	Q	I_3	S	Y
u	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	0	$\frac{1}{3}$
d	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$\frac{1}{3}$
s	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	-1	$-\frac{2}{3}$

Finally, we note that since the electric charge is always conserved, the conservation of I_3 implies Y-conservation and vice versa. To summarize, for hadronic interactions

$$\begin{aligned} \Delta|\mathbf{I}|^2 &= 0 \\ \Delta(Q,\ B,\ Y) &= 0. \end{aligned}$$

for electromagnetic interaction, we have the selection rules:

$$\Delta I_3 = 0, \qquad \Delta Y = 0, \qquad \Delta B = 0$$

$$\Delta|\mathbf{I}|^2 \neq 0.$$

Thus for weak interactions, we have the selection rules:

$$\Delta|\mathbf{I}|^2 \neq 0, \qquad \Delta Y \neq 0, \qquad \Delta B = 0.$$

Particle	B	Y	Q	I_3	I
$\Lambda(1116)$	1	0	0	0	0
p(938)	1	1	1	1/2	1/2
n(940)	1	1	0	−1/2	1/2
$K^-(494)$	0	1	1	1/2	1/2
$K^0(498)$	0	1	0	−1/2	1/2
$\pi^-(140)$	0	0	1	1	1
$\pi^0(135)$	0	0	0	0	1
$\pi^-(140)$	0	0	−1	−1	1

Summery of conservation of internal quantum numbers in interactions

Quantum Number	Hadronic	Electromagnetic	Weak
Q	Yes	Yes	Yes
B	Yes	Yes	Yes
S or Y	Yes	Yes	No
Isospin	Yes	No	No

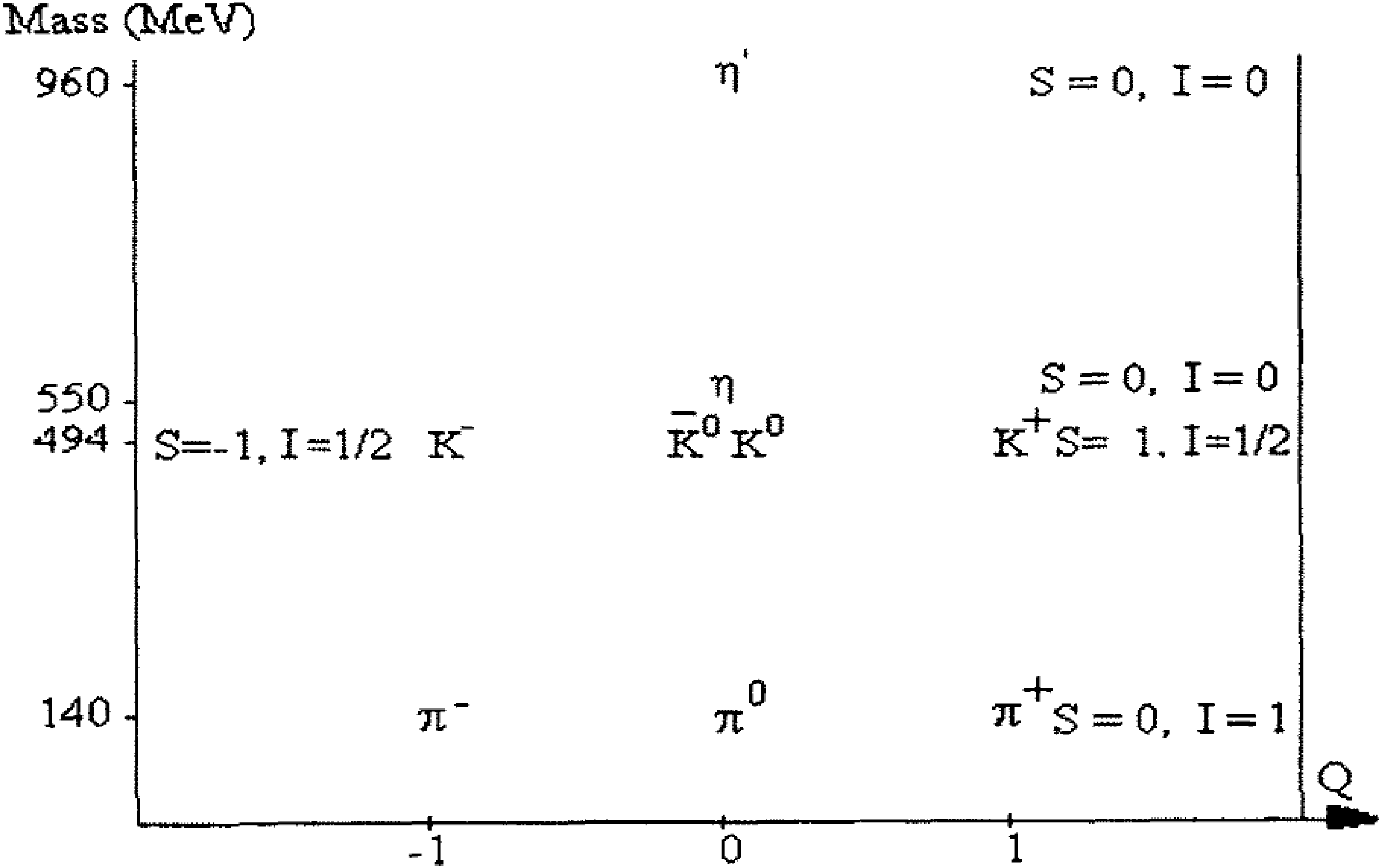


Figure 1 Lowest lying pseudoscalar mesons ($J^P = 0^-$).

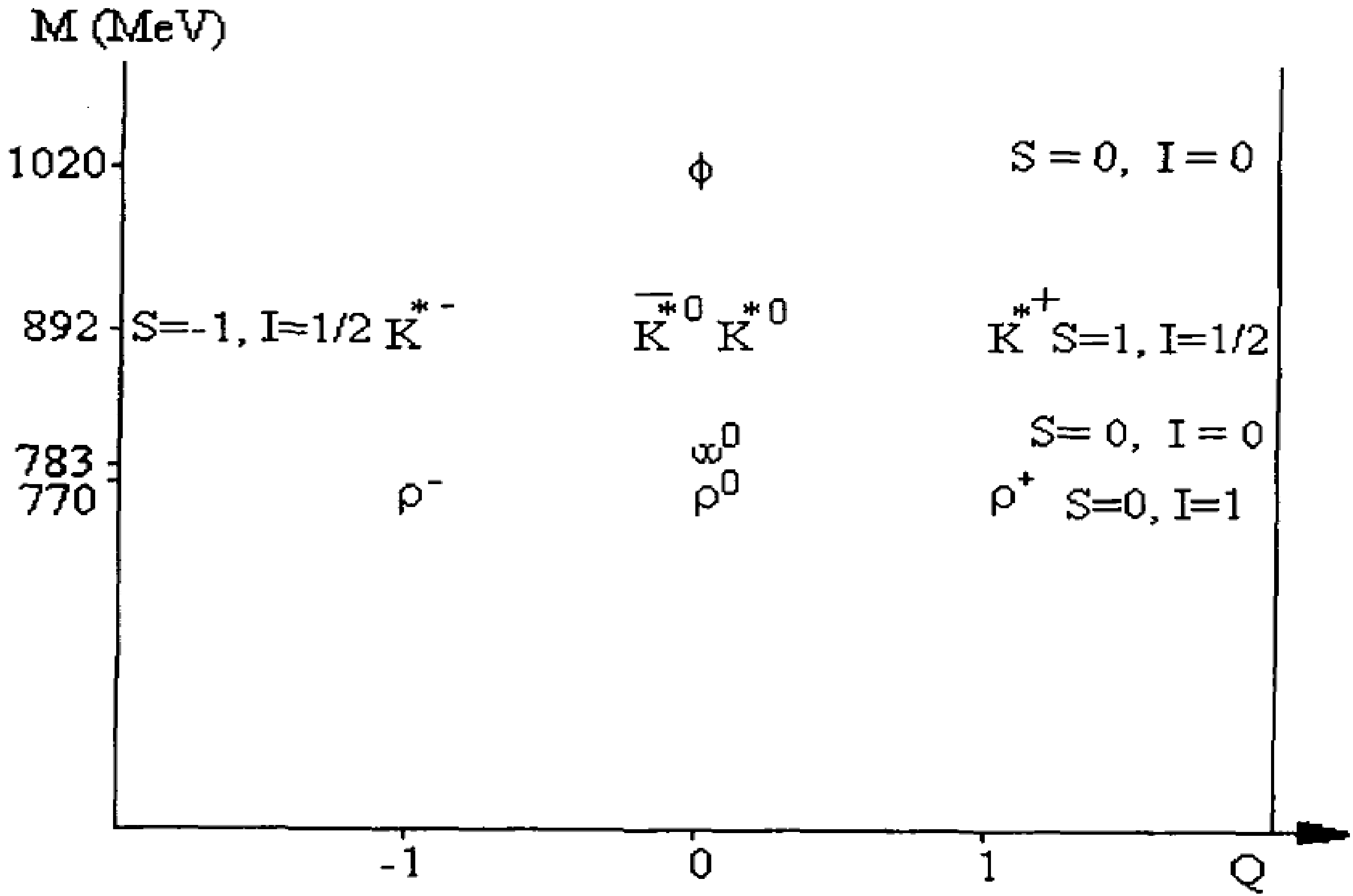


Figure 2 Lowest lying vector mesons ($J^P = 1^-$).

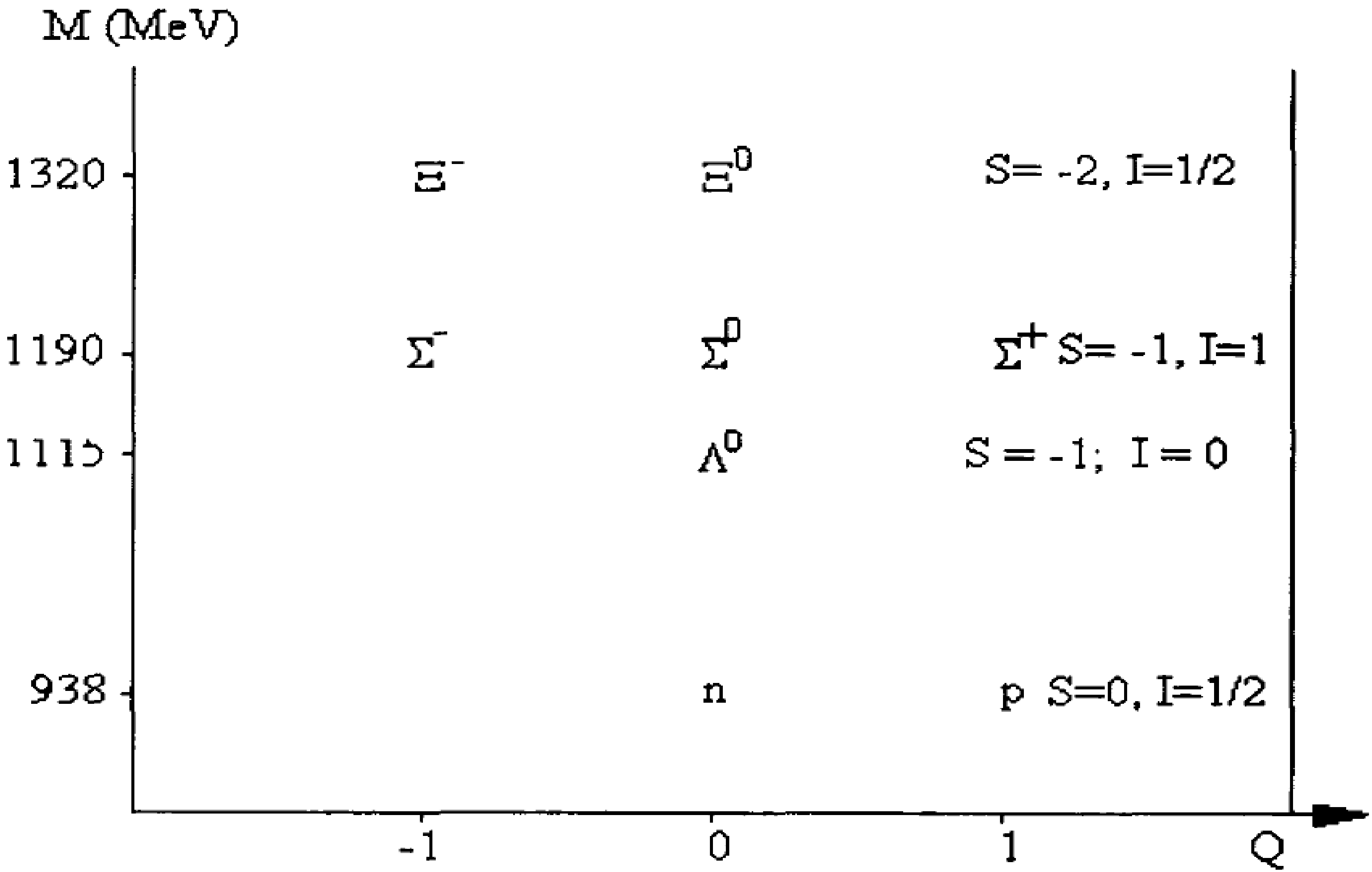


Figure 3 Lowest lying $1/2^+$ baryons.

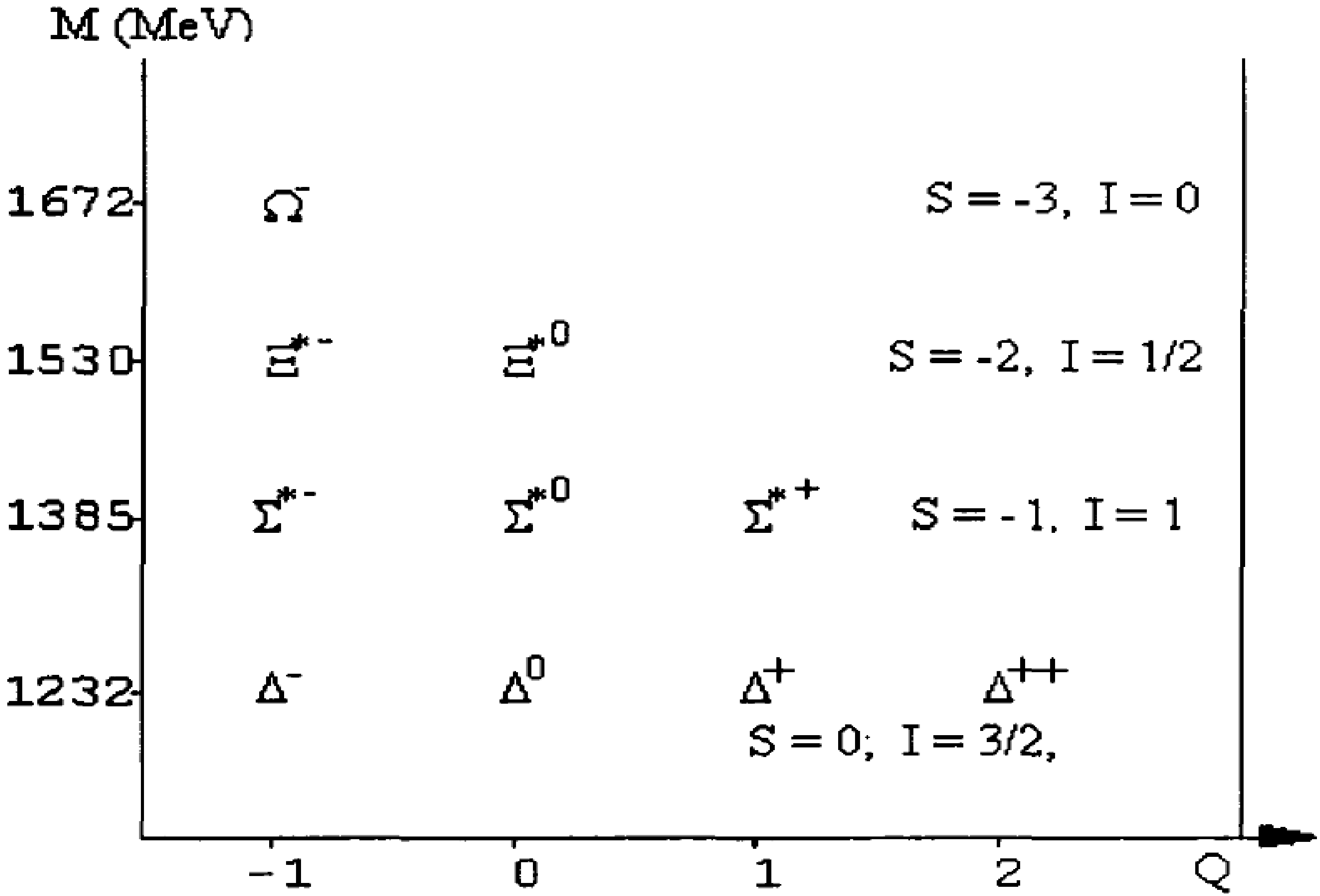
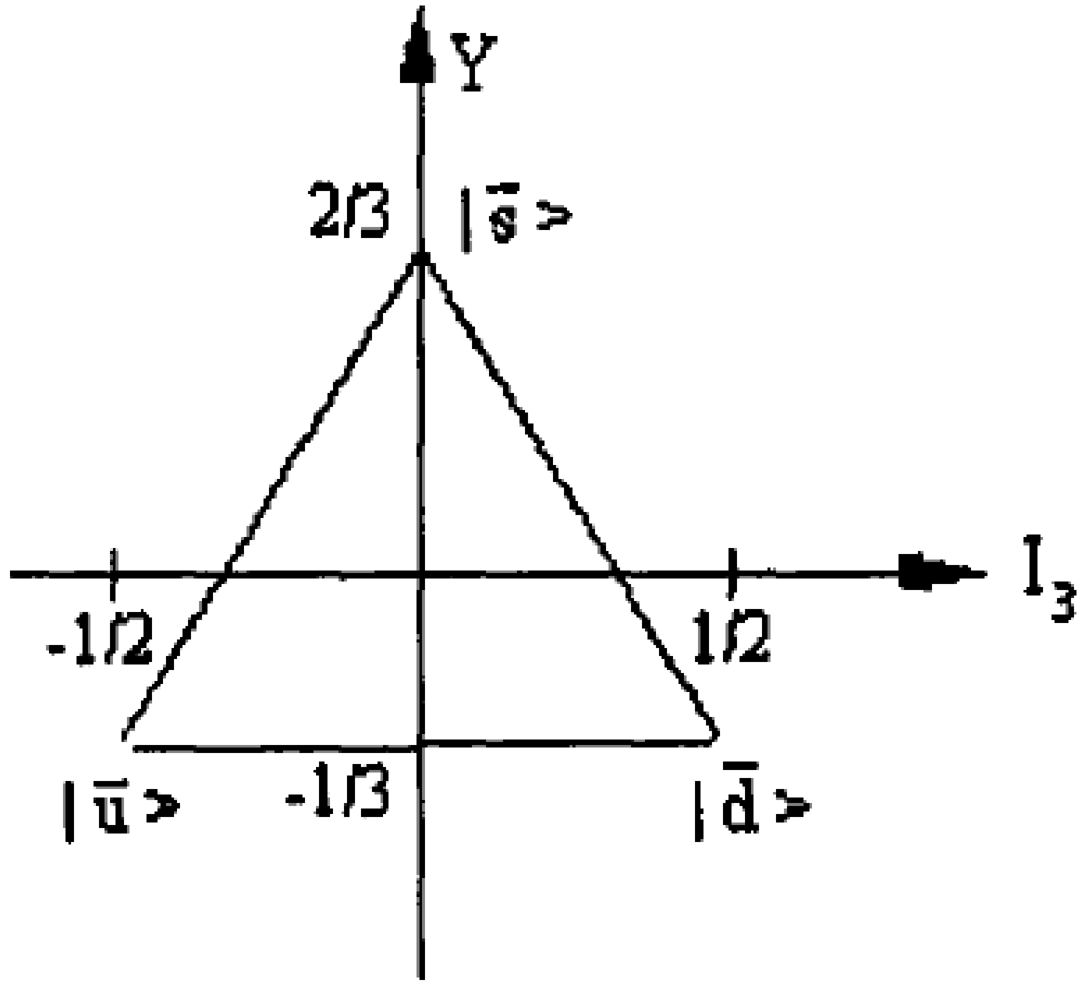
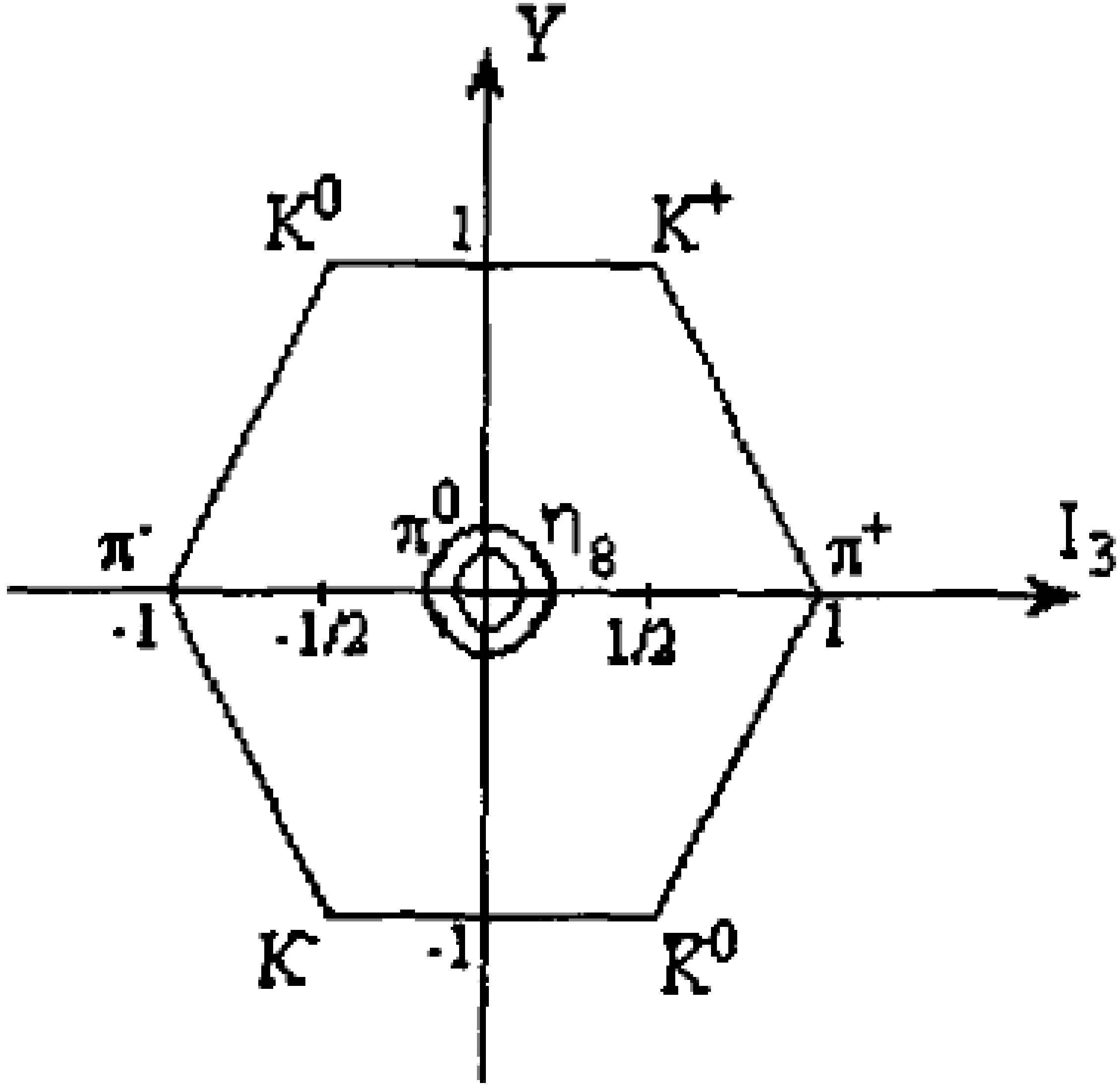
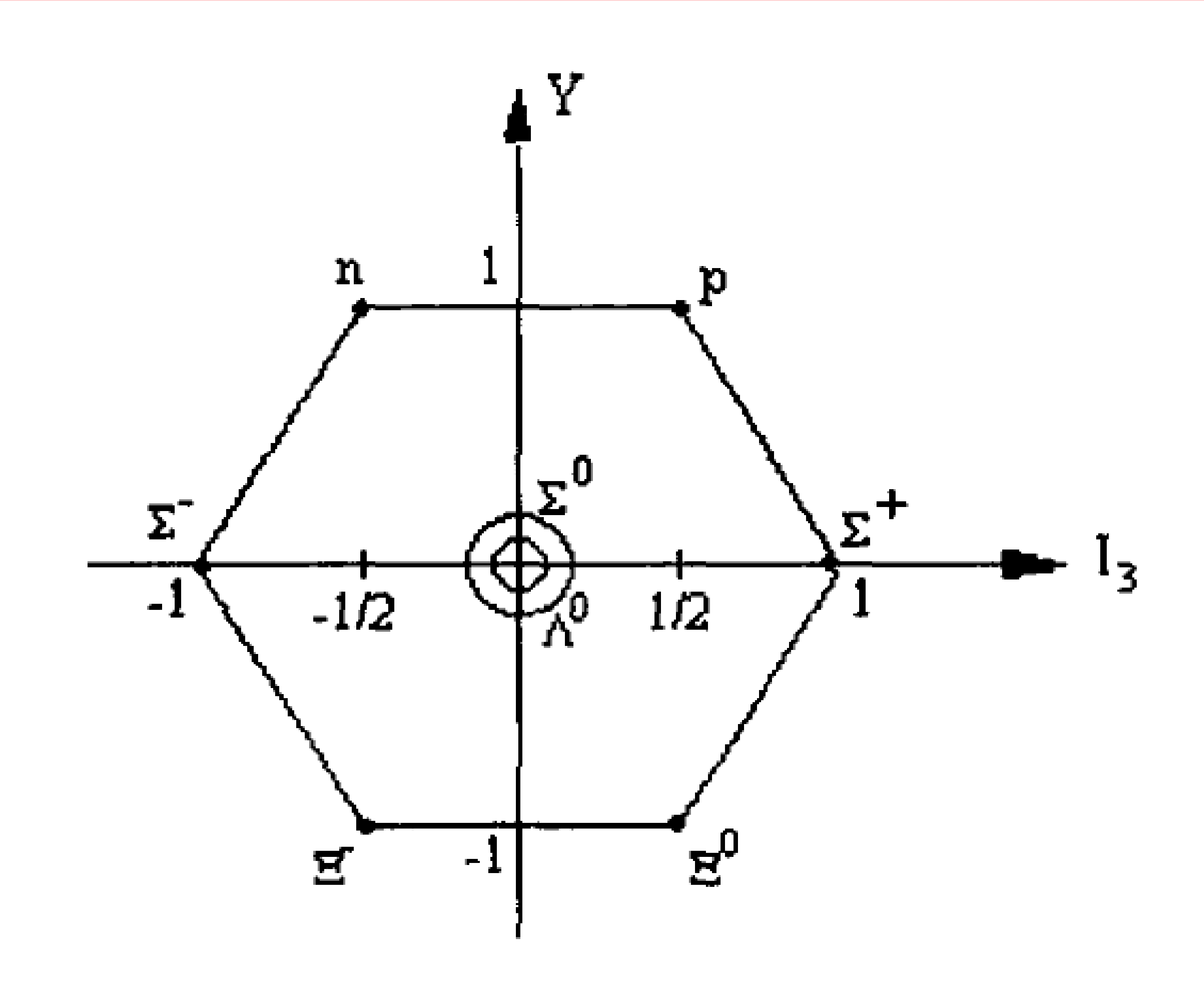


Figure 4 Lowest lying $3/2^+$ baryons.

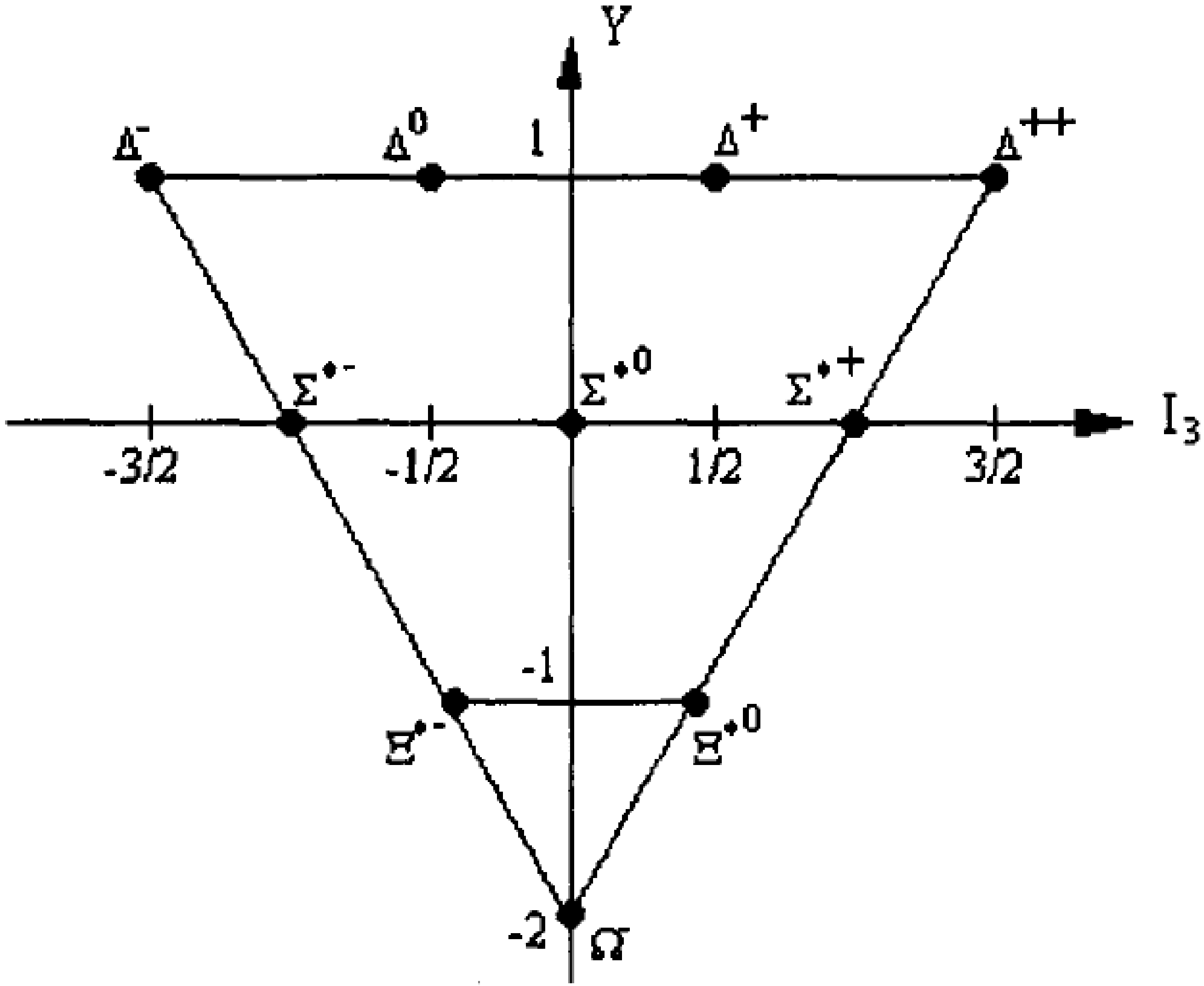

Y
2/3
|s̄ >
I3
-1/2
1/2
|ū >
-1/3
|d̄ >



Y
K^0
1
K^+
π^0
η_8
π^-
π^+
I_3
-1
-1/2
1/2
1
K^-
-1
$\bar{K}^0$



Y
I3
n
1
p
Σ0
Σ-
Σ+
-1
-1/2
Λ0
1/2
1
Ξ-
-1
Ξ0


Y
I3
Δ-
Δ0
1
Δ+
Δ++
Σ*-
Σ*0
Σ*+
-3/2
-1/2
1/2
3/2
-1
Ξ*-
Ξ*0
-2
Ω

Charge Conjugation

Charge Conjugation can be defined as

$$U_c \left| particle \right\rangle = \left| antiparticle \right\rangle$$

$$\left[Q, U_c \right]_+ = 0$$

$$\left[Q, U_c \right] \neq 0$$

As we have discussed, γ, π^0 and η^0 can be eigenstates of U_c. We now determine the C-parity of these states. Now under U_c, the electromagnetic current j_μ^{em}:

$$j_\mu^{em} \xrightarrow{U_c} - j_\mu^{em}.$$

But the electromagnetic field A_μ satisfies the equation

$$\Box^2 A_\mu = j_\mu^{em}.$$

it follows that

$$A_\mu \xrightarrow{U_c} - A_\mu.$$

Since a photon is a quantum of electromagnetic field, it follows that the C-parity of photon is -1 viz.

$$\eta_c \ (\gamma) = -1.$$

The decays $\pi^0 \to 2\gamma$ and $\eta^0 \to 2\gamma$ have been observed. Hence if these reactions proceed via electromagnetic interaction, it then follows from C-conservation that

$$\eta_c\left(\pi^0\right) = +1$$
$$\eta_c\left(\eta^0\right) = +1.$$

Since $\pi^0 \to 3\gamma$ and $\eta^0 \to 3\gamma$ can proceed via electromagnetic interaction, but have never been seen, these decays are strictly forbidden due to C-conservation in electromagnetic interaction.

$l = 0 = s$	${}^1S_0 \to 2\gamma$ ${}^1S_0 \to 3\gamma$	Allowed strictly forbidden
$l = 0$ $s = 1$	${}^3S_1 \to 2\gamma$ ${}^3S_1 \to 3\gamma$	strictly forbidden Allowed

Consider now the positronium, the bound states of e^- and e^+. Let us consider $e^- - e^+$ in definite $(l,\ s)$ state. Now e^- and e^+ are identical fermions which differ only in their electric charges. We can use a generalized Pauli principle for the positronium viz. "under total exchange of particles (which consists of changing simultaneously Q, $\mathbf{r}$ and s labels), the state should change sign or be antisymmetric". Under exchange of space co-ordinates, we get a factor $(-1)^l$, under spin co-ordinate exchange, we get a factor $(-1)^{s+1}$ ($s = 0$ for spin singlet state and $s = 1$, for spin triplet state), exchange of electric charge gives a factor η_c. We require the state to be antisymmetric, i.e.

$$(-1)^l\ (-1)^{s+1}\ \eta_c = -1$$

or

$$\eta_c = (-1)^{l+s}$$

which gives the charge conjugation parity of the positronium in (l, s) state.

Now for $(\pi^+ - \pi^-)$ system for which $B = 0,\ Y = 0,\ Q = 0$, generalized Pauli principle requires that the state should be symmetric (even) under total exchange of pions that is

$$(-1)^l\,\eta_c = 1$$

or

$$\eta_c = (-1)^l.$$

Similarly for $\pi^0 - \pi^0$ system we get $\eta_c = (-1)^l$. For this case, since two π^0's are identical particle, ordinary Pauli principle requires that $(-1)^l =$ even i.e. they must be in an orbital state with l even. Thus η_c must be $+1$ for $\pi^0 - \pi^0$ system, whereas η_c depends upon l value for $\pi^+\pi^-$ system.

G-Parity

For strong interactions, both isospin and C-parity are conserved. For hadrons, it is convenient to define a new operator $\hat{G}$ = charge conjugation $+180^o$ rotation around 2nd axis in isospin space. It follows that strong interactions are invariant under G, but

$$
\begin{aligned}
[\hat{G},\ H_{\mathrm{em}}] &\neq 0 \\
[\hat{G},\ H_{\mathrm{weak}}] &\neq 0
\end{aligned}
$$

i.e. electromagnetic and weak interactions are not invariant under G.

Under 180^o rotation around the 2nd axis in isospin space, we have

$$
\begin{aligned}
|\pi_1\rangle &\rightarrow -|\pi_1\rangle \\
|\pi_2\rangle &\rightarrow |\pi_2\rangle \\
|\pi_3\rangle &\rightarrow -|\pi_3\rangle \,.
\end{aligned}
$$

Therefore, we get

$$
\left|\pi^{\pm,0}\right\rangle \rightarrow -\left|\pi^{\mp,0}\right\rangle .
$$

Under charge conjugation

$$\left|\pi^{\pm}\right\rangle \xrightarrow{U_c} \left|\pi^{\mp}\right\rangle$$
$$\left|\pi^{0}\right\rangle \xrightarrow{U_c} \left|\pi^{0}\right\rangle.$$

Thus we have

$$\left|\pi^{\pm}\right\rangle \xrightarrow{\hat{G}} -\left|\pi^{\pm}\right\rangle$$

and

$$\left|\pi^{0}\right\rangle \xrightarrow{\hat{G}} -\left|\pi^{0}\right\rangle.$$

Thus the G-parity of pions is $G(\boldsymbol{\pi}) = -1$.

Thus for fermion-antifermion system, the G-parity is given by

$$G = (-1)^{l+s+I} = \eta_c (-1)^I.$$

For $(\pi^+\pi^-)$ system

$$G = (-1)^l (-1)^I = (-1)^{l+I} = 1.$$

As $\eta \to \pi^+\pi^-\pi^0$ is forbidden in strong interaction but is allowed by electromagnetic interaction.

Parity

Consider a transformation corresponding to space reflection:

$$\mathbf{x} \to \mathbf{x}' = -\mathbf{x}.$$

The corresponding unitary operator is denoted by $\widehat{P}$, which acting on a wave function gives

$$\widehat{P}\,\Psi(\mathbf{x},\ t) = \Psi(-\mathbf{x},\ t)\,.$$

Now

$$\widehat{P}^2 = 1,$$

so that $\widehat{P}$ has two eigenvalues ± 1. If

Under parity operator $\hat{P}$

$$\mathbf{x} \to -\mathbf{x}, \quad \mathbf{p} \to -\mathbf{p}$$

but the orbital angular momentum

$$\mathbf{L} = \mathbf{x} \times \mathbf{p} \to \mathbf{L},$$

so that

$$\mathbf{J} \to \mathbf{J}, \quad \boldsymbol{\sigma} \to \boldsymbol{\sigma}.$$

Such vectors are called axial vectors. Also under parity, the scalars:

$$\mathbf{x} \cdot \mathbf{p} \to \mathbf{x} \cdot \mathbf{p}$$

$$(\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_3 \to -(\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_3$$

$$\mathbf{J} \cdot \mathbf{p} \to -\mathbf{J} \cdot \mathbf{p}.$$

The scalars which change sign under parity are called pseudoscalars. All the three quantities are rotational invariant, but the last two have different behavior under $\hat{P}$.

Intrinsic parity

As far as orbital parity is concerned, it is independent of the species of particles and depends only on orbital angular momentum state of system of particles. When creation or annihilation of particles takes place, we have to assign an intrinsic parity to each particle. Consider, for example, a photon, the quantum of electromagnetic field represented by a vector potential.

$$\mathbf{A}(\mathbf{x}) = \boldsymbol{\varepsilon}\, f(x),$$

where $\boldsymbol{\varepsilon}$ is the polarization vector and $f(x)$ is a scalar function.

Now the interaction of a charged particle with electromagnetic field is introduced by the gauge invariant substitution:

$$\mathbf{p} \rightarrow \mathbf{p} - e\,\mathbf{A}(\mathbf{x}).$$

Since $\mathbf{x}$ and $\mathbf{p}$ change sign under $\widehat{P}$, it follows that

$$\mathbf{A}(\mathbf{x}) \longrightarrow -\mathbf{A}(-\mathbf{x})$$

i.e.

$$\widehat{P}\,\mathbf{A}(\mathbf{x})\,\widehat{P}^{-1} = -\mathbf{A}(-\mathbf{x}).$$

This means that under parity

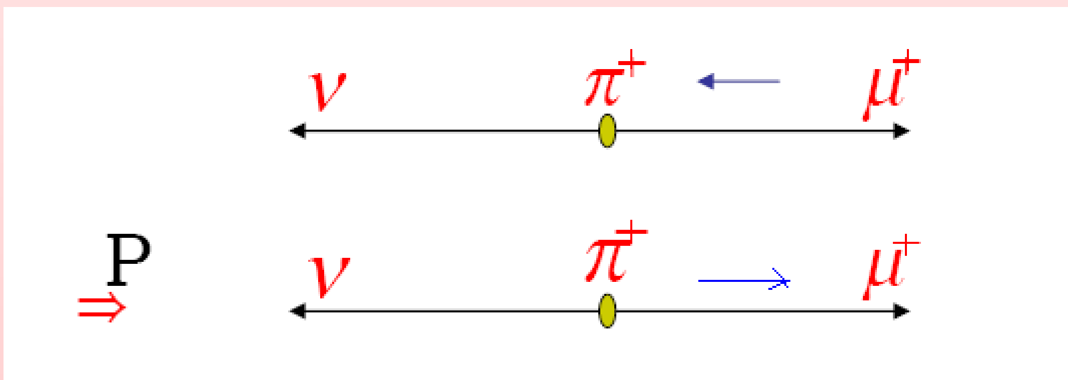
$$\boldsymbol{\varepsilon} \longrightarrow -\boldsymbol{\varepsilon}.$$

The behavior of the polarization vector $\boldsymbol{\varepsilon}$ characterizes what we call the intrinsic parity of a photon. Thus we say that intrinsic parity of a photon is odd.

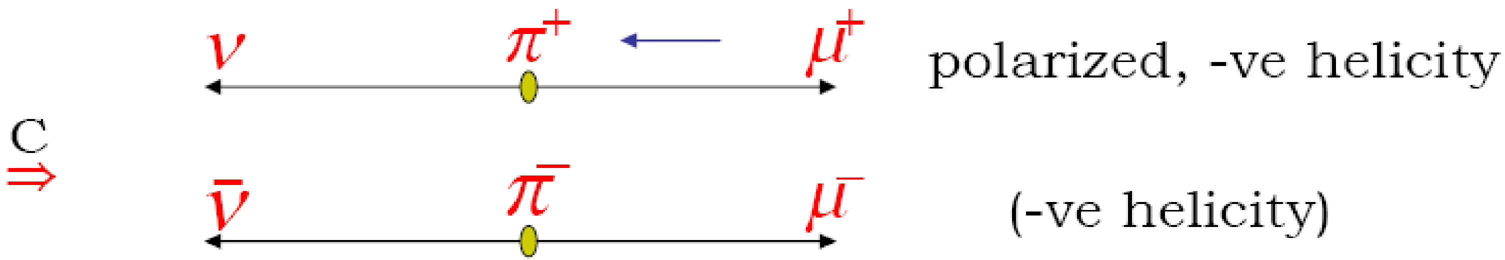
We shall assume that the spin of pion is zero (we shall show later, how it comes out to be zero). Consider first the decay $\pi^0 \to 2\gamma$. Here we have two polarization vectors $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ corresponding to two γ - rays, whose momenta we take as $\mathbf{k}_1$ and $\mathbf{k}_2$, such that (gauge invariance) $\mathbf{k}_1.\boldsymbol{\varepsilon}_1 = 0$, $\mathbf{k}_2.\boldsymbol{\varepsilon}_2 = 0$. We also note that $\boldsymbol{\varepsilon}_1.\boldsymbol{\varepsilon}_2 = 0$. Now only the momentum $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ is independent as $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 = 0$ in the rest frame of π^0. It is clear that the only invariant which we can form is $\mathbf{k} \cdot (\boldsymbol{\varepsilon}_1 \times \boldsymbol{\varepsilon}_2)$, which is a pseudoscalar, showing that intrinsic parity of π^0 is -1.

Parity Violation

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$



- It is easy to see that C must also be violated in weak interactions:

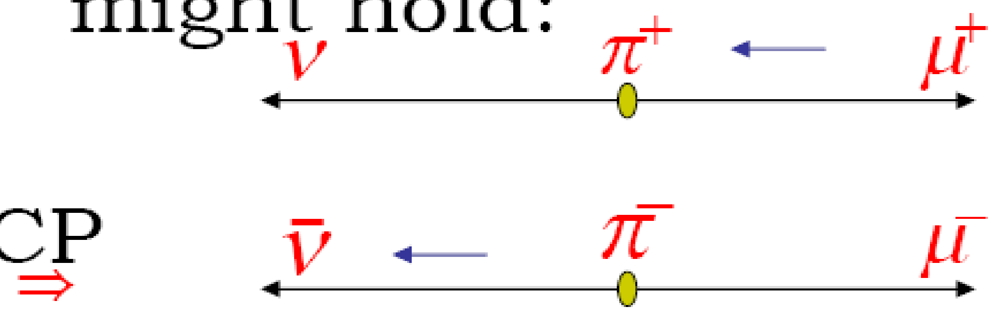


If invariant under C, rates will be same. Experimentally:

$$\Gamma_{\pi^+ \to \underset{(-)}{\mu^+} \bar{\nu}} \gg \Gamma_{\pi^- \to \underset{(-)}{\mu^-} \nu}$$

Thus C is violated.

What about CP invariance, which Landau, who independently expressed ideas similar to Salam, Lee – Yang about 2 – component neutrinos, suggested (1957) might hold:



Note that the momentum is reversed. Two Processes are observed with the same rate:

$$\Gamma_{\pi^+ \to \mu^+ \bar{\nu}} = \Gamma_{\pi^- \to \mu^- \nu}$$

$(-)$ $(+)$

We have seen that neither parity P nor charge conjugation C is conserved in weak interaction. Let us consider the decay $\pi^+ \to \mu^+ \nu$ in the rest frame of π where experimentally μ^+ is found to be polarized with helicity $\mathcal{H} = \mathbf{s} \cdot \mathbf{p} / |\mathbf{p}|$ to be negative. The application of charge conjugation operation C changes π^+ to π^-, $\mu^+ \to \mu^-$ and $\nu \to \bar{\nu}$ but does not change the helicity. Thus if weak interaction were invariant under C, one would find $\Gamma_{\pi^+ \to \mu^+(-)\nu} = \Gamma_{\pi^- \to \mu^-(-)\bar{\nu}}$, where $(-)$ denotes negative helicity. Experimentally $\Gamma_{\pi^+ \to \mu^+(-)\nu} \gg \Gamma_{\pi^- \to \mu^-(-)\bar{\nu}}$, showing that C is violated in weak interaction. If however, we now apply CP, then since helicity also changes sign we have $\Gamma_{\pi^- \to \mu^-(+)\bar{\nu}} = \Gamma_{\pi^+ \to \mu^+(-)\nu}$ as seen experimentally. Thus CP is conserved here.