Many-Body Interactions in QCD and its models

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CHEP

PU

Guidelines for models of QCD…

- Two-body interaction in perturbative QCD
- QQbar potential
- *Sum* in tree-level Feynman diagrams, and in a model: Born diagrams (Van der Walls)
- *Product* of two-body interactions in Loops and multi-quark Wilson loops (Lattice Gauge Theory)
- Beyond Product terms…

Two-body interaction in Tree-level Perturbative QCD

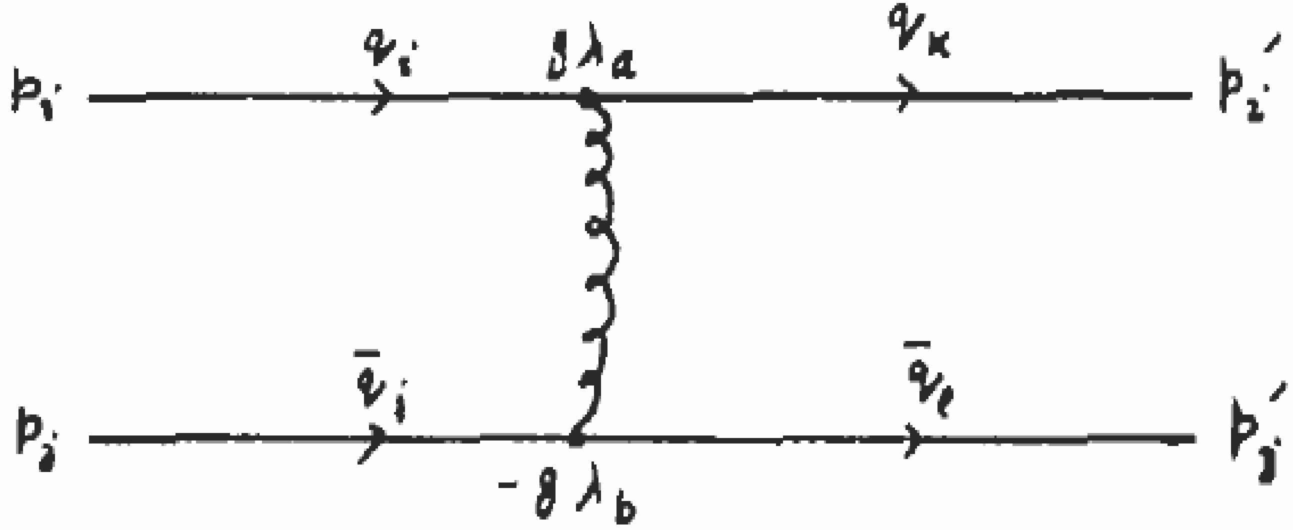


Fig. 2: Single gluon exchange between quark-antiquark pair.

http://prr.hec.gov.pk/chapters/2505-1.pdf

APPENDIX A: BUILDING THE EFFECTIVE $q\bar{q}$ POTENTIAL

$$\chi_s^\dagger \chi_{\bar{s}}^\dagger, V_{\rm eff}(\mathbf{P},\mathbf{r})\chi_s \chi_{\bar{s}} = \frac{1}{(2\pi)^3} \int d^3Q \, e^{i\mathbf{Q}\cdot\mathbf{r}} \overline{U}(\mathbf{p}',s')\overline{V}(-\mathbf{p},\bar{s}) I(Q^2) \times U(\mathbf{p},s)V(-\mathbf{p}',\bar{s}') \,, \tag{A1}$$

$$I(Q^2) = G(Q^2)(\gamma^\mu)_q(\gamma_\mu)_{\bar{q}} - S(Q^2)(1)_q(1)_{\bar{q}}$$

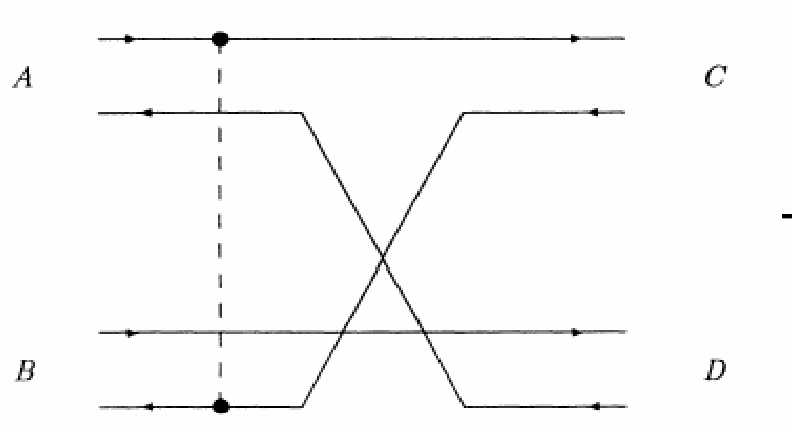
$$G(Q^2) = \frac{-4\alpha_s(Q^2)}{3}\frac{4\pi}{Q^2}$$

$$G(r) = -\frac{4\alpha_s(r)}{3r}$$

$$S(r) = br + c$$

S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189

Sum in Multi-quark "tree-level" Diagrams (and Born diagrams)



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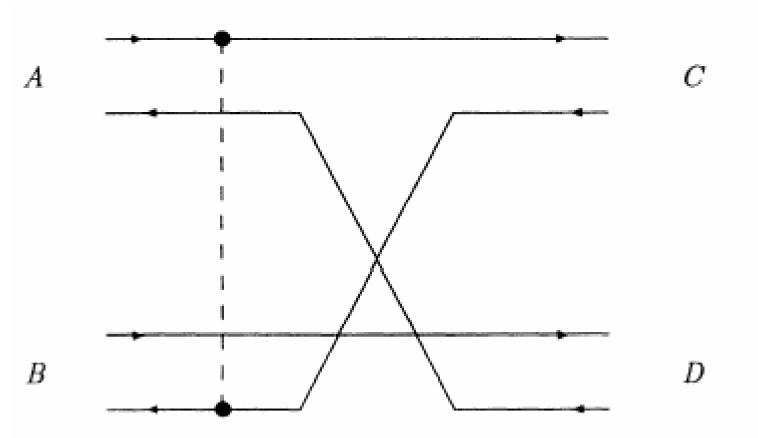

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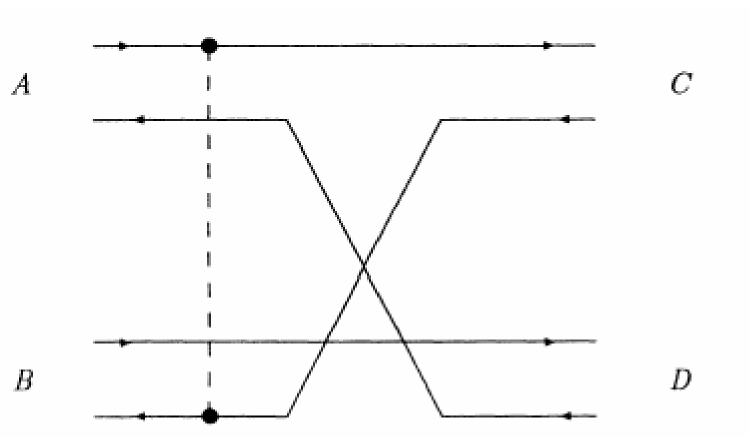

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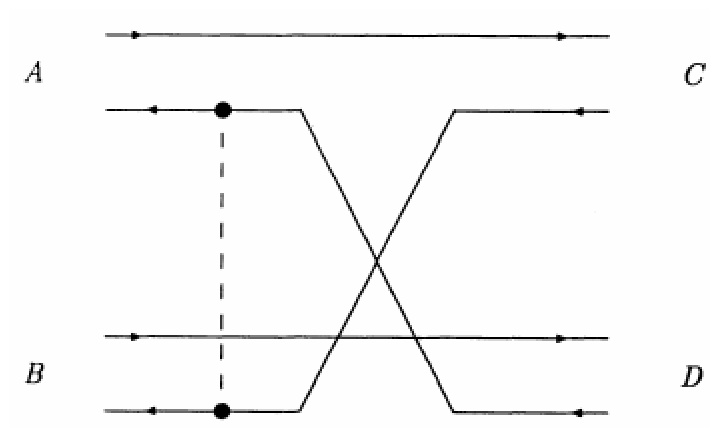


T. Barnes and E. Swanson, Phys. Rev. D46 (1992) 131

$Q\overline{Q}$ Potential replaces one gluon exchange in above, but *Sum.*

$$H = \sum_i -\frac{\hbar^2}{2m_i}\nabla_i^2 + \sum_{i<j} H_{ij} \ ,$$

$$H_{ij} = \frac{\lambda^a(i)}{2}\frac{\lambda^a(j)}{2}\left\{ + \frac{\alpha_s}{r_{ij}} - \frac{3a}{4} r_{ij} \right.$$

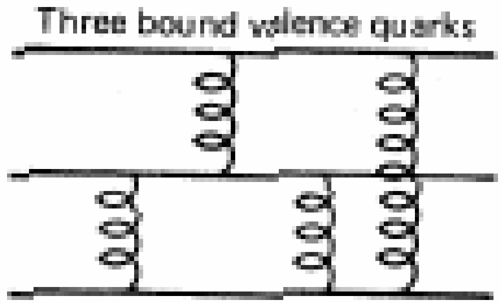
$$\left. - \frac{8\pi\alpha_s}{3m_i m_j}\mathbf{S}_i \cdot \mathbf{S}_j \ \delta(\mathbf{r}_{ij}) \right\} \ .$$

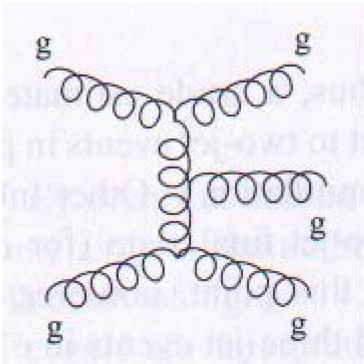
Faces a Van der Walls problem….

T. Barnes and E. Swanson, Phys. Rev. D46(1992) 131

But QCD is *not* limited to a sum..

- Product of 2-body interactions (or of gluon propagators) in higher order Feynman diagrams…





And QCD is not limited to a small coupling constant α_s

The running $\alpha_S(Q^2)$

- non-Abelian character of theory leads to :

$$\alpha_S(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\ln(Q^2/\Lambda^2)}$$

- this exhibits **asymptotic freedom** as long as $N_f < 17$

How can we calculate with large coupling α_s

The S-matrix expansion in powers of coupling or H_I

$$S = \sum_{n=0}^{\infty} \frac{(-\mathrm{i})^n}{n!} \int \ldots \int \mathrm{d}^4 x_1\, \mathrm{d}^4 x_2 \ldots \mathrm{d}^4 x_n\, \mathrm{T}\{\mathscr{H}_\mathrm{I}(x_1)\mathscr{H}_\mathrm{I}(x_2) \ldots \mathscr{H}_\mathrm{I}(x_n)\}, \quad (6.23)$$

can be written as

$$S = T \exp\left[-i \int d^4x\, \mathcal{H}_I(x)\right]$$

and remains well defined no matter how large is H_I

Path Integrals..

In scalar field theory

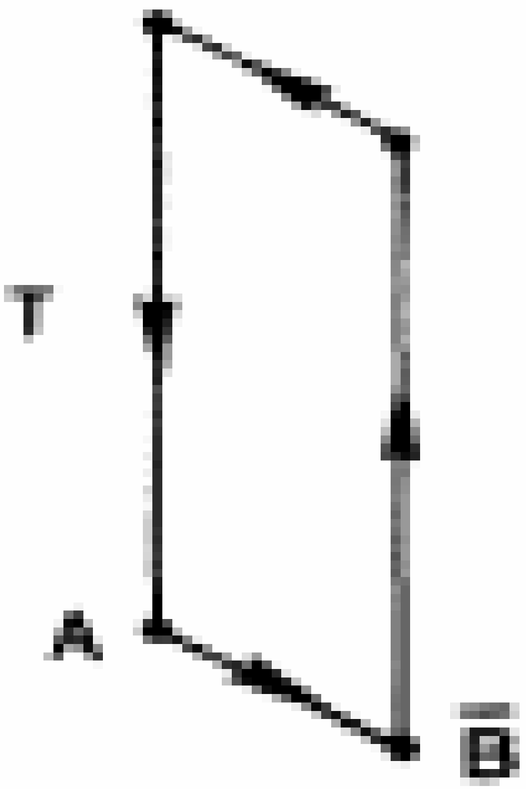
$$\langle 0| \mathcal{T}\phi(x_1)\phi(x_2)\ldots\phi(x_n)|0\rangle = \frac{1}{Z}\int \mathcal{D}\phi\, e^{iS[\phi]}\phi(x_1)\phi(x_2)\ldots\phi(x_n)$$

In QCD, scalar field $\phi(x)$ is replaced by colour *i*'d field $\psi_i(x)$ and the gluonic field $A_\mu^a(x)$

In lattice QCD, a pure gluonic path integral (using *adiabatic approximation*) with $U(x, x+\epsilon n) = \exp\left[ie\epsilon n^\mu A_\mu(x)\right]$ is

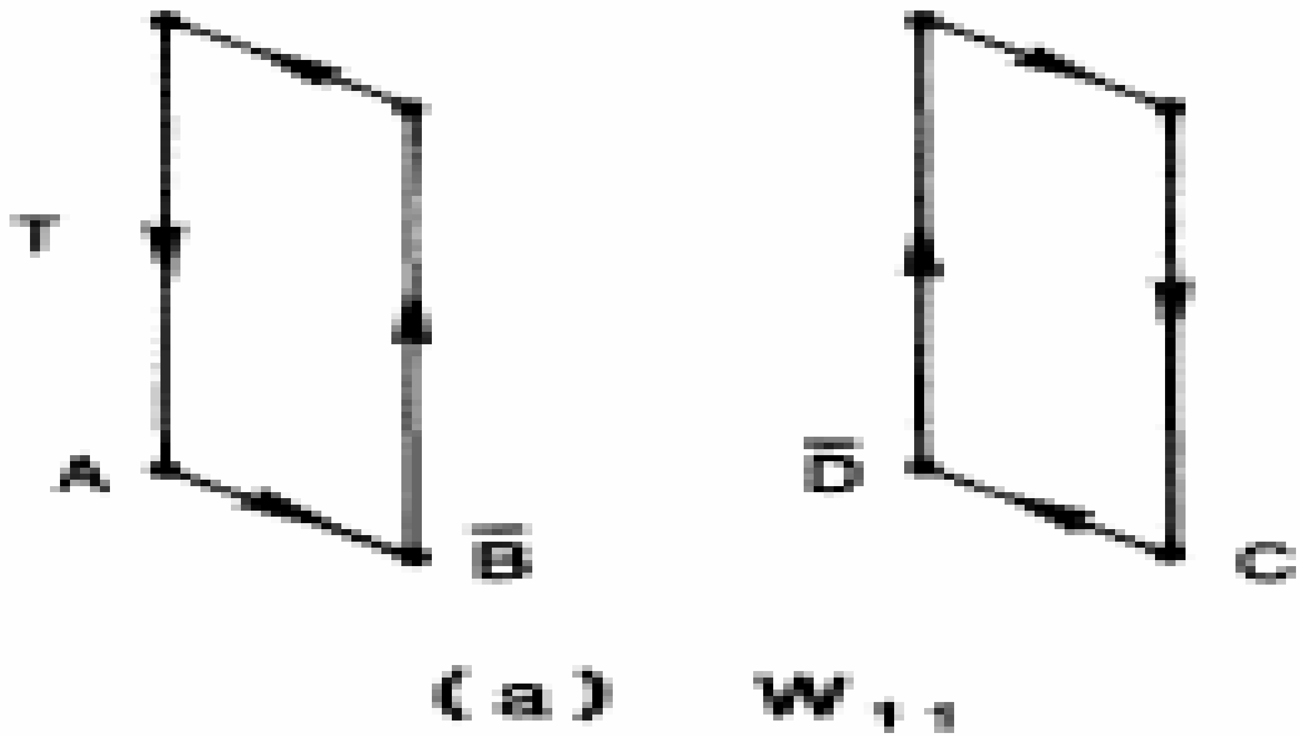
$$\langle O\rangle = \frac{\int DU\, O(U) e^{-S(U)}}{\int DU\, e^{-S(U)}}$$

An operator $O(U)$: Wilson loop for $Q\overline{Q}$(A and $\overline{B}$) time evolution



H. Matsuoka and D. Sivers, Phys. Rev. D33(1986)1441

For Mutiquarks ($A, B, \overline{C}, \overline{D}$) Wilson loops at least *Multiply…*(no sum)



H. Matsuoka and D. Sivers, Phys. Rev. D33(1986)1441

Hence a correction (*f-model*): The sum-of-two-body approach

$$V(q_1 q_2 \bar{q}_3 \bar{q}_4) = \sum_{ij} \boldsymbol{F}_i \cdot \boldsymbol{F}_j v_{ij},$$

giving

$$V = \begin{pmatrix} -\frac{4}{3}(v_{1\bar{3}} + v_{2\bar{4}}) & \frac{4}{9}\begin{pmatrix} v_{12} + v_{\bar{3}\bar{4}} - v_{1\bar{3}} \\ -v_{2\bar{4}} - v_{1\bar{4}} - v_{2\bar{3}} \end{pmatrix} & \frac{2}{3\sqrt{3}}\begin{pmatrix} -2(v_{1\bar{3}} + v_{2\bar{4}}) + v_{1\bar{4}} \\ +v_{2\bar{3}} - v_{12} - v_{\bar{3}\bar{4}} \end{pmatrix} \\ & -\frac{4}{3}(v_{1\bar{4}} + v_{2\bar{3}}) & \frac{2}{3\sqrt{3}}\begin{pmatrix} 2(v_{1\bar{4}} + v_{2\bar{3}}) + v_{12} \\ +v_{\bar{3}\bar{4}} - v_{2\bar{4}} - v_{1\bar{3}} \end{pmatrix} \\ \text{symmetric} & & -\frac{1}{3}\begin{pmatrix} 2(v_{12} + v_{\bar{3}\bar{4}}) + v_{1\bar{3}} \\ +v_{2\bar{4}} + v_{1\bar{4}} + v_{2\bar{3}} \end{pmatrix} \end{pmatrix}.$$

in a (redundant basis)

$$|A\rangle = |\mathbf{1}_{1\bar{3}}\mathbf{1}_{2\bar{4}}\rangle, \qquad |B\rangle = |\mathbf{1}_{1\bar{4}}\mathbf{1}_{2\bar{3}}\rangle, \qquad |C\rangle = |\bar{\mathbf{3}}_{12}\mathbf{3}_{\bar{3}\bar{4}}\rangle$$

was replaced by…

$$V=\begin{pmatrix} -\frac{4}{3}\left(v_{1\bar{3}}+v_{2\bar{4}}\right) & \frac{4f}{9}\begin{pmatrix} v_{12}+v_{\bar{3}\bar{4}}-v_{1\bar{3}} \\ -v_{2\bar{4}}-v_{1\bar{4}}-v_{2\bar{3}} \end{pmatrix} & \frac{2f}{3\sqrt{3}}\begin{pmatrix} -2(v_{1\bar{3}}+v_{2\bar{4}})+v_{1\bar{4}} \\ +v_{2\bar{3}}-v_{12}-v_{\bar{3}\bar{4}} \end{pmatrix} \\ & -\frac{4}{3}\left(v_{1\bar{4}}+v_{2\bar{3}}\right) & \frac{2f}{3\sqrt{3}}\begin{pmatrix} 2(v_{1\bar{4}}+v_{2\bar{3}})+v_{12} \\ +v_{\bar{3}\bar{4}}-v_{2\bar{4}}-v_{1\bar{3}} \end{pmatrix} \\ \text{symmetric} & & -\frac{1}{3}\begin{pmatrix} 2(v_{12}+v_{\bar{3}\bar{4}})+v_{1\bar{3}} \\ +v_{2\bar{4}}+v_{1\bar{4}}+v_{2\bar{3}} \end{pmatrix} \end{pmatrix}$$

in the corresponding basis including the gluonic field dependence

$$f=\exp\left(-\bar{k}\sum_{i<j} r_{ij}^2\right)$$

a *product* of two-body terms tried *initially*

B. Masud et. al. Nuclear Physics A528 (1991) 477–512

Dynamical calculations: RGM

$$|\Psi(q_1, q_2, \bar{q}_3, \bar{q}_4; g)\rangle = \sum_{kI} |k\rangle_g |kI\rangle_s |k\rangle_f \psi_c(\mathbf{R}_c)$$

$$\times \chi_{kI}(\mathbf{R}_k)\xi_k(\mathbf{y}_k)\zeta_k(\mathbf{z}_k),$$

$$\langle\delta\Psi|H - E_c|\Psi\rangle = \sum_{kIlJ} \int d^3\mathbf{R}_c d^3\mathbf{R}_k d^3\mathbf{y}_k d^3\mathbf{z}_k \psi_c(\mathbf{R}_c)\delta\chi_{kI}(\mathbf{R}_k)\xi_k(\mathbf{y}_k)\zeta_k(\mathbf{z}_k)$$
$$\times {}_f\langle k|_s\langle kI|_g\langle k|H - E_c|l\rangle_g|lJ\rangle_s|l\rangle_f \psi_c(\mathbf{R}_c)\chi_{lJ}(\mathbf{R}_l)\xi_l(\mathbf{y}_l)\zeta_l(\mathbf{z}_l) = 0.$$

$$\sum_{lJ} \int d^3\mathbf{y}_k d^3\mathbf{z}_k \xi_k(\mathbf{y}_k)\zeta_k(\mathbf{z}_k)$$

$$\times {}_f\langle k|\,_s\langle kI|\,_g\langle k|H - E_c|l\rangle_g|lJ\rangle_s|l\rangle_f$$

$$\times \chi_{lJ}(\mathbf{R}_l)\xi_l(\mathbf{y}_l)\zeta_l(\mathbf{z}_l) = 0,$$

solved, for a given E_c , for *the only unknown* $\chi_{kI}(\mathbf{R}_k)$

T-matrix read off

$$\chi_1(p_1) = \frac{\delta(p_1 - p_c(1))}{p_c^2(1)} - \frac{1}{\Delta_1(p_1)}[Q_9^{(1)}A_1 + Q_{10}^{(1)}B_1 + Q_{11}^{(1)}A_2 + Q_{12}^{(1)}B_2], \quad \text{(B.21)}$$

$$\chi_2(p_2) = -\frac{1}{\Delta_2(p_2)}[Q_{11}^{(2)}A_1 + Q_{12}^{(2)}B_1 + Q_9^{(2)}A_2 + Q_{10}^{(2)}B_2], \quad \text{(B.22)}$$

$$T_{1,1} = 2\mu_{12}\frac{\pi}{2}p_c(1)[Q_9^{(1)}A_1 + Q_{10}^{(1)}B_1 + Q_{11}^{(1)}A_2 + Q_{12}^{(1)}B_2],$$

$$T_{2,1} = 2\mu_{34}\frac{\pi}{2}p_c(1)\sqrt{\frac{v_2}{v_1}}[Q_{11}^{(2)}A_1 + Q_{12}^{(2)}B_1 + Q_9^{(2)}A_2 + Q_{10}^{(2)}B_2]$$

Phase shifts (meson-meson interaction) decrease if a *Product* term *f* multiplies off-diagonal elements

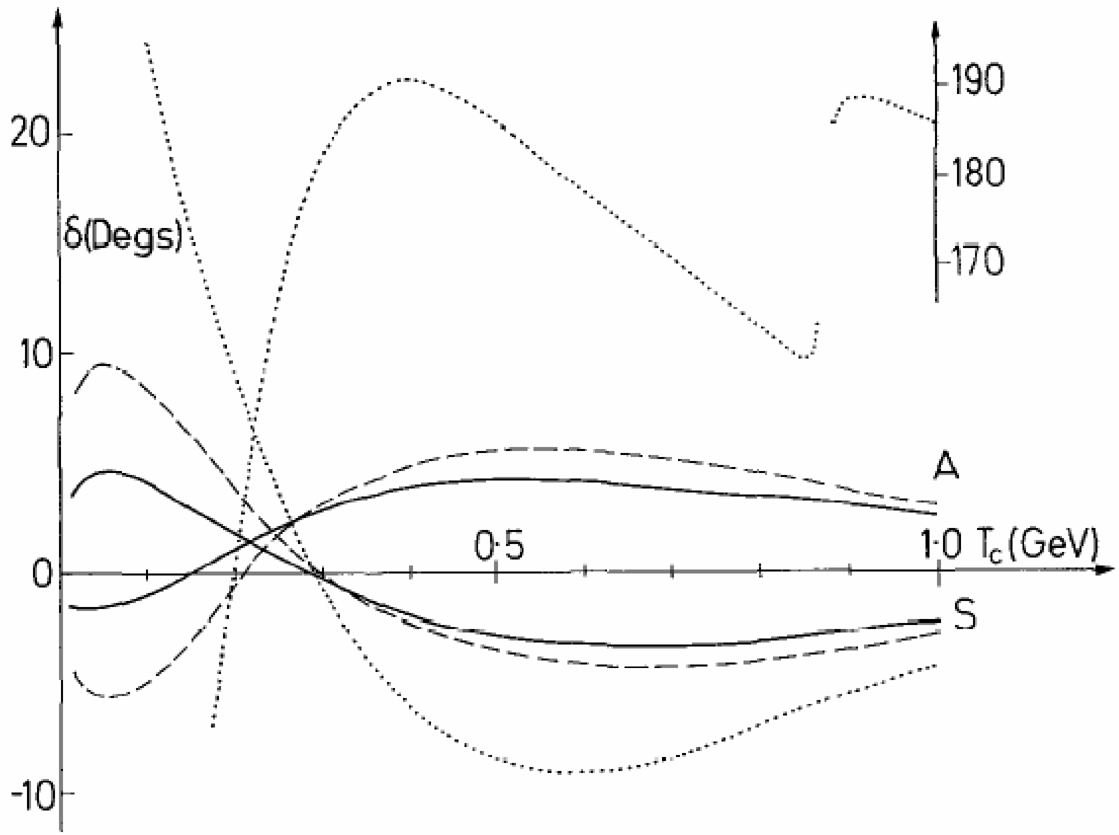
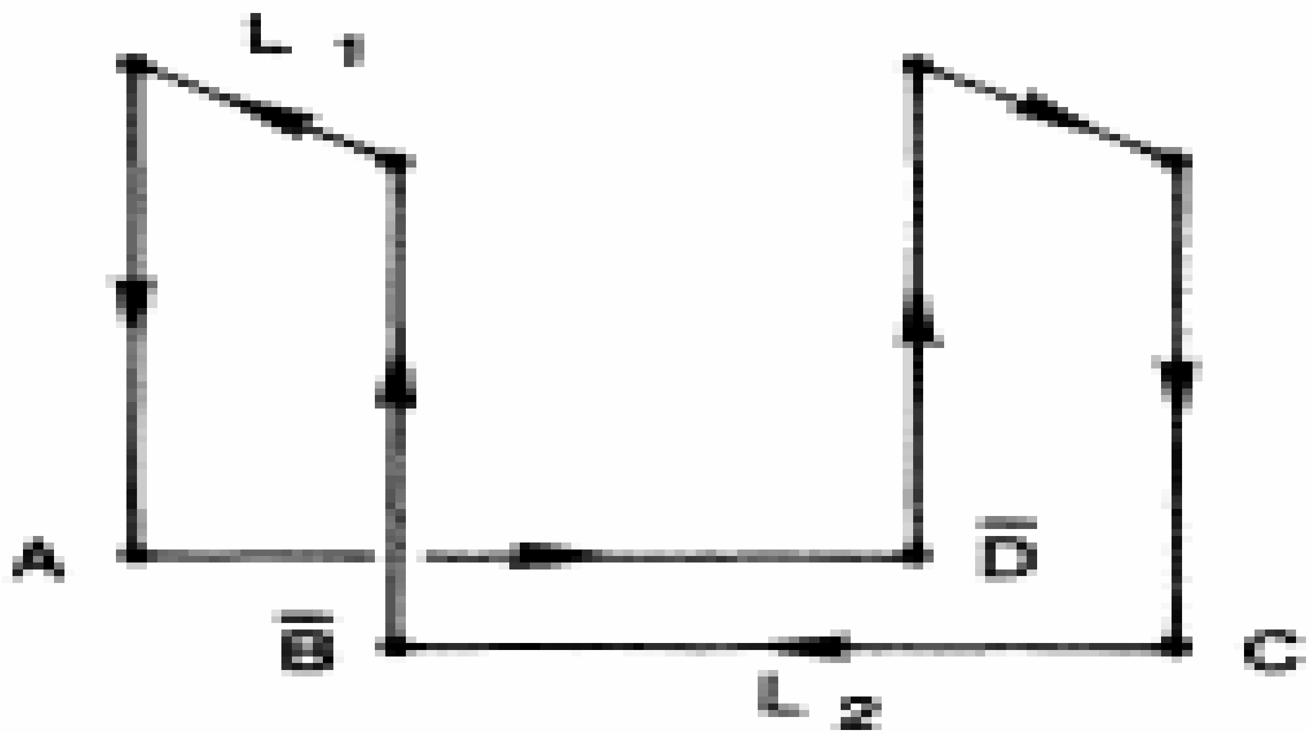


Fig. 6. Same as fig. 4 but for different $\bar{k}$ – i.e. $M_l = 624$ MeV and $D = 1.5$ GeV. Solid line: $\bar{k} = \frac{1}{6}\,\text{fm}^{-2}$. Dashed line: $\bar{k} = \frac{1}{9}\,\text{fm}^{-2}$. Dotted line: $\bar{k} = 0\,\text{fm}^{-2}$ – a resonance occurring at 860 MeV.

Beyond Product, starting from Wilson loops in Lattice QCD



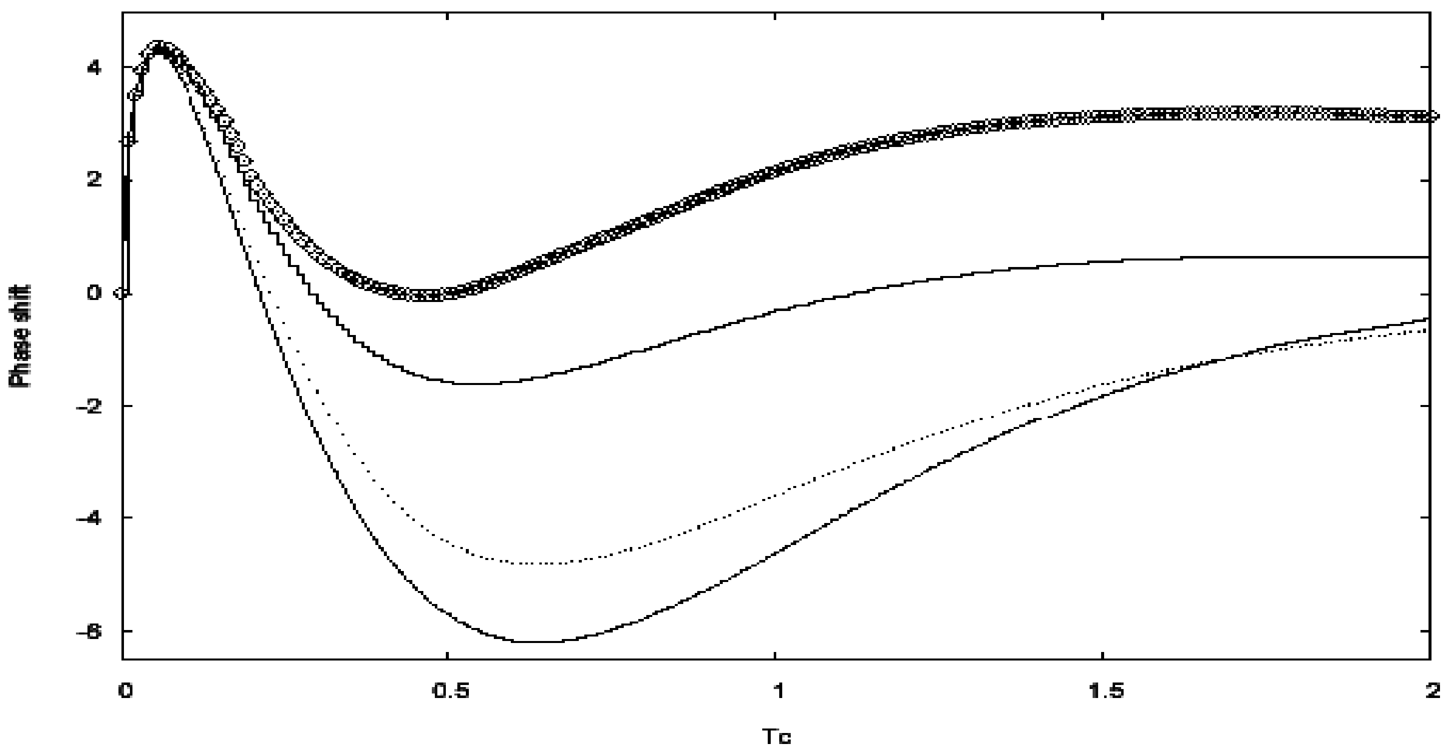
(b) W_{12}

H. Matsuoka and D. Sivers, Phys. Rev. D33(1986)1441

The form of *f closer to lattice QCD*

$$f = \exp(-b_s E A)$$

New calculations give a new *qualitative effect,*
ANGLE DEPDENCE OF PHASE SHIFTS..



Conclusion

- The lattice-gauge-theory motivated models of the hadronic physics may have *many-body* interaction terms (like a $Q^2\overline{Q}^2$ *area* dependence) that have no counterpart in perturbative QCD, resulting in measurable effects (like an angle dependence of the meson-meson phase shifts).

(May be compared with linear potential/confinement, that has no counterpart in PQCD but is a two-body term.)