Pion – A Closer Look

NJL Model – A Closer Look

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Facts and Challenges

- Dynamical mass generation for massless quarks; (dynamical chiral symmetry breaking). No degeneracy between $J^{P=+}$ and $J^{P=-}$.

- Color degrees … l gluons) are not observable (co…

$$N(\tfrac{1}{2}^+, 938) = N(\tfrac{1}{2}^-, 1535),$$
$$\pi(0^-, 140) = \sigma(0^+, 600),$$
$$\rho(1^-, 770) = a_1(1^+, 1260)$$

- Strong interac… ude of bound states whose complete quantitative understanding continues to be a challenge.

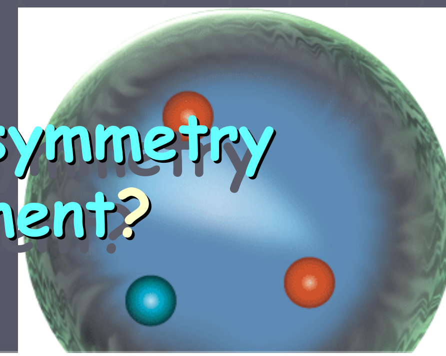
- Studying QCD: lattice, Schwinger-Dyson and Bethe-Salpeter equations. Then there are effective models such as the NJL model.

Facts and Challenges

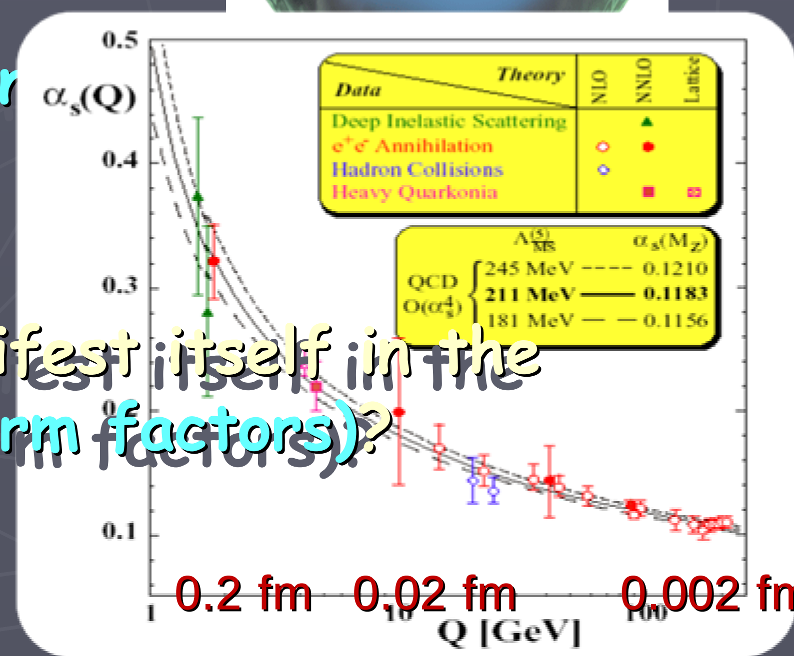
- What is the connection between the current quark mass and the constituent quark mass.

$m_{current} \sim 5$ MeV $\to m_{constituent} \sim 200$ MeV

- What is the relation between chiral symmetry breaking and the light quark confinement?



- How does the QCD coupling con infrared domain?

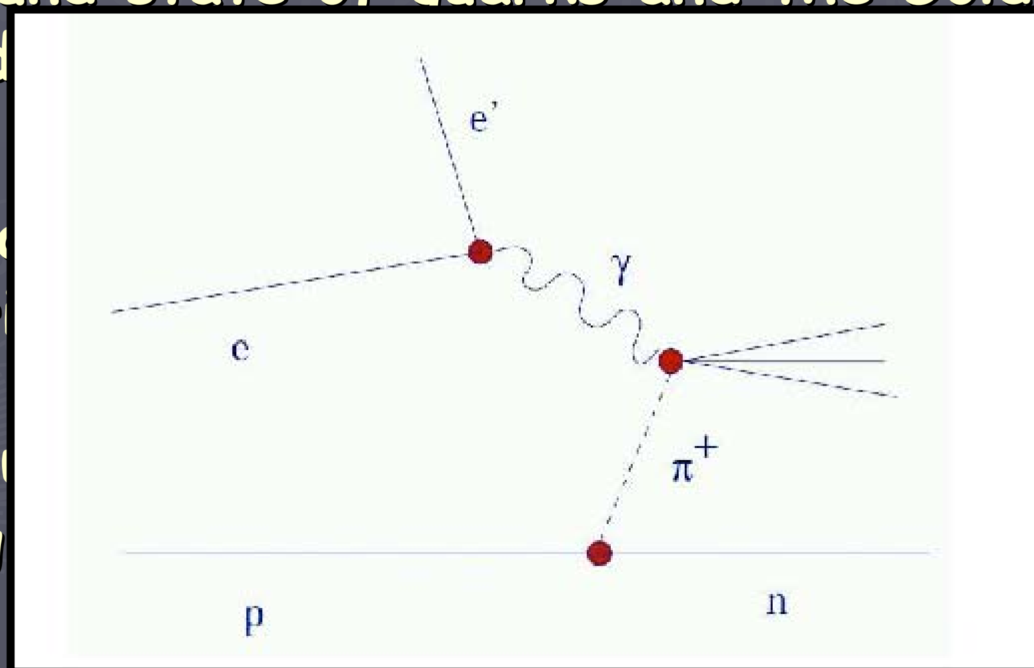


- How does this connection manifest itself in the hadronic observables (e.g., form factors)?

Facts and Challenges

- At higher values, it is measured through high-energy electro-production of pions on a nucleon.

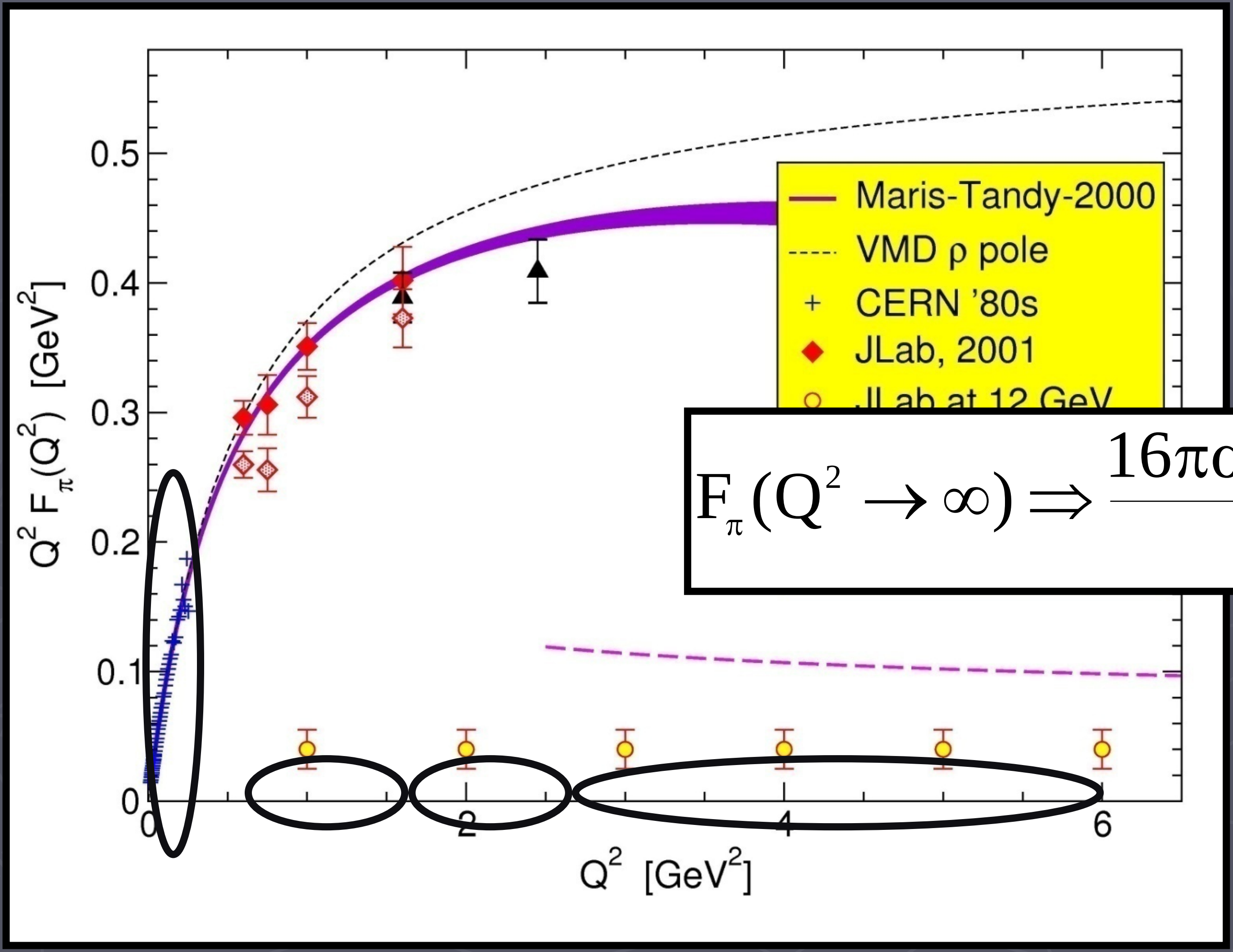
- It is a bound state of quarks and the Goldstone mode associated ... breaking.



- Its electr... much studied and experi...

- At low val... hrough the scattering...

Facts and Challenges



$$F_\pi(Q^2 \to \infty) \Rightarrow \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

1980's 2001 2006

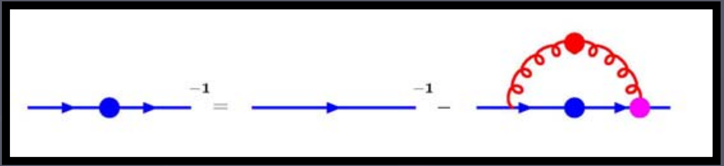
2013?

NJL Model Description of the Quark

- U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27 195 (1991).
- T. Hatsuda and T. Kunihiro, Phys. Rept. 247 221 (1994).
- I.C. Cloet, W. Bentz and A. W. Thomas, Phys. Lett. B642 210 (2006); Phys. Rev. Lett. 95 052302 (2005).
- H. Mineo, W. Bentz, N. Ishi, A.W. Thomas and K. Yazaki, Nucl. Phys. A735 482 (2004).
- I.C. Cloet, W. Bentz and A. W. Thomas, Phys. Lett. B621 246 (2005).

NJL Model Description of the Quark

The SDE for the quark propagator in Euclidean space



$$S^{-1}(p) = Z_2(i\gamma\cdot p + m) + \Sigma(p)$$

where

$$\Sigma(p) = Z_1 \int^{\Lambda} g^2 D_{\mu\nu}(p-q)\, \frac{\lambda^a}{2}\, \gamma_\mu\, S(q)\, \Gamma^a_\nu(q,p)$$

A simple *ansatz* :

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu}\, \frac{1}{m_G^2}$$

$$\Gamma^a_\nu(p,q) = \frac{\lambda^a}{2}\, \gamma_\nu$$

yields NJL model gap equation:

$$S^{-1}(p) = (i\gamma\cdot p + m) + \frac{1}{3\pi^2 m_G^2} \int_q^{\Lambda} \gamma_\mu\, S(q)\, \gamma_\mu$$

NJL Model Description of the Quark

Define

$$S(p) = -i\gamma\cdot p\,\sigma_V(p^2) + \sigma_S(p^2) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

The gap equation gives the solution: $Z(p) = 1$

And constituent mass:

$$M = m + \frac{M}{3\pi^2 m_G^2}\int_0^{\Lambda^2} ds\,\frac{s}{s+M^2}$$

In chiral limit

M

$$M = \frac{M}{3\pi^2 m_g^2}\left[\Lambda^2 - M^2 \mathrm{Log}\left(1+\frac{\Lambda^2}{M^2}\right)\right]$$

Thus for m … only

solution ac…

$$M(p^2) = m\left(1 - \frac{\alpha}{\pi}\,\mathrm{Log}\left[\frac{p^2}{m^2}\right] + \cdots\right)$$

NJL Model Description of the Quark

For

$M \neq 0$

$$1 = \frac{1}{3\pi^2 m_g^2}\left[\Lambda^2 - M^2 \mathrm{Log}\left(1 + \frac{\Lambda^2}{M^2}\right)\right] \equiv \frac{1}{3\pi^2 m_g^2}\,\mathcal{C}(M,\Lambda)$$

Let Λ=1

$$1 = \frac{1}{3\pi^2 m_g^2}\left[1 - M^2 \mathrm{Log}\left(1 + \frac{1}{M^2}\right)\right] = \frac{1}{3\pi^2 m_g^2}\,\mathcal{C}(M,1)$$

- The maximum value of C(M,1) is 1.
- It is a monotonically decreasing function of M

Chirally asymmetric solution exists only when

S $m_g^2 < (0.2\mathrm{GeV})^2$ le

interaction strength is proprtional to $1/m_g^2$.

Λ^2

NJL Model Description of the Quark

$$M = m + \frac{M}{3\pi^2 m_g^2}\left[\Lambda^2 - M^2 \mathrm{Log}\left(1 + \frac{\Lambda^2}{M^2}\right)\right]$$

Weak coupling

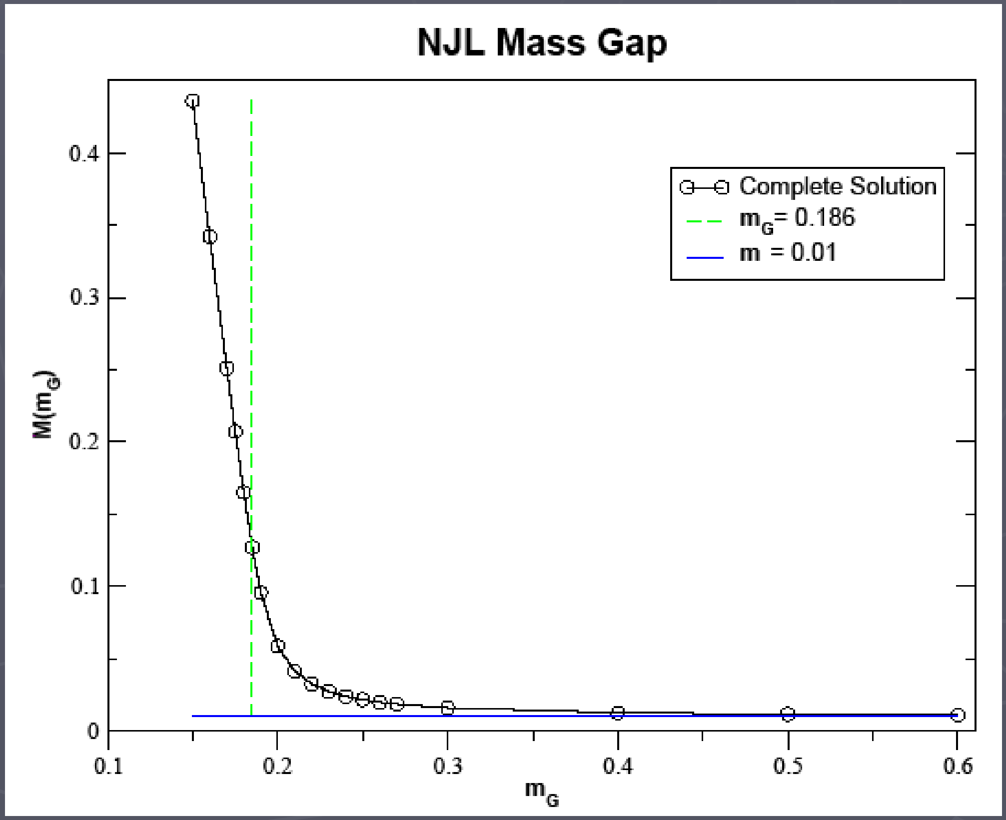
➡ Large m_G

➡ $M \sim m$

Strong coupling

➡ Small m_G

➡ $M >> m$



NJL Model Description of the Quark

Thus NJL model describes DCSB like QCD.

Does it exhibit confinement?

The fully dressed NJL quark propagator is:

$$S^{\mathrm{NJL}}(p) \;=\; \frac{[Z(p^2)=1]}{i\gamma\cdot p+[M(p^2)=M]}$$

It corresponds to a free particle with a shifted mass.
Thus it does not exhibit confinement.

NJL Model Description of the Quark

We can improve on the NJL model to make it quark confining.

We can use proper time regularization which guarantees confinement and is backed by hadron phenomenology.

$$\frac{1}{s+M^2} \quad \int^{r_{IR}^2=1/\Lambda_{IR}^2} \cdots \quad \frac{Z(s)}{s+M^2}$$

$$\frac{1}{X^n} \rightarrow \frac{1}{(n-1)!} \int_{r_{UV}^2}^{r_{IR}^2} d\tau \, \tau^{n-1} \, \mathrm{e}^{-\tau X}$$

with

$$Z(s) = \mathrm{e}^{-(s+M^2) r_{UV}^2} - \mathrm{e}^{-(s+M^2) r_{IR}^2}$$

NJL Model Description of the Quark

With proper time regularization NJL Model incorporates both DCSB and confinement.

It has 4 free parameters: m, Λ_{UV}, Λ_{IR}, m_G.

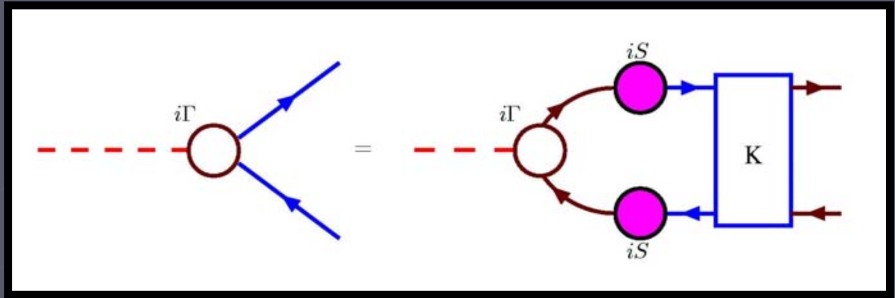
If we take following phenomenology based values for these parameters:

$$m = 0,\ \Lambda_{IR} = 0.24\text{GeV},\ \Lambda_{UV} = 0.823\text{GeV},\ m_g = 0.11\text{GeV}$$

Constituent Mass — Chiral Condensate

$$M = 0.4\ \text{GeV} \qquad \text{and} \qquad <\bar{q}q> = -(0.22\ \text{GeV})^3$$

The Bethe-Salpeter Equation



$$i\Gamma_\pi(P) = \int d^4q\, iS(q+P)\, i\Gamma_\pi(P)\, iS(q)\, \mathcal{K}$$

In the NJL Model:

$$\Gamma_\pi(P) = \gamma_5\left[iE_\pi(P) + \frac{\gamma\cdot P}{M}F_\pi(P)\right]$$

$$\mathcal{N}^2 P_\mu = \frac{3}{4\pi^2}\mathrm{Tr}\int_q^\Lambda \Gamma_\pi(-P)\left[\frac{\partial S(q_+)}{\partial P_\mu}\Gamma_\pi(P)S(q_-) + S(q_+)\Gamma_\pi(P)\frac{\partial}{\partial P_\mu}S(q_-)\right]$$

$$\Gamma_\pi^C = \frac{1}{\mathcal{N}}\,\Gamma_\pi(P)$$

Axial Vector Ward Takahashi Identity

$$P_\mu \Gamma_{5\mu}(q_+, q) \quad = \quad S^{-1}(q_+) i\gamma_5 + i\gamma_5 S^{-1}(q)$$

Dressed quark propagator:

$$S(p) \quad = \quad \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

QCD GT-Relations:

$$f_\pi \; E_\pi(k; P = 0) \quad = \quad B(p^2)$$

$$H_R(k;0) + 2 f_\pi \; H_\pi(k;0) \quad = \quad 0$$

GT-Relations in NJL Model:

$$f_\pi \; E_\pi = M \qquad\qquad 2\,\frac{F_\pi}{E_\pi} + F_R = 1$$

Axial Vector Ward Takahashi Identity

WTI and GT relations are not valid in the NJL Model.

These facts become more apparent if we consider the pseudovector component of the Pion BS amplitude.

However, pseudovector components dominate ultraviolet behavior of the Pion electromagnetic form factor.

	f_π^{C}	f_π^{GT}	f_π^{WT}
F_π [illegible]	[illegible]	[illegible]	[illegible]
$F_\pi = 0$	0.0923286	0.0923286	0.0923286

Remedy is to regularize NJL Model as to ensure that there are no quadratic or logarithmic divergences.

[illegible]

WTI and GT relations are now satisfied exactly in the NJL model. It now preserves basic symmetries of QCD.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$

Consider an incoming pion with momentum p_1 that absorbs a photon with spacelike momentum q so that it departs the interaction region with momentum $p_2 = p_1 + q$. We work in the Breit frame, which means we choose $p_1 = K - q/2$.

In the chiral limit,

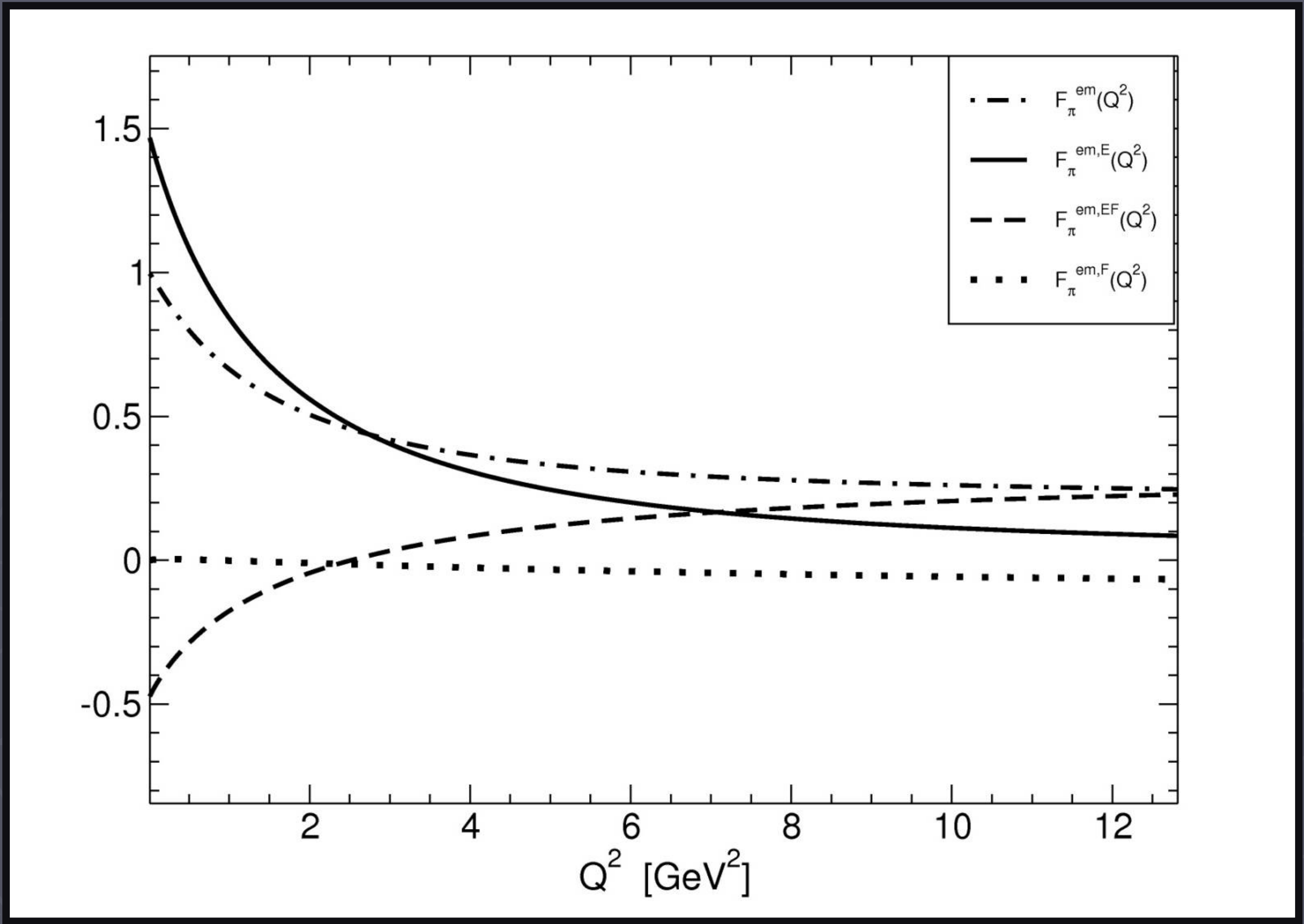
$$p_1^2 = (K - q/2)^2 = 0 = p_2^2 = (K + q/2)^2 \to K \cdot q = 0,\ K^2 = -q^2/4$$

so that we can choose

$$q = (0, 0, Q, 0),\ K = (0, 0, 0, iQ/2)$$

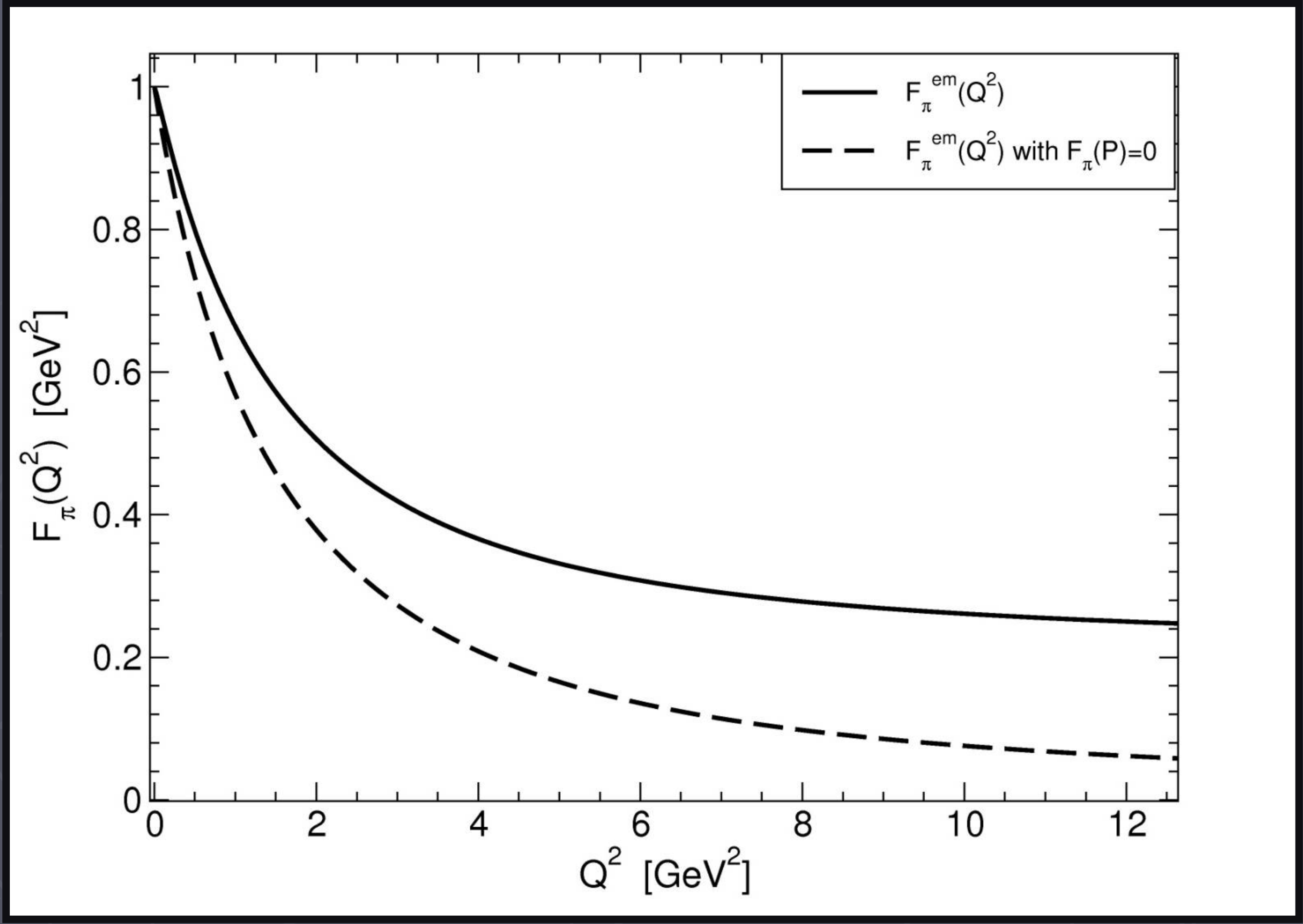
$$2K_\mu F_\pi^{\text{em}} = \frac{3}{2\pi^2} \int_t^\Lambda \text{Tr}_\text{D} \left[i\Gamma_\pi^C(-p_2)\, S(t + p_2)\, i\gamma_\mu\, S(t + p_1)\, i\Gamma_\pi^C(p_1)\, S(t) \right]$$

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



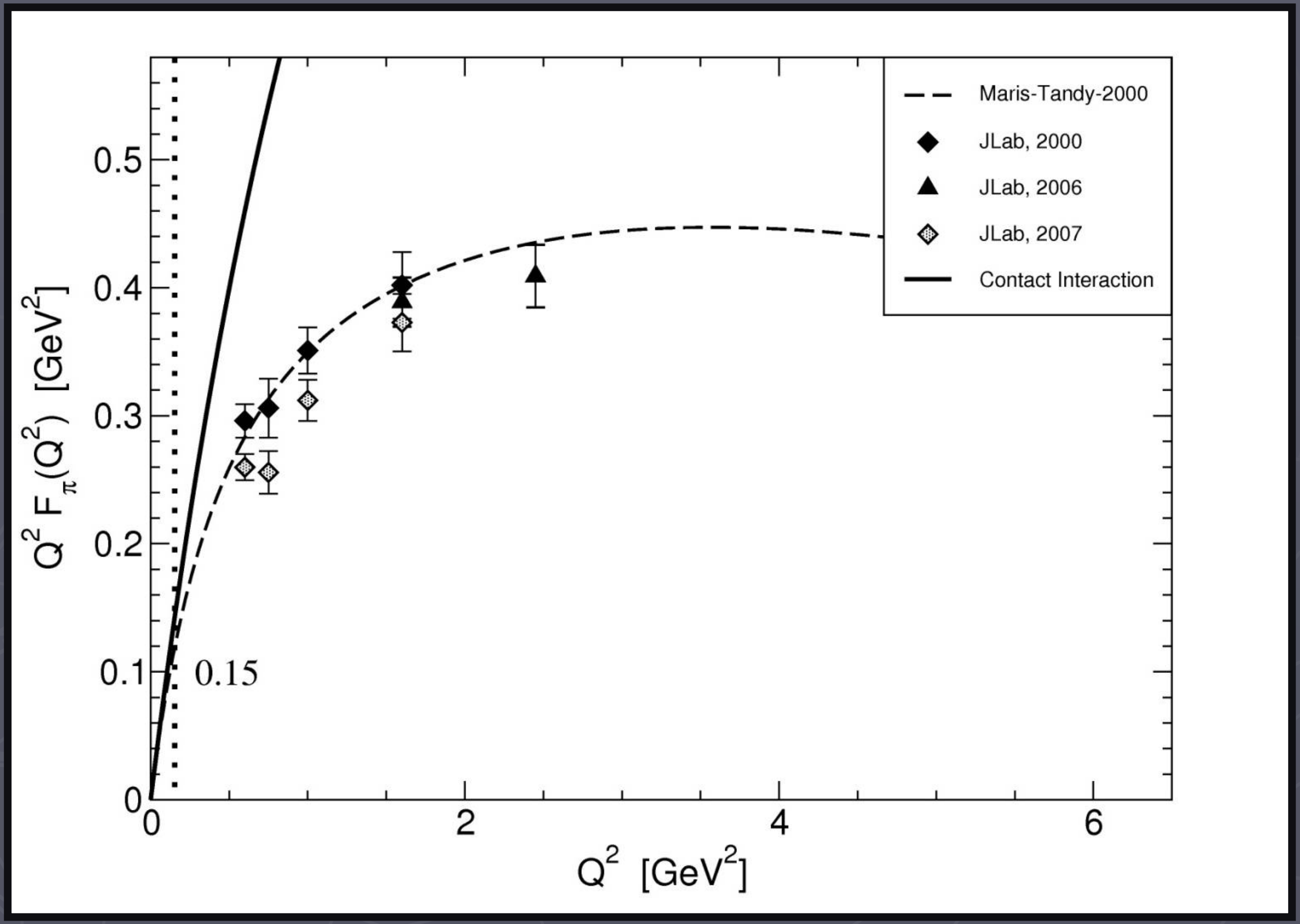
Interplay of pseudo scalar and pseudo vector components.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



With and without the pseudo vector component.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



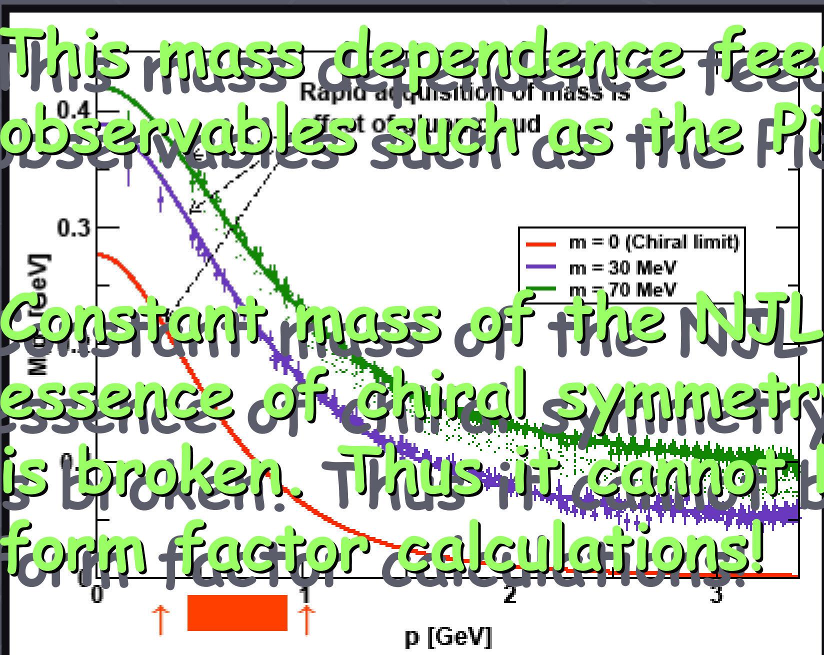
Comparison with other models and experiments.

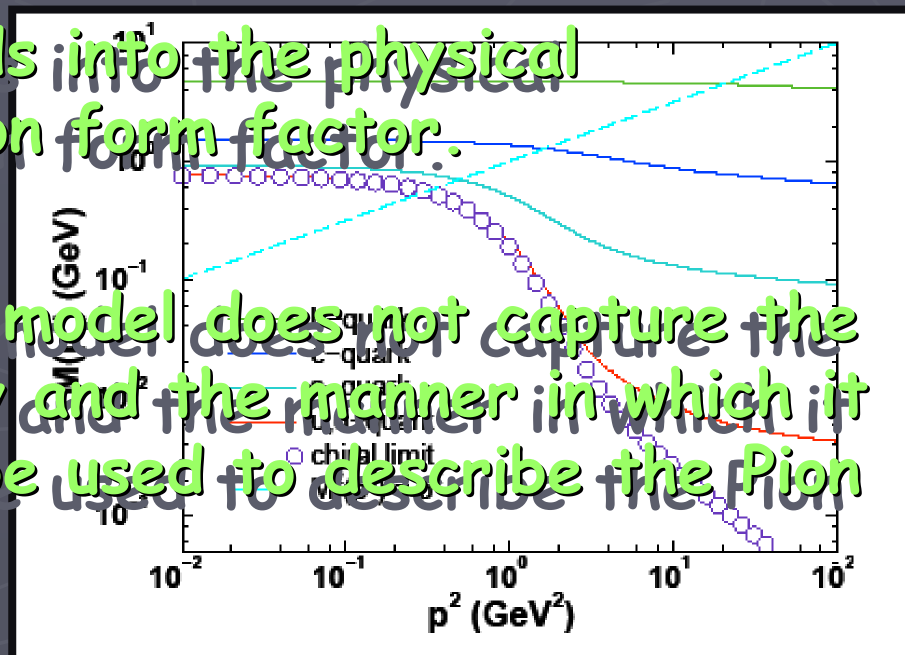
Conclusions

In real QCD, interaction is momentum dependent.

It implies that tha mass of a quark is a mass dependent function.

This mass dependence feeds into the physical observables such as the Pion form factor.





Constant mass of the NJL model does not capture the essence of chiral symmetry and the manner in which it is broken. Thus it cannot be used to describe the Pion form factor calculations!

Conclusions

- We study the electromagnetic pion form factor in an NJL model.
- An NJL model in which the pion has no pseudo vector component does not respect WTI for arbitrary values of P and its corollaries such as the GT-relations.
- One can implement proper regularization scheme and add the pseudo vector component of the pion to make NJL a consistent and a symmetry preserving model.
- We study the consequences of symmetry preserving NJL model in its role as an effective description of QCD at low Q^2 through a study of the pion form factor.