General Analysis for the Decay $B \to K_1 l^+ l^-$.

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- Introduction and Motivation
- What is effective Hamiltionian?
- How we can go beyond the SM?
- How Ward Identities relate the form factors?
- What is the Forward Backward Asymmetry mean?
- Interesting results.

Introduction and Motivation I

- Flavor Changing Neutral Current transitions are not allowed at tree level but are induced by the Glashow-Iliopoulos-Miani (GIM) amplitudes at loop level.
- Additionally these are also suppressed in the Standard Model (SM) due to their dependence on the weak mixing angles of the quark-flavor rotation matrix- the Cabibo-Kobayashi-Maskawa (CKM) matrix.

GIM conjectured that full charged weak current is given by

$$J_{\mu}(x) = \overline{u}(x)\gamma^{\mu}(1+\gamma^{\mu})d_c(x) + \overline{c}(x)\gamma^{\mu}(1+\gamma^{\mu})s_c(x)$$

where

$$\begin{aligned} d_c(x) &= \cos\theta d(x) + \sin\theta s(x) \\ s_c(x) &= -\sin\theta d(x) + \cos\theta s(x) \end{aligned}$$

or in a matrix notation

$$J_{\mu}(x) = \overline{U}(x)\gamma^{\mu}(1+\gamma^{\mu})CD(x)$$

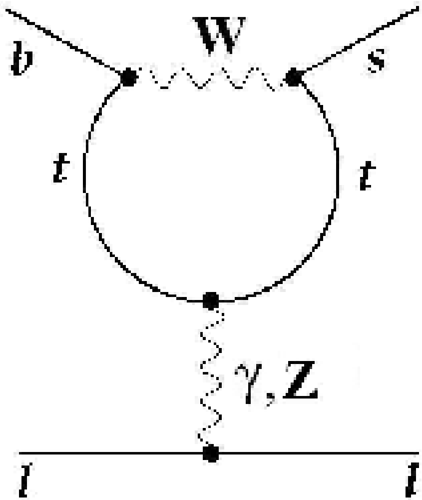
with

$$U = \begin{pmatrix} u \\ c \end{pmatrix};\ D = \begin{pmatrix} d \\ s \end{pmatrix};\ C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The important point is that, the current J_3, given by the commutator of J and $J^\dagger$, is diagonal in flavor space. As a result in a gauge theory the neutral current, which is a linear superposition of J_3 and electromagnetic current, will also be diagonal. This ensures that FCNC processes will not be generated in the tree approximation.

- CKM Matrix.

for the Decay
$B \longrightarrow K_1 l^+ l^-$.
quark level transition is
$b \longrightarrow s$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$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

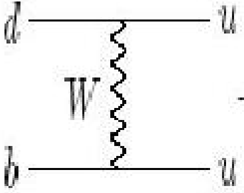
$$\simeq \begin{pmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.008 & 0.040 & 0.999 \end{pmatrix}$$

These two circumstancess make the FCNC decays relatively rare and provides potentially stringent tests of the SM. Hence FCNC transitions $b \to s$ are important for the presence of new physics, i.e., physics beyond SM.

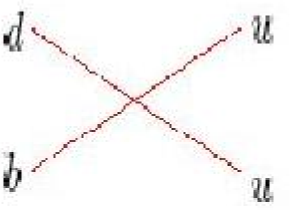
Effective Hamiltonian I

To understand the concept, one can consider the following Feynman diagram.

Effective Hamiltonian II



$$+\mathrm{QCD} \simeq C\left(\frac{M_W}{\mu}, \alpha_s\right) \cdot$$



Here, a key feature is provided by the fact that the W mass M_W is very much heavier than the other momentum scales.

$$M_W \gg m_b, m_c \gg \Lambda_{QCD} \gg m_u, m_d, (m_s)$$

Ignoring QCD, the corresponding tree-level W-exchange amplitude is given by

$$\begin{aligned} A(b \to cs\bar{u}) &= -\frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \frac{M_W^2}{k^2 - M_W^2} (\bar{d}u)_{V-A} (\bar{u}u)_{V-A} \\ &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \underbrace{(\bar{d}u)_{V-A} (\bar{u}u)_{V-A}}_{\text{local operator}} + \mathcal{O}\left(\frac{k^2}{M_W^2}\right) \end{aligned}$$

where

$$(\bar{q}_1 q_2)_{V-A} \equiv \bar{q}\gamma_\mu (1 - \gamma_5) q_2.$$

Since k, the momentum transfer through the W propagator, is very small as compared to M_W, we can safely neglect the terms $\mathcal{O}\left(k^2/M_W^2\right)$. The W propagator then shrinks to a point and we obtain an effective four fermion interaction. If we include also short distance QCD or electroweak corrections more operators have to be added to the effective Hamiltonian which we generalize to

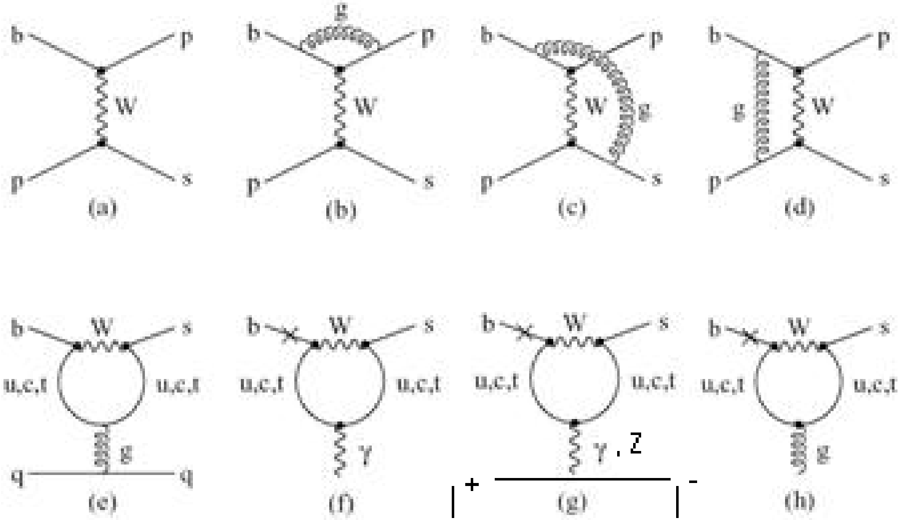
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i^{10} V_{\text{CKM}}^i C_i(\mu) O_i(\mu)$$

The operators which describe the $b \to s$ transitions are given

$$
\begin{aligned}
Q_1 &= (\bar{d}_i u_i)_{V-A}(\bar{u}_j b_j)_{V-A} \\
Q_2 &= (\bar{d}_i u_j)_{V-A}(\bar{u}_j b_i)_{V-A} \\
Q_3 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A} \\
Q_4 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A} \\
Q_5 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A} \\
Q_6 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A} \\
Q_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu}(1+\gamma_5) b_i F_{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
Q_8 &= \frac{g_s}{8\pi^2} m_b \overline{s}_i \sigma^{\mu\nu}(1+\gamma_5) T^a_{ij} b_j G^a_{\mu\nu} \\
Q_9 &= \overline{s}_i \gamma^\mu (1-\gamma_5) b_i (\bar{l}\gamma^\mu l) \\
Q_{10} &= \overline{s}_i \gamma^\mu (1-\gamma_5) b_i (\bar{l}\gamma^\mu \gamma_5 l)
\end{aligned}
$$

these operators originate from the following diagrams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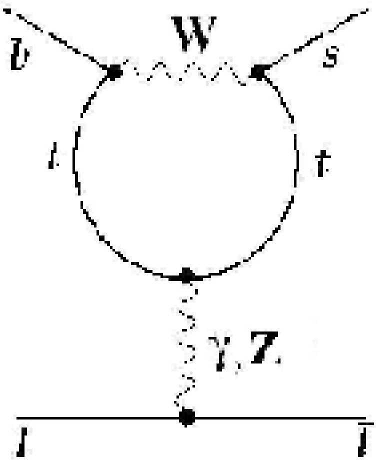
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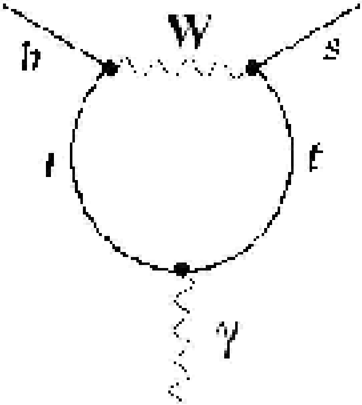
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Now the transition

$$b \to s l^+ l^-$$

only C_7, $C9$, and C_{10} operators are relevent. The effective hamiltonian for this process is given

$$\begin{aligned} H_{SM} &= \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} [(C_9^{eff} - C_{10}) \bar{s}_L \gamma^\mu b_L \bar{l}_L \gamma^\mu l_L \\ & + (C_9^{eff} + C_{10}) \bar{s}_L \gamma^\mu b_L \bar{l}_R \gamma^\mu l_R \\ & - 2 C_7^{eff} \bar{s} i \sigma^{\mu\nu} \frac{q^\nu}{q^2} (m_b R) b \bar{l} \gamma^\mu l], \end{aligned}$$

Where

$$\begin{aligned} L &= \gamma^\mu (1 - \gamma^5) \text{ and} \\ R &= \gamma^\mu (1 + \gamma^5) \end{aligned}$$

There are two ways to go beyond the SM

- Include new operators which are not present in the SM
- Modification in the Wilson Coefficients

There are ten independent local four-Fermi interactions which may contribute to the process. H_{NEW} is a function of the coefficients of local four-Fermi interactions and is defined as

$$\begin{aligned} H_{NEW} &= \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} [C_{LL} \bar{s}_L \gamma^\mu b_L \bar{l}_L \gamma^\mu l_L \\ &+ C_{LR} \bar{s}_L \gamma^\mu b_L \bar{l}_R \gamma^\mu l_R \\ &+ C_{RL} \bar{s}_R \gamma^\mu b_R \bar{l}_L \gamma^\mu l_L \\ &+ C_{RR} \bar{s}_R \gamma^\mu b_R \bar{l}_R \gamma^\mu l_R \\ &+ C_{LRLR} \bar{s}_L b_R \bar{l}_L l_R \\ &+ C_{RLLR} \bar{s}_R b_L \bar{l}_L l_R \\ &+ C_{LRRL} \bar{s}_L b_R \bar{l}_R l_L \\ &+ C_{RLRL} \bar{s}_R b_L \bar{l}_R l_L \\ &+ C_T \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma^{\mu\nu} l \\ &+ i C_{TE} \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma_{\alpha\beta} l \epsilon^{\mu\nu\alpha\beta}] \end{aligned}$$

Form Factors I

The exclusive decay $B \to K_1 l^+ l^-$ invloves the hadronic matrix elements of quark operators which one can be parametrized in terms of the form factors as follows

$$\begin{aligned}
\langle K_1(k,\varepsilon)\,|V_\mu|\,B(p)\rangle &= \varepsilon^*_\mu\,(M_B + M_{K_1})\,V_1(s) \\
&\quad -(p+k)_\mu\,(\varepsilon^*\cdot q)\,\frac{V_2(s)}{M_B + M_{K_1}} \\
&\quad -q_\mu\,(\varepsilon\cdot q)\,\frac{2M_{K_1}}{s}\,[V_3(s) - V_0(s)] \qquad (1) \\
\langle K_1(k,\varepsilon)\,|A_\mu|\,B(p)\rangle &= \frac{2i\epsilon_{\mu\nu\alpha\beta}}{M_B + M_{K_1}}\varepsilon^{*\nu}p^\alpha k^\beta A(s) \qquad (2)
\end{aligned}$$

Form Factors II

where $V_{\mu} = \bar{s}\gamma_{\mu}b$ and $A_{\mu} = \bar{s}\gamma_{\mu}\gamma_{5}b$ are the vectors and axial vector currents respectively and ε_{μ}^{*} is the polarization vector for the final state axial vector meson and $q^2 = s$. In Eq.(1) we have

$$V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}} V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}} V_2(s)$$

with

$$V_3(0) = V_0(0)$$

Form Factors III

In addition to the above form factors we have Penguin form factors as well, these are

$$\begin{aligned} \langle K_1(k,\varepsilon)\,|\bar{s}i\sigma_{\mu\nu}q^{\nu}b|\,B(p)\rangle &= \left[\begin{array}{c}\left(M_B^2-M_{K_1}^2\right)\varepsilon_{\mu}- \\ (\varepsilon\cdot q)(p+k)_{\mu}\end{array}\right]F_2(s) \\ &\quad +(\varepsilon^{*}\cdot q)\left[\begin{array}{c} q_{\mu}- \\ \frac{s}{M_B^2-M_{K_1}^2}(p+k)_{\mu}\end{array}\right]F_3(s) \quad (3) \\ \langle K_1(k,\varepsilon)\,|\bar{s}i\sigma_{\mu\nu}q^{\nu}\gamma_5 b|\,B(p)\rangle &= -i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^{\alpha}k^{\beta}F_1(s) \quad (4) \end{aligned}$$

with $F_1(0)=2F_2(0)$.

In addition to the above mentioned martix elements there is an additional matrix element

$$\left\langle K_1(k,\varepsilon)\left|\bar{s}(1\pm\gamma^5)b\right|B(p)\right\rangle \quad (5)$$

Form Factors IV

which is not present in the SM calculation. One can obtain the matrix element given in Eq.(5) by mupltiplying both sides of Eq.(**??**) with q_μ and using equation of motion. By neglecting the strange quark mass, we get

$$\left\langle K_1(k,\varepsilon)\left|\bar{s}(1\pm\gamma^5)b\right|B(p)\right\rangle = \frac{1}{m_b}\{\mp 2iM_{K_1}(\varepsilon^*\cdot q)V_0(s)\}$$

- Since the form factors are non perturbatvie quantities and they are functions of four momentum transfer square. Differnt models are used in literature to work out these form factors
- The form factors we used here in the analysis of physical observable like Branching ratio and forward asymmetry have been calculated using Ward Identities.

Ward Identities I

We start with the relation
$\langle A(k,\varepsilon)|\bar{s}i\sigma_{\mu\nu}q^{\nu}\gamma_5 b|B(p)\rangle e^{-i(p-k).x} =$
$-\langle A(k,\varepsilon)|\partial_\nu(\bar{s}(x)\sigma^{\mu\nu}\gamma_5 b(x))|B(p)\rangle$ ———— (1A)
We can replace ∂_ν by the covariant derivative D_ν to take into account the strong interaction of the quark field and using the following relations

$$\begin{aligned}\sigma^{\mu\nu} &= \frac{i}{2}[\gamma^\mu,\gamma^\nu] \\ 2g^{\mu\nu} &= \{\gamma^\mu,\gamma^\nu\}\end{aligned}$$

$$-\langle A(k,\varepsilon)|D_\nu\bar{s}(x)[-i\gamma^\nu\gamma^\mu + ig^{\mu\nu}]\gamma_5 b(x)|B(p)\rangle$$
$$-\langle A(k,\varepsilon)|\bar{s}(x)[i\gamma^\mu\gamma^\nu - ig^{\mu\nu}]\gamma_5 D_\nu b(x)|B(p)\rangle$$

Ward Identities II

using the Dirac eqauation

$$\not{D}b(x) = -im_b b(x), \ \overline{s}(x)\not{D} = im_s\overline{s}(x)$$

the relation becomes

$$\begin{aligned} & (m_b - m_s) \langle A(k,\varepsilon) | \overline{s}(x)\gamma^\mu\gamma_5 b(x) | B(p)\rangle - \\ & -i \langle A(k,\varepsilon) | D^\mu \overline{s}(x)\gamma_5 b(x) | B(p)\rangle \\ & +i \langle A(k,\varepsilon) | \overline{s}(x)\gamma_5 D^\mu b(x) | B(p)\rangle \\ = \ & (m_b - m_s) \langle A(k,\varepsilon) | \overline{s}\gamma^\mu\gamma_5 b | B(p)\rangle e^{-iq.x} \\ & -i \langle A(k,\varepsilon) | D^\mu(\overline{s}(x)\gamma_5 b(x)) | B(p)\rangle \\ & +2i \langle A(k,\varepsilon) | \overline{s}(x)\gamma_5 D^\mu b(x) | B(p)\rangle \end{aligned} \quad (6)$$

Ward Identities III

Using now the linear momentum commutation relation

$$[\widehat{P}^{\mu}, O(x)] = -iD^{\mu}O(x), \;\; \widehat{P}^{\mu}_{q} = \int d^3x : q^{\dagger}(x)D^{\mu}q(x) :$$

the last two terms of Eq.(6) become

$$\left\langle A(k,\varepsilon)\left|(\widehat{P}^{\mu}\overline{s}(x)\gamma_5 b(x) - \overline{s}(x)\gamma_5 b(x)\widehat{P}^{\mu})\right| B(p)\right\rangle$$
$$-2\left\langle A(k,\varepsilon)\left|\overline{s}(x)\gamma_5(-iD^{\mu}b(x))\right| B(p)\right\rangle$$

and using

$$\begin{aligned} \langle A(k,\varepsilon)|\widehat{P}^{\mu} &= k^{\mu}\langle A(k,\varepsilon)| \\ \widehat{P}^{\mu}|B(p)\rangle &= p^{\mu}|B(p)\rangle, \end{aligned}$$

Ward Identities IV

$$-q^{\mu}\left\langle A(k,\varepsilon)\right|\bar{s}\gamma_5 b\left|B(p)\right\rangle e^{-iq.x}$$
$$+2\left\langle A(k,\varepsilon)\right|\bar{s}\gamma_5 b p_b^{\mu}\left|B(p)\right\rangle e^{-iq.x} - - - (2A)$$

where in the last term we have use that $\widehat{P}_b^{\mu}|A(k,\varepsilon)\rangle = 0$ as $V(k,\varepsilon)$ does not contain the quark b.
In the heavy quark effective theory m_b is taken to infinity and the four momentum of the light degree of freedom are neglected compared with m_b. This enable us to identify with

$$p_b^{\mu} \sim p^{\mu} \text{ and } 2p - q = p + k$$

so,

$$\left\langle A(k,\varepsilon)\right|\bar{s}i\sigma_{\mu\nu}q^{\nu}\gamma_5 b\left|B(p)\right\rangle = (m_b - m_s)\left\langle A(k,\varepsilon)\right|\bar{s}\gamma^{\mu}\gamma_5 b\left|B(p)\right\rangle + (p^{\mu}+k^{\mu})\left\langle A(k,\varepsilon)\right|\bar{s}\gamma_5 b\left|B(p)\right\rangle \quad (7)$$

Ward Identities V

Similarly,

$$\langle A(k,\varepsilon)|\bar{s}i\sigma^{\mu\nu}q_\nu b|B(p)\rangle = -(m_b+m_s)\langle A(k,\varepsilon)|\bar{s}\gamma^\mu b|B(p)\rangle + (p^\mu+k^\mu)\langle A(k,\varepsilon)|\bar{s}b|B(p)\rangle \tag{8}$$

Using the Ward Identitiy (7) in Eq. (2) and Eq.(4), and comparing the coefficient, we obtain

$$F_1(s) = -\frac{m_b - m_s}{M_B + M_{K_1}} 2A(s) \tag{9}$$

Ward Identities VI

Again, using the Ward Identity (8) in Eq.(1) and Eq.(3), and comparing the coefficients we obtain

$$\begin{aligned} F_2(s) &= -\frac{m_b + m_s}{M_B - M_{K_1}} 2V_1(s), \\ F_3(s) &= \frac{2M_{K_1}}{s}(m_b + m_s)[V_3(s) - V_0(s)] \end{aligned} \tag{10}$$

These are model independent results derived by using Ward Identities.

Ward Identities VII

The final expressions of the form factors that we have used for the numerical work are

$$
\begin{aligned}
A(s) &= \frac{A(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)} \\
V_1(s) &= \frac{V_1(0)}{(1 - s/M_{B_A^*}^2)(1 - s/M_{B_A^*}'^2)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right) \\
V_2(s) &= \frac{\tilde{V}_2(0)}{(1 - s/M_{B_A^*}^2)(1 - s/M_{B_A^*}'^2)} \\
&\quad - \frac{2M_{K_1}}{M_B - M_{K_1}} \frac{V_0(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)}
\end{aligned}
\tag{11}
$$

Ward Identities VIII

with

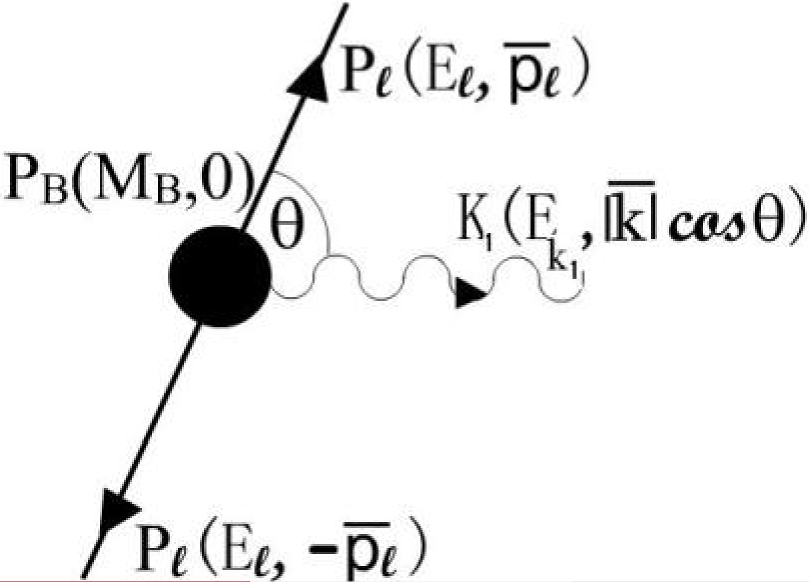
$$\begin{aligned} A(0) &= -(0.52 \pm 0.05) \\ V_1(0) &= -(0.24 \pm 0.02) \\ \tilde{V}_2(0) &= -(0.39 \pm 0.05) \end{aligned} \tag{12}$$

Forward-Backward-Asymmetry (FBA) I

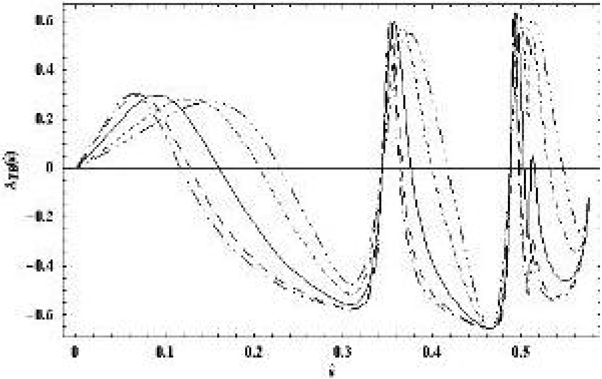
The forward-backward asymmetry is find out by the given farmula

$$A_{FB}(s)=\frac{\int_0^1 d\cos\theta\frac{d\Gamma}{dsd\cos\theta}-\int_{-1}^0 d\cos\theta\frac{d\Gamma}{dsd\cos\theta}}{\int_0^1 d\cos\theta\frac{d\Gamma}{dsd\cos\theta}+\int_{-1}^0 d\cos\theta\frac{d\Gamma}{dsd\cos\theta}}$$

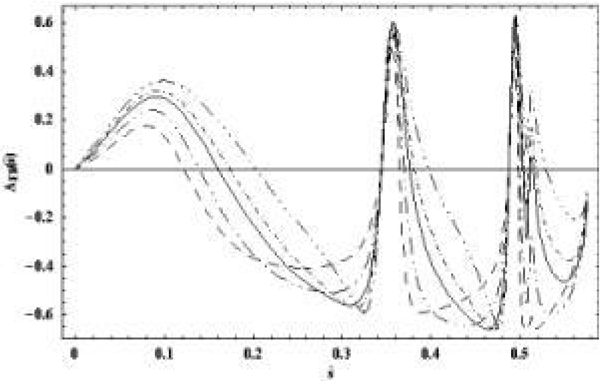
Forward-Backward-Asymmetry (FBA) II



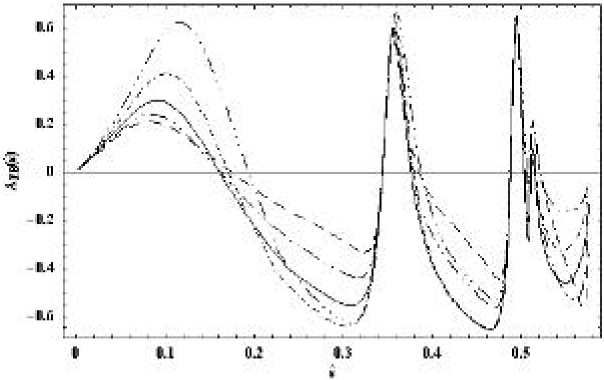
Results



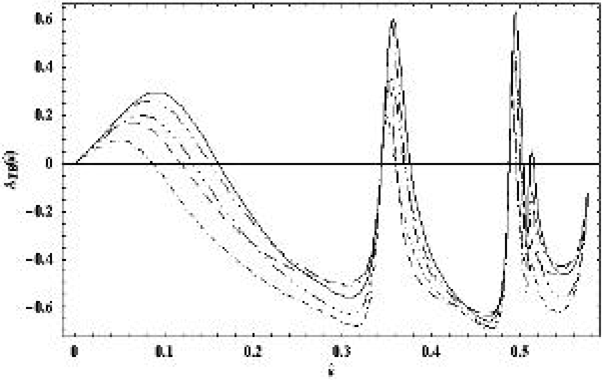
The FB asymmetry for the SM (solid line), CLL is -C10 (dashed-double dotted line), CLL is -0.7×C10 (dashed line), CLL is C10 (dashed-triple dotted line), CLL is 0.7×C10 (dashed single dotted line). The coefficients of the other interactions are set to zero.



The FB asymmetry for the SM (solid line), CLR is -C10 (dashed-double dotted line), CLR is -0.7×C10 (dashed line), CLR is C10 (dashed-triple dotted line), CLR is 0.7×C10 (dashed single dotted line). The coefficients of the other interactions are set to zero.



The FB asymmetry for the SM (solid line), CRL is -C10 (dashed-double dotted line), CRL is -0.7×C10 (dashed line), CRL is C10 (dashed-triple dotted line), CRL is 0.7×C10 (dashed single dotted line). The coefficients of the other interactions are set to zero.



The FB asymmetry for the SM (solid line), CRR is -C10 (dashed-double dotted line), CRR is -0.7×C10 (dashed line), CRR is C10 (dashed-triple dotted line), CRR is 0.7×C10 (dashed single dotted line). The coefficients of the other interactions are set to zero.

Conclusion

- Our analysis showed that the precise measurements of the forward backward asymmetry, we can determine the existence of new physics beyond the SM, and in particular we can obtain information about the values and the signs of various new Wilson coefficients
- The new facilities to explore B physics, like LHCb, CMS and ATLAS experiments at CERN are expected to increase the data and statistics in rare B-decays. Therefore, these experiments are expected to provide the appropriate number of events needed to measure the physical observable in rare B-decays. The observation of the branching ratio and the zero (and its shift due to new physics) of the forward backward asymmetry will provide useful probe of any possible new physics as well as suggest to pick the values of new Wilson coefficients.


THANKS!