

First School on LHC Physics

Elementary Particle Physics

Lecture#3

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Strong Color Charge

We have seen that the quarks form hadrons; the baryons and mesons in the ground state are composites of $(qqq)_{L=0}$ and $(q\bar{q})_{L=0}$. Quarks and (anti-quarks) are spin 1/2 fermions. Now q and \bar{q} spins may be combined to form a total spin S , which is 0 or 1. Total spin for qqq system is 3/2 or 1/2. Further as q and \bar{q} have opposite intrinsic parities, the parity of the $q\bar{q}$ system is $P = (-1)(-1)^L = -1$ for the ground state. Thus we have for the ground states.

Mesons	Baryons
$(q\bar{q})_{L=0}$	$(qqq)_{L=0}$
$S = 0, 1$	$S = 1/2, 3/2$
$J^P = 0^-, 1^-$	$J^P = 1/2^+, 3/2^+$

Mesons	Baryons
π, ρ	p, Δ
$\pi^+ = (u\bar{d}) \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$	$p (S = 1/2, S_Z = 1/2)$ $= (uud) (\uparrow\uparrow\downarrow)$
$\rho^+ (S = 1, S_Z = 0) :$ $(u\bar{d}) \frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow)$ $\rho^+ (S = 1, S_Z = 1) :$ $(u\bar{d}) (\uparrow\uparrow)$	$\Delta^{++} (S = 3/2, S_Z = 3/2)$ $= (uuu) (\uparrow\uparrow\uparrow)$

There is a difficulty with the above picture; consider, for example the state, $|\Delta^{++} (S_Z = 3/2)\rangle \sim |u^\uparrow u^\uparrow u^\uparrow\rangle$. This state is symmetric in quark flavor and spin indices (\uparrow). The space part of the wave function is also symmetric ($L = 0$). Thus, the above state being totally symmetric violates the Pauli principle for fermions. Therefore, another degree of freedom (called color) must be introduced to distinguish the otherwise identical quarks: each quark flavor carries three different strong color charges, red(r), yellow(y) and blue(b) i.e.

$$q = q_a \qquad a = r, y, b$$

[Leptons do not carry color and that is why they do not take part in strong interactions].

Including the color, we write e.g.

$$|\Delta^{++} (S_z = 3/2)\rangle = \frac{1}{\sqrt{6}} \sum \varepsilon_{abc} |u_a^\dagger u_b^\dagger u_c^\dagger\rangle.$$

so that the wave function is now antisymmetric in color indices and satisfies the Pauli principle. Other examples, as far as quark content is concerned, are

$$|p\rangle = \frac{1}{\sqrt{6}} \sum \varepsilon_{abc} |u_a u_b d_c\rangle,$$
$$|\pi^+\rangle = \frac{1}{\sqrt{3}} \sum_a |u_a \bar{d}_a\rangle,$$

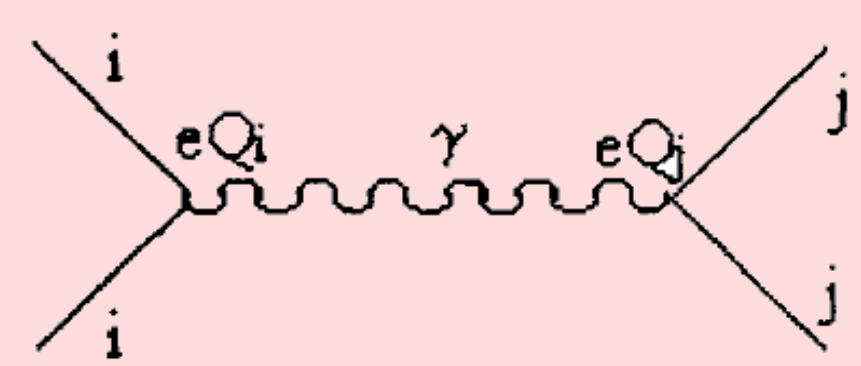
i.e. these states are color singlets. In fact, all known hadrons are color singlets. Thus, the color quantum number is hidden. This is the postulate of color confinement mentioned earlier and explains the non-existence of free quark (q) or such systems as (qq) , $(q\bar{q}q)$, and $(qqqq)$.

Electromagnetic Force Between 2 Electrically Charged Particles

We deal with electrically neutral atoms.

Mediator of the electromagnetic force is electrically neutral massless spin 1 photon, the quantum of the electromagnetic field.

Exchange of photon gives the electric potential:

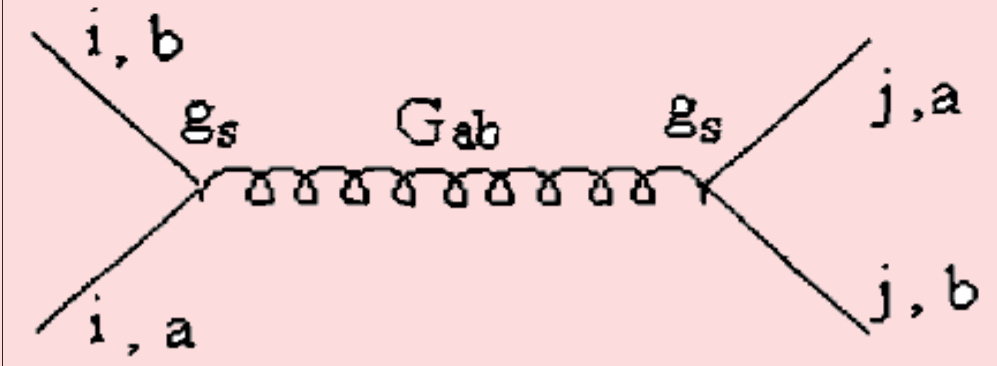


Strong Color Force Between 2 Quarks

We deal with color singlet systems i.e. hadrons.

Mediators are eight massless spin 1 color carrying gauge vector bosons, called gluons.

Exchange of gluons gives the color electric potential:



$$V_{ij}^e = \frac{e^2}{4\pi} \frac{Q_i Q_j}{r}, r = |\mathbf{r}_i - \mathbf{r}_j|$$

For an electron and proton

$$V_{ij} = -\frac{\alpha}{r}, \alpha = \frac{e^2}{4\pi}$$

This attractive potential is responsible for the binding of atoms.

The theory here is called quantum electrodynamics (QED).

$$V_{ij}^{q\bar{q}} = -\frac{4}{3}\alpha_s \frac{1}{r}, \alpha_s = \frac{g_s^2}{4\pi},$$

for $\bar{q}q$ color singlet system (mesons) while for qqq color singlet system (baryons).

$$V_{ij}^{qq} = -\frac{2}{3}\alpha_s \frac{1}{r}$$

Note the very important fact that in both cases, we get an attractive potential. Without color, V_{ij}^{qq} would have been repulsive.

The theory here is called quantum chromodynamics (QCD).

Due to quantum (radiative) corrections, $\alpha(\sqrt{Q^2})$ increases with increasing momentum transfer Q^2 , for example

$$\alpha(m_e) \approx \frac{1}{137},$$
$$\alpha(m_W) \approx \frac{1}{128}$$

Due to quantum (radiative) corrections, $\alpha(\sqrt{Q^2})$ decreases with increasing Q^2 [this is brought about by the self interaction of gluons (cf. Table 1)], for example

$$\alpha(m_\tau) \approx 0.35,$$
$$\alpha_s(m_\Upsilon \simeq 10 \text{ GeV}) \approx 0.16,$$
$$\alpha_s(m_Z) \approx 0.125.$$

That the effective coupling constant decreases at short distances is called the asymptotic freedom property of QCD.

Evidence for Color

As we have discussed in the introduction in order that 3 quark wave function of lowest lying baryons satisfy the Pauli principle, each quark flavor carries three color charges, red (r), yellow (y) and blue (b) i.e.

$$q_a \quad a = r, y, b.$$

Leptons do not carry color and that is the reason why they do not experience strong interactions. Thus each quark belongs to a triplet representation of color $SU(3)$, which we write as $SU_C(3)$. Now $SU(3)$ has the remarkable property that $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$ and $\bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{1}$, so that baryons which are bound states of 3 quarks belong to the singlet representation, which is totally antisymmetric as required by the Pauli principle and mesons which are bound states of $q\bar{q}$ belong to the singlet representation which is totally symmetric. This assignment takes into account the fact that all known hadrons are color singlets. Thus the color is hidden. This is the postulate of color confinement and explains the non-existence of free quarks.

Evidence for color also comes from $\pi^0 \rightarrow 2\gamma$ decay. Since π^0 is bound state of $q\bar{q}$ i.e. $|\pi^0\rangle = \frac{1}{\sqrt{2}} |u\bar{u} - d\bar{d}\rangle$, one can imagine that the decay takes place as shown in Fig. 1. The matrix elements M for the π^0 - decay, without and with color [where we have to sum over the 3 colors for the quarks in the above diagrams] are respectively proportional to

$$M \propto \frac{1}{\sqrt{2}} \left[\left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2 \right] e^2 = \frac{1}{\sqrt{2} \cdot 3} e^2$$

$$M \propto \frac{1}{\sqrt{2}} \left[\left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2 \right] 3e^2 = \frac{1}{\sqrt{2}} e^2.$$

In fact the above quark triangle diagrams predict

$$M = e^2 F = \frac{e^2}{2\pi^2} \frac{S_\pi}{f_\pi}$$

where

$$S_\pi = \begin{cases} \frac{1}{3\sqrt{2}} & \text{without color} \\ \frac{1}{\sqrt{2}} & \text{with color} \end{cases}$$

and f_π is the pion decay constant and is determined from the decay $\pi^+ \rightarrow \mu^+ + \nu_e$ its value is 132 MeV. Hence the decay rate is given by

$$\begin{aligned} \Gamma(\pi^0 \rightarrow 2\gamma) &= 4\pi\alpha^2 |F|^2 \frac{m_{\pi^0}^3}{16} \\ &= \frac{\alpha^2}{16\pi^3 f_\pi^2} S_\pi^2 m_{\pi^0}^3. \end{aligned}$$

With $S_\pi = \frac{1}{\sqrt{2}}$, this gives $\Gamma(\pi^0 \rightarrow 2\gamma) = 7.58$ eV in very good agreement with the experimental value $\Gamma_{exp} = 7.74 \pm 0.50$. Without color Γ_{th} will be a factor of 9 less in complete disagreement with the experimental value.

Another evidence for color comes from measuring the ratio of e^-e^+ annihilation processes

$$R = \frac{\sigma(e^-e^+ \rightarrow \text{hadrons})}{\sigma(e^-e^+ \rightarrow \mu^-\mu^+)}$$

in the large center of mass energy $\sqrt{s} = \sqrt{(p_1 + p_2)^2}$ limit, where p_1 and p_2 are the momenta of e^- and e^+ respectively.

The Mass Spectrum

The one gluon exchange potential is obtained by summing over all possible quark indices in V_{ij}^G in a multiquark system like $q\bar{q}$ and qqq . Thus

$$\begin{aligned} V^G &= \frac{1}{2} \sum_{i \neq j} V_{ij}^G \\ &= \frac{1}{2} \left[\sum_{i > j} V_{ij}^G + \sum_{i < j} V_{ij}^G \right] \\ &= \frac{1}{2} \left[\sum_{i > j} (V_{ij}^G + V_{ji}^G) \right] \\ &= \sum_{i > j} V_{ij}^G. \end{aligned}$$

Now our Hamiltonian, including the rest masses of the quarks can be written as

$$H(\mathbf{r}) = \sum_i m_i + \sum_i \frac{\hat{\mathbf{p}}_i^2}{2m_i} + V_C(r) + V_G(r),$$

where

$$\hat{\mathbf{p}}_i^2 = -\hbar^2 \nabla_i^2.$$

Here $V_C(r)$ is the confining potential, $V_G(r)$ is the one gluon exchange potential, i is the quark flavor index, i.e. $i = u, d, s$ for ordinary hadrons.

For s -wave, we write the space function as $\Psi_s(\mathbf{r})$.
Let us first take the expectation value of $H(\mathbf{r})$ with respect to $\Psi_s(\mathbf{r})$,

$$\begin{aligned}
 M &\equiv \langle \Psi_s | H | \Psi_s \rangle \\
 &= \sum_i m_i + \sum_i \frac{a}{m_i} + A_0 \\
 &\quad + k_s \alpha_s \left[b - \frac{c}{m_i m_j} - d \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16 \mathbf{s}_i \cdot \mathbf{s}_j}{3 m_i m_j} \right) \right],
 \end{aligned}$$

the mass operator for S -wave mesons can be written as

$$\begin{aligned}
 M &= M_0 + m_1 + m_2 + a \left[\frac{1}{m_1} + \frac{1}{m_2} \right] + \frac{\bar{c}}{m_1 m_2} \\
 &\quad + \bar{d} \left[\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16 \mathbf{s}_1 \cdot \mathbf{s}_2}{3 m_1 m_2} \right],
 \end{aligned}$$

where

$$M_0 = A_0 + k_s \alpha_s b$$

$$\bar{c} = -k_s \alpha_s c$$

$$\bar{d} = -k_s \alpha_s d.$$

Indices 1 and 2 refer to the constituent antiquark and quark respectively. For vector gluon $k_s = -\frac{4}{3}$. Now

$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \begin{cases} \frac{1}{4} & \text{spin triplet state } S = 1: \text{ vector meson} \\ -\frac{3}{4} & \text{spin singlet state } S = 0: \text{ pseudoscalar meson.} \end{cases}$$

Thus if $k_s = -\frac{4}{3}$, as for vector gluons, it is clear that

$$m(^3S_1) > m(^1S_0)$$

i.e. vector meson mass is greater than the corresponding pseudoscalar mass in agreement with experimental observation. If gluons were scalar particles, then $\mathbf{s}_1 \cdot \mathbf{s}_2$ term would be absent so that $m(^3S_1) = m(^1S_0)$ in disagreement with the experimental observation. For pseudoscalar gluons, $k_s = \frac{4}{3}$, since pseudoscalar coupling is the same for antiquarks. In this case we would have $m(^3S_1) < m(^1S_0)$, again in disagreement with the experimental result. We conclude that the experimental results about meson spectrum support the fact that gluons are vector particles and are thus quanta of QCD.

In order to discuss the mass spectrum of the baryons, it is convenient to first calculate the matrix elements of the spin operator

$$\Omega_{ss} = \sum_{i>j} \frac{1}{m_i m_j} \mathbf{s}_i \cdot \mathbf{s}_j$$

between spin states. The eigenvalues of $\mathbf{s}_i \cdot \mathbf{s}_j$ are $1/4$ and $-3/4$ for spin triplet and singlet states respectively.

we get

$$\begin{aligned} \Omega_{ss} |\Lambda\rangle &= -\frac{3}{4m_u^2} |\Lambda\rangle \\ \Omega_{ss} |\Sigma^0\rangle &= \frac{1}{4} \left(\frac{1}{m_u^2} - \frac{4}{m_u m_s} \right) |\Sigma^0\rangle \\ \Omega_{ss} |\Xi^0\rangle &= \frac{1}{4} \left(\frac{1}{m_u^2} - \frac{4}{m_u m_s} \right) |\Xi^0\rangle, \end{aligned}$$

where we have used

$$\begin{aligned} |\Lambda\rangle &= -|uds\rangle \chi_{MS}^{1/2} \\ |\Sigma^0\rangle &= |uds\rangle \chi_{MS}^{1/2} \\ |\Xi^0\rangle &= |ssu\rangle \chi_{MS}^{1/2}. \end{aligned}$$

$$\Omega_{ss} |\Sigma^{*+}\rangle = \frac{1}{4} \left(\frac{2}{m_u m_s} + \frac{1}{m_u^2} \right) |\Sigma^{*+}\rangle$$

Similarly we get

$$\Omega_{ss} |\Xi^{*0}\rangle = \frac{1}{4} \left(\frac{2}{m_u m_s} + \frac{1}{m_s^2} \right) |\Xi^{*0}\rangle$$

$$\Omega_{ss} |\Omega^-\rangle = \frac{3}{4} \frac{1}{m_s^2} |\Omega^-\rangle,$$

where we have used

$$\begin{aligned} |\Sigma^{*+}\rangle &= |uus\rangle |\uparrow\uparrow\uparrow\rangle \\ |\Xi^{*0}\rangle &= |ssu\rangle |\uparrow\uparrow\uparrow\rangle \\ |\Omega^-\rangle &= |sss\rangle |\uparrow\uparrow\uparrow\rangle. \end{aligned}$$

Since the spin-spin interaction term is, $\frac{16}{3} \left(-\frac{2}{3} \alpha_s \right) (-d) \Omega_{ss}$,

we have:

$$m \left(J = \frac{3}{2} \right) > m \left(J = \frac{1}{2} \right)$$

in agreement with experimental observations. For gluons with color, $k_s = -2/3$; if gluons do not carry color, then $k_s = 1$ instead of $-2/3$ and we would get results in contradiction with experimental values. This supports that the vector gluons carry color.

Heavy Flavours

Charm Quark

The J/Ψ was discovered in 1974 in the reaction

$$p + Be \longrightarrow e^+e^- + X$$

at $\sqrt{s} = 7.6$ GeV. A narrow peak at $m(e^+e^-) = 3.1$ GeV was found. It was also seen in e^+e^- collision at $\sqrt{s} = 3.105$ GeV in the following reactions

$$e^-e^+ \longrightarrow e^-e^+$$

$$e^-e^+ \longrightarrow \mu^-\mu^+$$

$$e^-e^+ \longrightarrow \text{hadrons.}$$

The width of the resonance was very narrow. It was less than the energy spread of the beam, $\Gamma \leq 3$ MeV. For this reason, the width cannot be read off directly from resonance curve. The resonant cross section for any final state f :

$$e^-e^+ \longrightarrow J/\Psi \longrightarrow f$$

is given by the Breit-Wigner formula:

$$\sigma_{ef} = \frac{\pi}{k^2} \frac{2J + 1}{(2s_1 + 1)(2s_2 + 1)} \frac{\Gamma_e \Gamma_f}{(\sqrt{s} - m)^2 + \frac{\Gamma^2}{4}}$$

where J is the spin of the resonance, m is its mass, $s_1 = s_2 = 1/2$ is the spin of electron or positron and

$$s = E_{cm}^2 = 4(k^2 + m_e^2) \approx 4k^2.$$

Here $k = |\mathbf{k}|$ is the center of mass momentum. Γ is the total width, Γ_e and Γ_f are the partial widths into e^-e^+ and f respectively.

The experimental values for these decay widths are given below:

$$\begin{aligned} m(J/\Psi) &= 3096.88 \pm 0.04 \text{ MeV}, \\ \Gamma_e &= \Gamma_\mu = 5.26 \pm 0.37 \text{ keV}, \\ \Gamma &= 87 \pm 5 \text{ keV}. \end{aligned}$$

The J/Ψ spin-parity can be determined from a study of the interference between $e^-e^+ \longrightarrow \gamma \longrightarrow \mu^-\mu^+$ and $e^-e^+ \longrightarrow \Psi \longrightarrow \mu^-\mu^+$

The cross section for the QED process $e^-e^+ \longrightarrow \gamma \longrightarrow \mu^-\mu^+$ is well known and is given by ($s \gg m_e^2, m_\mu^2$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2 \theta)$$
$$\sigma = \frac{4\pi\alpha^2}{s}$$

If the spin-parity of J/Ψ is that of photon viz. 1^- , then the angular distribution would not change by the interference between QED amplitude and the resonant amplitude. In fact, experimentally, it was found to be $(1 + \cos^2 \theta)$ near the resonance, clearly establishing the spin-parity of J/Ψ to be 1^- .

Although J/Ψ itself does not carry any new quantum number, its unusually narrow width in spite of large available phase space suggests that it is a bound state of $c\bar{c}$, where c is a quark with a flavor which is outside the three flavors u, d and s of $SU(3)$. This new flavor is called charm. The quark c is assigned a new quantum number $C = 1$ and $C = 0$ for u, d and s quarks. Thus to take this quantum number into account, the Gell-Mann-Nishijima relation would be modified to

$$Q = I_3 + \frac{1}{2}(Y + C).$$

For the charmed quark c , $C = 1$, $I_3 = 0$, $Y = B = 1/3$. Thus the charge of charmed quark is $2/3$ and its mass $m_c \approx \frac{1}{2}m_{J/\Psi} = 1.55$ GeV.

The narrow width of J/Ψ (87 keV compared to 100 MeV for ρ) can be qualitatively understood by the OZI rule, just as the suppression of $\phi \rightarrow 3\pi$ compared to $\phi \rightarrow K\bar{K}$ is explained by this rule.

Thus the decay depicted in Fig. 1 is allowed but that shown in Fig. 2 is suppressed by *OZI* rule. But the decay $J/\psi \rightarrow D\bar{D}$ shown in Fig. 1 is not allowed energetically since $m_{J/\psi} < 2m_D$.

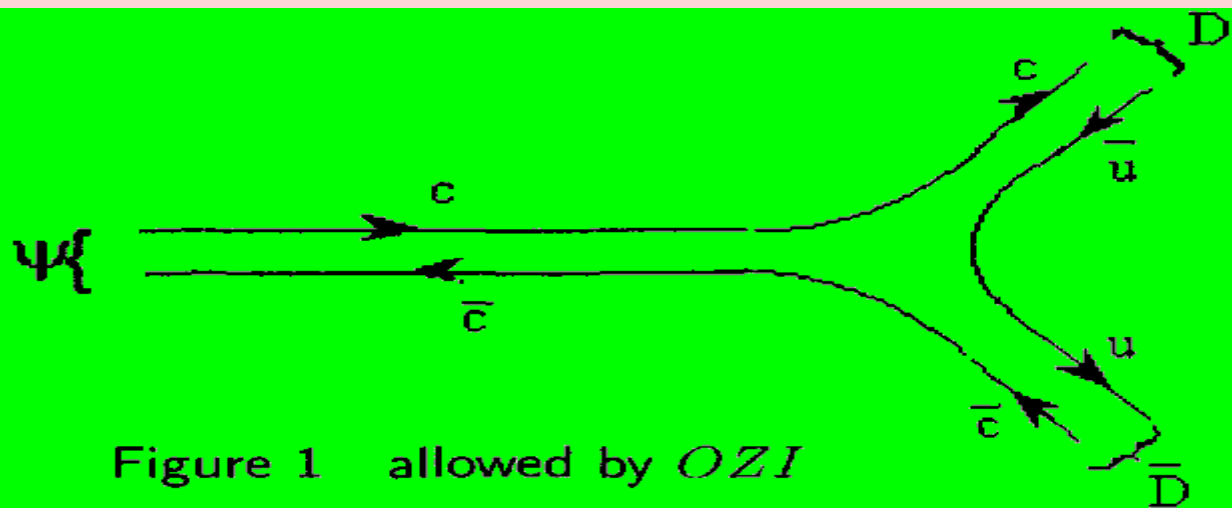


Figure 1 allowed by *OZI*

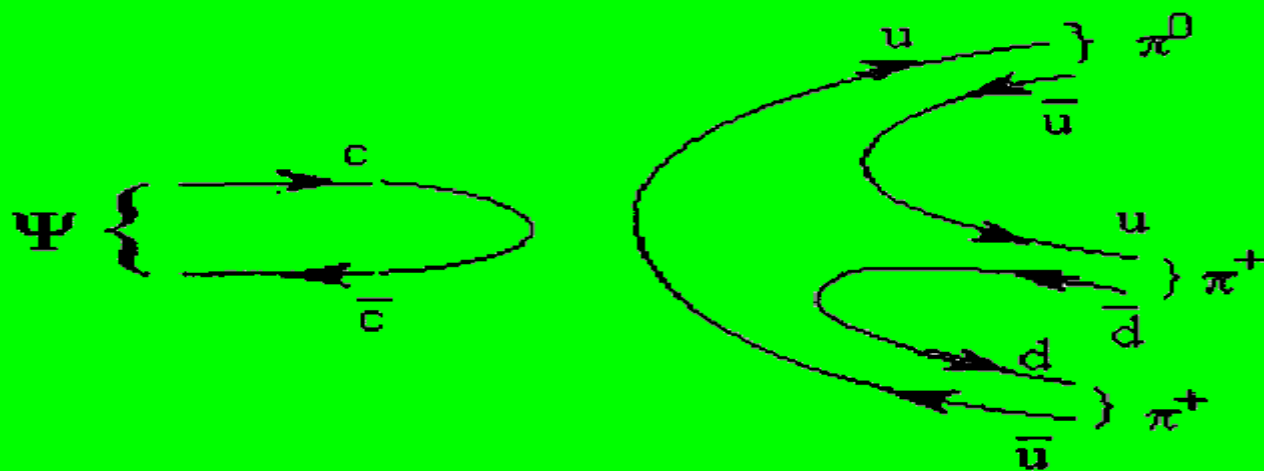


Figure 2 J/ψ suppressed by *OZI* rule.

Charmed Mesons

The charmed quark c can form bound states with \bar{q} , where $q = u, d, s$. The low lying bound states such as $c\bar{q}$ have been found experimentally.

The $C = 1$ states $(D^+, D^0) D_s^+$ form an SU(3) triplet ($\bar{\mathbf{3}}$); (D^+, D^0) form an isospin doublet. Similarly $C = -1$ states $q\bar{c}$: $(\bar{D}^0, D^-) D_s^-$ form an SU(3) triplet ($\mathbf{3}$).

The states D^* and D_2^* are unstable and decay strongly and radiatively. For example

$$\begin{aligned} D^* &\rightarrow D\pi \\ &\rightarrow D\gamma \\ D_2^* &\rightarrow D\pi, D^*\pi \\ &\rightarrow D\gamma \end{aligned}$$

Bottom Quark

Fifth quark was discovered, when in 1977 the upsilon meson $\Upsilon(J^{PC} = 1^{--})$ was found experimentally as a narrow resonance at Fermi Lab. with mass ~ 9.5 GeV. This was later confirmed in e^+e^- experiments at DESY and CERN which determined its mass to be 9460 ± 10 MeV and also its width. The updated parameters of this resonance [from the Particle Data Group Tables] are mass 9460.37 ± 0.21 MeV and width 52.5 ± 1.8 keV. Again the narrow width in spite of large phase space available suggests the existence of a fifth quark flavor called beauty, with a new quantum number $B = -1$ for the bottom (b) quark. With this assignment the formula $Q = I_3 + 1/2(Y + B + C)$ would give the charge of b quark the value $-1/3(I_3 = 0)$. The mass of b quark is expected to be around 4.9 GeV as suggested by the Υ mass which is regarded as a 3S_1 bound state of $b\bar{b}$.

Thus one would expect particles with $B = \pm 1$, such as $b\bar{q}$ or $q\bar{b}$. The lowest lying bound states $b\bar{q}$ and $q\bar{b}$ have been found experimentally. The $B = -1$ states $(\bar{B}^0, B^-)\bar{B}_s^0$ form an SU(3) triplet ($\bar{\mathbf{3}}$) and $B = +1$ states $(B^+, B^0)B_s^0$ form another triplet ($\mathbf{3}$).

Top Quark

The top quark t with $Q = 2/3$ and new flavor $T = 1$ was expected on theoretical grounds. It was first found experimentally in 1996; its mass is $m_t = 175 \pm 6$ GeV. Since (t, b) form a weak doublet, it decays weakly to $W^+ + b$, i.e.

$$t \rightarrow W^+ + b.$$

The predicted decay rate is

$$\Gamma(t \rightarrow W^+ + b) = \frac{G_F}{8\pi\sqrt{2}} m_t^3 \left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + 2\frac{m_W^2}{m_t^2}\right)$$

where we have neglected the b quark mass compared to m_W and m_t . Taking $m_t = 175$ GeV, $m_W = 80$ GeV, $G_F = 1.166 \times 10^{-5}$ GeV⁻², we get

$$\Gamma \approx 1.56 \text{ GeV}.$$

If QCD correction is taken into account, then

$$\Gamma \approx 1.43 \text{ GeV}$$

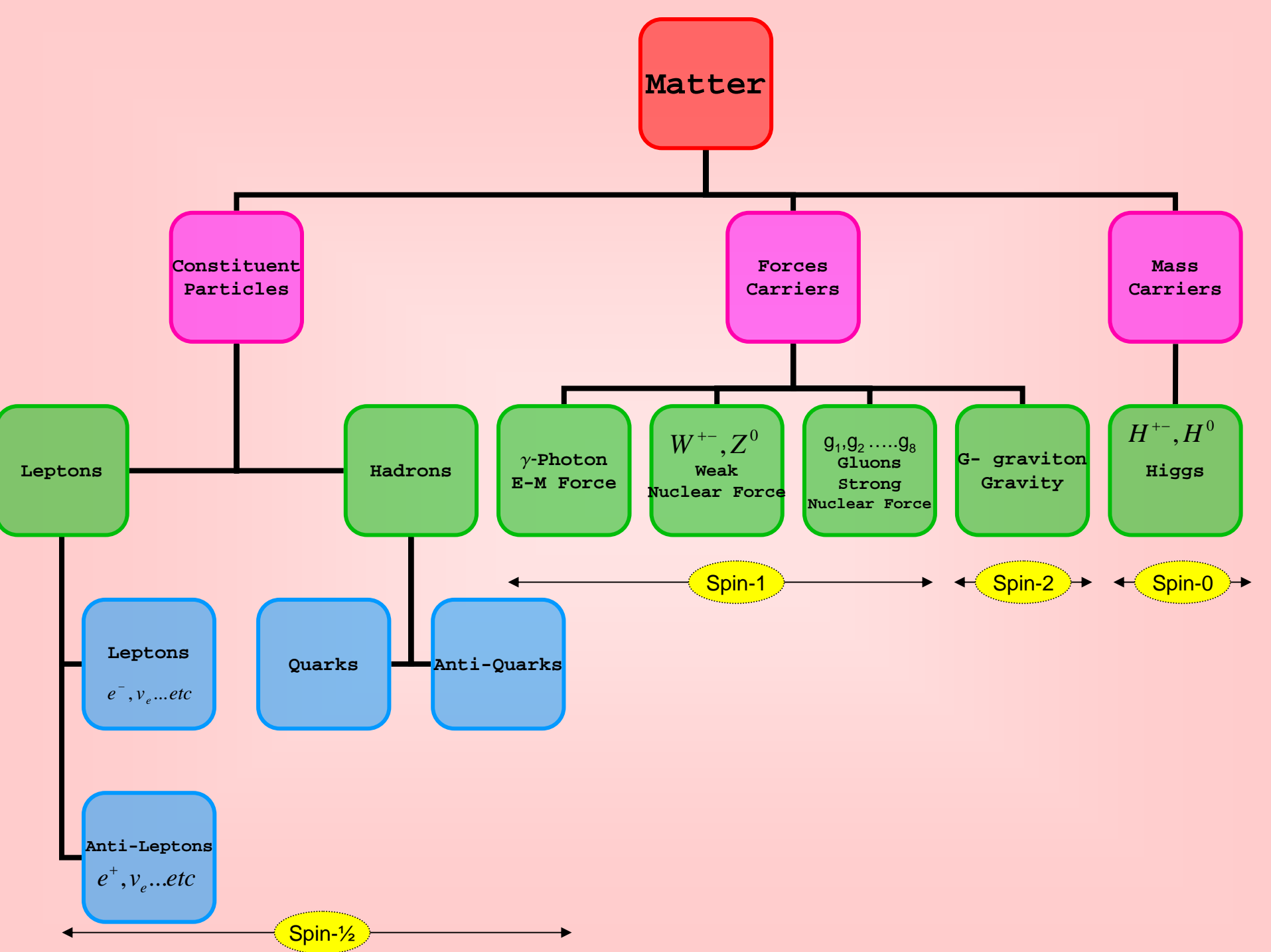
which gives the life time of τ to be

$$\tau = 4.60 \times 10^{-25} \text{ s}.$$

Thus t quark decays before it can form bound states such as $t\bar{t}$ and $t\bar{q}$.

Summery

- Production
- Characterization, Mass, Spin, Parity, Isospin, Hypercharge, G-Parity.
- Classification
- Families
- Flavors $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} e \\ \nu_e \end{pmatrix}$ $\begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$ $\begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$
- Color
- Standard Model Particles
- Gauge Particles $\gamma, W^{+-}, Z, 8$ mass less
Gluons. $SU_L(2) \times U(1) \times SU_c(3)$
- Mass Particles, Higgs



Thanks