Exotica

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- It is well known that in quark model the mesons and baryons are composed of $q\bar{q}$ and qqq, respectively and it provides a convenient framework in the classification of hadrons. Most experimentally observed hadronic states fit in it quite nicely.
- The states which are beyond this quark model are non-conventional hadrons and are termed "exotics" .
- The exotic states in the spectrum of charmonium have been found experimentally, where few of them are labelled as X, Y and Z states.
  There are three different frameworks suggested to accommodate these exotic states
- $D - D^*$ molecule
- $c\bar{c}g$ hybrids
- Diquark-antidiquark or four quark states.

- Motivation to explain the state $X\,(3872)$ observed by the Belle by the hadronic molecule is that the mass of this state is very close to the $D^0\bar{D}^{*0}$ threshold.
- Dates back to the work by Fermi and Yang: The pion is interpreted as a nucleon-nucleon bound state.
- Problem: The $D^0\bar{D}^{*0}$ molecule containing the $X\,(3872)$ state would be characterized by the extremely small binding energy. This makes it odd that such a loosely bound molecule could be produced promptly (i.e. not from B decays, used by Belle and BABAR) in high energy hadron collision environment.
- Grinstein et al. have estimated the prompt production cross section of $X\,(3872)$ by considering it as a $D^0\bar{D}^{*0}$ hadron molecule
- It is about two orders of magnitude smaller than the minimum production cross section one can extract from CDF data.

- Favors an alternate scenario in which these exotic states are roughly of hadronic size.
- Maiani *et al.* have proposed that instead of a molecule one can study these states by considering the bound system of a diquark and an antidiquark in a tetraquark scheme.
- Jaffe and Wilckez interpretation of pentaquark baryons that can be formed from "antidiquark-antidiquark-quark".
- Tetraquark states: Diquark-antidiquark pairs are in color $\bar{3}$ and 3 configuration, respectively, bound together by the color forces.
- Distinctive from the $D - D^*$ molecules.
- Demerits of this picture is that there is no set of selection rule available to explain why many of the states predicted in this picture are not yet seen.

- The purpose is to discuss;
- Spin-spin interactions in the constitutent quark model and give the spectrum of bottom diquark-antidiquark states both for $L_{Q\bar{Q}} = 0, 1$.
- The estimate of the different decay modes of these states.
- Its potential to look at on going and future experiments.

- **Spectrum of bottom diquark-antidiquark states** The mass spectrum of the systems $[bq][\overline{bq'}]$ with $q = u, d, s$ and c can be described in terms of the constituent diquark masses, spin-spin interactions inside the single diquark, spin-spin interaction between quark and antiquark belonging to two diquarks, spin-orbit and purely orbital term, i.e.

$$H = 2m_Q + H_{SS}^{(QQ)} + H_{SS}^{(Q\bar{Q})} + H_{SL} + H_{LL} \tag{1}$$

where:

$$\begin{aligned} H_{SS}^{(QQ)} &= 2(\mathcal{K}_{bq})_{\bar{3}}[(\mathbf{S}_b \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{b}} \cdot \mathbf{S}_{\bar{q}})], \\ H_{SS}^{(Q\bar{Q})} &= 2(\mathcal{K}_{b\bar{q}})(\mathbf{S}_b \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{b}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{b\bar{b}}(\mathbf{S}_b \cdot \mathbf{S}_{\bar{b}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}), \\ H_{SL} &= 2A_Q(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}), \\ H_{LL} &= B_Q \frac{L(L+1)}{2}. \end{aligned} \tag{2}$$

The parameters involved in the above Hamiltonian (2) can be obtained from the known meson and baryon masses by resorting to constituent quark model

$$H = \sum_i m_i + \sum_{i<j} 2\mathcal{K}_{ij}(\mathbf{S}_i \cdot \mathbf{S}_j) \tag{3}$$

where the sum runs over the hadron constituents. For instant applied to $L = 0$ mesons $K - K^*$ gives

$$M = m_q + m_s + \mathcal{K}_{s\bar{q}}\left[J(J+1) - \frac{3}{2}\right]. \tag{4}$$

Writing the similar equations for $\pi - \rho$, $D - D^*$, $D_s - D_s^*$ and $J/\Psi - \eta_c$ complex one can obtain the values of color singlet $\mathcal{K}_{ij}$ for u, d, s and $c-$quark flavor which together with spin-spin interaction in color antitriplet state that comes from baryon masses.

Table: Constituent quark masses derived from $L = 0$ mesons and baryons.

Constituent mass (MeV)	q	s	c	b
Mesons	305	490	1670	5008
Baryons	362	546	1721	5050

Table: Spin-Spin couplings for quark-antiquark pairs in in the color singlet state from known mesons.

Spin-spin couplings	$q\bar{q}$	$s\bar{q}$	$s\bar{s}$	$c\bar{q}$	$c\bar{s}$	$c\bar{c}$	$b\bar{q}$	$b\bar{s}$	$b\bar{c}$	$b\bar{b}$
$\left(\mathcal{K}_{ij}\right)_0$(MeV)	318	200	129	71	72	59	23	23	20	36

Table: Spin-Spin couplings for quark-quark in color $\bar{3}$ state from known baryons.

Spin-Spin couplings	qq	sq	cq	cs	ss	bq	bs	bc
$\left(\mathcal{K}_{ij}\right)_{\bar{3}}$(MeV)	98	65	22	24	72	6	25	10

To calculate the spin-spin interaction of the $Q\bar{Q}$ quantum state explicitly, we use the following non-relativistic notation

$$\left|S_Q, S_{\bar{Q}};\ J\right\rangle = \left|\Gamma, \Gamma';\ J\right\rangle \tag{5}$$

where, S_Q and $S_{\bar{Q}}$ are the spin of diquark and antidiquark, J is the total angular momentum and the Γ^α are 2×2 matrices in spinor space. Using pauli matrices they can be written as:

$$\Gamma^0 = \frac{\sigma_2}{\sqrt{2}};\ \Gamma^i = \frac{1}{\sqrt{2}}\sigma_2\sigma_i \tag{6}$$

and we define:

$$\begin{aligned}
\left|0_Q, 0_{\bar{Q}};\ 0_J\right\rangle &= \frac{1}{2}\left(\sigma_2\right) \otimes \left(\sigma_2\right), \\
\left|1_Q, 1_{\bar{Q}};\ 0_J\right\rangle &= \frac{1}{2\sqrt{3}}\left(\sigma_2\sigma^i\right) \otimes \left(\sigma_2\sigma^i\right), \\
\left|0_Q, 1_{\bar{Q}};\ 1_J\right\rangle &= \frac{1}{2}\left(\sigma_2\right) \otimes \left(\sigma_2\sigma^i\right), \\
\left|1_Q, 0_{\bar{Q}};\ 1_J\right\rangle &= \frac{1}{2}\left(\sigma_2\sigma^i\right) \otimes \left(\sigma_2\right), \\
\left|1_Q, 1_{\bar{Q}};\ 1_J\right\rangle &= \frac{1}{2\sqrt{2}}\varepsilon^{ijk}\left(\sigma_2\sigma^j\right) \otimes \left(\sigma_2\sigma^k\right).
\end{aligned} \tag{7}$$

Next step is the diagonalization of the Hamiltonian (1) using the basis of states with definite diquark and antidiquark spin and total angular momentum. There are two different possibilities:

- **Lowest lying $[bq][\bar{b}\bar{q}]$ states $(L_{Q\bar{Q}} = 0)$**
  Conventionally the states can be classified in terms of the diquark and antidiquark spin, S_Q and $S_{\bar{Q}}$, total angular momentum J, parity, P and charge conjugation, C. Considering both good and bad diquraks and having $L_{Q\bar{Q}} = 0$ we have six possible states:

- **Two states with $J^{PC} = 0^{++}$**

$$\begin{aligned} \left|0^{++}\right\rangle &= \left|0_Q, 0_{\bar{Q}};\ 0_J\right\rangle ; \\ \left|0^{++\prime}\right\rangle &= \left|1_Q, 1_{\bar{Q}};\ 0_J\right\rangle . \end{aligned} \tag{8}$$

- **Three states with $J = 1$**

$$\begin{aligned} \left|1^{++}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|0_Q, 1_{\bar{Q}};\ 1_J\right\rangle + \left|1_Q, 0_{\bar{Q}};\ 1_J\right\rangle\right) ; \\ \left|1^{+-}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|0_Q, 1_{\bar{Q}};\ 1_J\right\rangle - \left|1_Q, 0_{\bar{Q}};\ 1_J\right\rangle\right) ; \\ \left|1^{+-\prime}\right\rangle &= \left|1_Q, 1_{\bar{Q}};\ 1_J\right\rangle . \end{aligned} \tag{9}$$

  All these states have positive parity because both good and bad diquarks have positive parity and $L_{Q\bar{Q}} = 0$. The difference is of charge conjugation, the state $\left|1^{++}\right\rangle$ is even under charge conjugation, where as $\left|1^{+-}\right\rangle$ and $\left|1^{+-\prime}\right\rangle$ are odd.

- **One state with $J^{PC} = 2^{++}$**

$$\left|2^{++}\right\rangle = \left|1_Q, 1_{\bar{Q}};\ 2_J\right\rangle . \tag{10}$$

Keeping in view that for $L_{Q\bar{Q}} = 0$ there is no spin-orbit and purely orbital term, the Hamiltonian (1) takes the form

$$\begin{aligned} H &= 2m_{[bq]} + 2(\mathcal{K}_{bq})_{\bar{3}}[(\mathbf{S}_b \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{b}} \cdot \mathbf{S}_{\bar{q}})] + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}) \\ &\quad +2(\mathcal{K}_{b\bar{q}})(\mathbf{S}_b \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{b}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{b\bar{b}}(\mathbf{S}_b \cdot \mathbf{S}_{\bar{b}}). \end{aligned} \quad (11)$$

The diagonalization of Hamiltonian (11) with the states defined above gives its eignevalues which are needed to estimate the masses of these states. It is straight forward to see that for 1^{++} and 2^{++} states the Hamiltonian is diagonal with the eigenvalues

$$M\left(1^{++}\right) = 2m_{[bq]} - (\mathcal{K}_{bq})_{\bar{3}} + \frac{1}{2}\mathcal{K}_{q\bar{q}} - \mathcal{K}_{b\bar{q}} + \frac{1}{2}\mathcal{K}_{b\bar{b}}, \quad (12)$$

$$M\left(2^{++}\right) = 2m_{[bq]} + (\mathcal{K}_{bq})_{\bar{3}} + \frac{1}{2}\mathcal{K}_{q\bar{q}} + \mathcal{K}_{b\bar{q}} + \frac{1}{2}\mathcal{K}_{b\bar{b}}. \quad (13)$$

To find the numerical values everything is known except the mass of the constituent diquark which can be estimated as

$$m_{[bq]} = m_{[cq]} + (m_b - m_c) \quad (14)$$

$$= 5.267 \text{ GeV, for } q = u, d \quad (15)$$

$$= 5.451 \text{ GeV, for } q = s \quad (16)$$

with $m_{[cq]} = 1933$ MeV, $m_{[cs]} = m_{[cq]} + (m_s - m_q)$ and the value of m_b, m_c and m_q are given in Table I.

The color octet couplings can be estimated with the help of the one gluon exchange model, where neglecting spin, we can write the diquark in $\bar{3}$ color channel as: $Q^i = [bq]^i \equiv \varepsilon^{ijk} b_j q_k$, where i, j

and k are the color indices in the fundamental representation of $SU(3)$. Hence, the color neutral hadron is

$$[bq][\bar{b}\bar{q}] = \varepsilon^{ijk}\varepsilon_{ilm}(b_j q_k)\left(\bar{b}^l\bar{q}^m\right) = \left(b_j\bar{b}^j\right)\left(q_k\bar{q}^k\right) - \left(b_j\bar{q}^k\right)\left(\bar{b}^j q_k\right). \quad (17)$$

To re-arrange the color indices in the last term of Eq. (17) one can use the $SU(N)$ identity for the Lie algebra generators

$$\sum_{a=1}^{N^2-1}\lambda^a_{ij}\lambda^a_{kl} = 2\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\right) \quad (18)$$

which for $N = 3$ gives

$$\begin{aligned}[bq][\bar{b}\bar{q}] &= \left(b_j\bar{b}^j\right)\left(q_k\bar{q}^k\right) - \left[\frac{1}{2}\left(\bar{b}^i\lambda^a_{ij}b^j\right)\left(\bar{q}^k\lambda^a_{kl}q^l\right) + \frac{1}{3}\left(b_j\bar{b}^j\right)\left(q_k\bar{q}^k\right)\right] \\ &= \frac{2}{3}\left(b_j\bar{b}^j\right)\left(q_k\bar{q}^k\right) - \frac{1}{2}\left(\bar{b}^i\lambda^a_{ij}b^j\right)\left(\bar{q}^k\lambda^a_{kl}q^l\right). \quad (19)\end{aligned}$$

This gives the relative weights of a color singlet and a color octet state in a diquark-antidiquark system. Quite easily one can see that the probability to find a particular $q\bar{q}$ pair in color octet state is twice the probability of finding it in color singlet state. So for $\mathcal{K}_{b\bar{b}}$

$$\mathcal{K}_{b\bar{b}}\left([bq][\bar{b}\bar{q}]\right) = \frac{1}{3}\left(\mathcal{K}_{b\bar{b}}\right)_0 + \frac{2}{3}\left(\mathcal{K}_{b\bar{b}}\right)_8 \quad (20)$$

where $(\mathcal{K}_{b\bar{b}})_0$ is reported in Table II. The only thing that we do not know is the $(\mathcal{K}_{b\bar{b}})_8$ which can be derived from the one gluon exchange model by using the relation:

$$(\mathcal{K}_{b\bar{b}})_{\mathbf{X}} \sim \left(C^2(\mathbf{X}) - C^2(\mathbf{3}) - C^2(\bar{3})\right) \tag{21}$$

with $C^2(\mathbf{X}) = 0, 4/3, 4/3, 3$ for $\mathbf{X} = \mathbf{0}, \mathbf{3}, \bar{3}, \mathbf{8}$ respectively. Finally, Eq. (20) gives

$$\mathcal{K}_{b\bar{b}}([bq][\bar{b}\bar{q}]) = \frac{1}{4}(\mathcal{K}_{b\bar{b}})_0 \tag{22}$$

Now, we have all the input parameters to calculate the mass spectrum numerically. Putting everything together the masses for 1^{++} and 2^{++} states becomes

$$\begin{aligned} M(1^{++}) &= 10.504 \text{ GeV, for } q = u, d && (23)\\ &= 10.849 \text{ GeV, for } q = s && (24)\\ &= 13.217 \text{ GeV, for } q = c && (25)\\ M(2^{++}) &= 10.520 \text{ GeV, for } q = u, d && (26)\\ &= 10.901 \text{ GeV, for } q = s && (27)\\ &= 13.239 \text{ GeV, for } q = c && (28) \end{aligned}$$

For the 0^{++} and 1^{+-} states, the Hamiltonina is not diagonal and corresponding to them we have 2×2 matrices which are

$$M(0^{++}) = \begin{pmatrix} -3(\mathcal{K}_{bq})_{\bar{3}} & \frac{\sqrt{3}}{2}\left(\mathcal{K}_{q\bar{q}} + \mathcal{K}_{b\bar{b}} - 2\mathcal{K}_{b\bar{q}}\right) \\ \frac{\sqrt{3}}{2}\left(\mathcal{K}_{q\bar{q}} + \mathcal{K}_{b\bar{b}} - 2\mathcal{K}_{b\bar{q}}\right) & (\mathcal{K}_{bq})_{\bar{3}} - \left(\mathcal{K}_{q\bar{q}} + \mathcal{K}_{b\bar{b}} + 2\mathcal{K}_{b\bar{q}}\right) \end{pmatrix}, \tag{29}$$

$$M\left(1^{+-}\right)=\begin{pmatrix} -(\mathcal{K}_{bq})_{\bar{3}}+\mathcal{K}_{b\bar{q}}-\frac{(\mathcal{K}_{q\bar{q}}+\mathcal{K}_{b\bar{b}})}{2} & \mathcal{K}_{q\bar{q}}-\mathcal{K}_{b\bar{b}} \\ \mathcal{K}_{q\bar{q}}-\mathcal{K}_{b\bar{b}} & (\mathcal{K}_{bq})_{\bar{3}}-\mathcal{K}_{b\bar{q}}-\frac{(\mathcal{K}_{q\bar{q}}+\mathcal{K}_{b\bar{b}})}{2} \end{pmatrix}. \quad (30)$$

To estimate the masses of these two states, one has to diagonalize the above matrices.

- **Higher mass** $[bq][\bar{b}\bar{q}]$ **states** $\left(L_{Q\bar{Q}}=1\right)$
The orbital excitation comes when $L_{Q\bar{Q}}=1$ and both good and bad diquarks are considered.

$$\begin{aligned} |A\rangle &= \left|0_Q, 0_{\bar{Q}};\ 1_J\right\rangle; \\ |B\rangle &= \frac{\left|1_Q, 0_{\bar{Q}};\ 1_J\right\rangle + \left|0_Q, 1_{\bar{Q}};\ 1_J\right\rangle}{\sqrt{2}}; \\ |C\rangle &= \left|1_Q, 1_{\bar{Q}};\ 1_J\right\rangle. \end{aligned} \tag{31}$$

Now both the good and the bad diquarks have positive parity, therefore, the state $|B\rangle$ has $P=C=-1$, only for $L_{Q\bar{Q}}=1$. Since $C_{Q\bar{Q}}\,(-1)^{L_{Q\bar{Q}}+S_{Q\bar{Q}}}=1$, therefore, states $|A\rangle$ and $|C\rangle$ have $C_{Q\bar{Q}}=-1$ provided that $L_{Q\bar{Q}}=1$ and $S_{Q\bar{Q}}=0\,,2$.
Using the notations defined in Eq. (7) one can easily diagonalize the Hamiltonian (1) and the mass shift due to spin-spin interaction terms H_{SS} becomes

$$\Delta M_{SS}=\begin{pmatrix} -3\left(\mathcal{K}_{bq}\right)_{\bar{3}} & 0 & 0 \\ 0 & \begin{matrix} -\left(\mathcal{K}_{bq}\right)_{\bar{3}}-\mathcal{K}_{b\bar{q}} \\ +\left(\mathcal{K}_{q\bar{q}}+\mathcal{K}_{b\bar{b}}\right)/2 \end{matrix} & 0 \\ 0 & 0 & \begin{matrix} -\left(\mathcal{K}_{bq}\right)_{\bar{3}}-\mathcal{K}_{b\bar{q}} \\ -\left(\mathcal{K}_{q\bar{q}}+\mathcal{K}_{b\bar{b}}\right)/2 \end{matrix} \end{pmatrix}$$

The eigenvalues of spin-orbit and angular momentum operators given in Eq. (1) were calculated by Polosa et al.

Table: Eigenvalues of spin-orbit and angular momentum operator in Eq. (1) with all the states having $J = L_{Q\bar{Q}} + S_{Q\bar{Q}} = 1$.

$\left\vert S_Q, S_{\bar{Q}}, S_{Q\bar{Q}}, L\right\rangle$	$a\left(S_Q, S_{\bar{Q}}, S_{Q\bar{Q}}, L\right)$	$b\left(s_Q, S_{\bar{Q}}, S_{Q\bar{Q}}, L\right)$
$\vert 0, 0, 0, 1\rangle$	0	1
$\vert 1, 0, 1, 1\rangle$	-2	1
$\vert 1, 1, 2, 1\rangle$	-6	1
$\vert 1, 1, 1, 1\rangle$	-2	1
$\vert 1, 1, 0, 1\rangle$	0	1

Hence the four states having quantum numbers 1^{--} are:

$$\begin{aligned} M^{(1)}\left(S_Q = 0,\ S_{\bar{Q}} = 0,\ S_{Q\bar{Q}} = 0,\ L_{Q\bar{Q}} = 1\right) &= 2m_{[bq]} + \lambda_1 + B_{\mathcal{Q}}, \\ M^{(2)}\left(S_Q = 1,\ S_{\bar{Q}} = 0,\ S_{Q\bar{Q}} = 1,\ L_{Q\bar{Q}} = 1\right) &= 2m_{[bq]} + \delta + \lambda_2 - 2A_Q + B_{\mathcal{Q}}, \\ M^{(3)}\left(S_Q = 1,\ S_{\bar{Q}} = 1,\ S_{Q\bar{Q}} = 0,\ L_{Q\bar{Q}} = 1\right) &= 2m_{[bq]} + 2\delta + \lambda_3 + B_{\mathcal{Q}}, \\ M^{(4)}\left(S_Q = 1,\ S_{\bar{Q}} = 1,\ S_{Q\bar{Q}} = 2,\ L_{Q\bar{Q}} = 1\right) &= 2m_{[bq]} + 2\delta + \lambda_3 - 6A_Q + B_{\mathcal{Q}}, \end{aligned} \tag{32}$$

The δ is the mass difference of the good and the bad diquark, i.e.

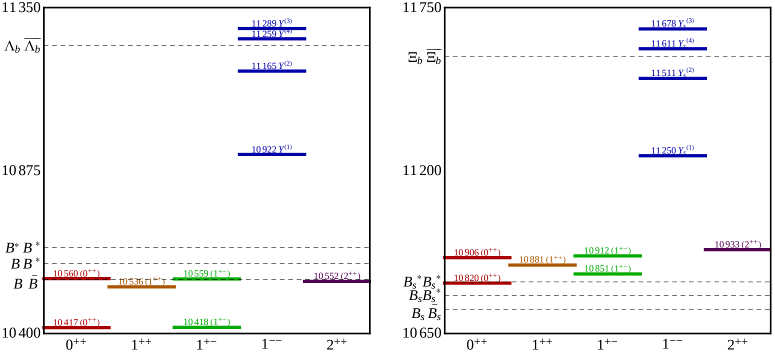
$$\delta = m_Q\left(S_Q = 1\right) - m_Q\left(S_Q = 0\right). \tag{33}$$

In order to calculate the numerical values of these states, we have to calculate the value of A_Q, B_Q and δ which are the only unknowing remaining in this calculation.

$$
\begin{aligned}
A_Q &= 5 \text{ MeV, for } q = u, d, \\
A_Q &= 3 \text{ MeV, for } q = s, \\
A_Q &= 3 \text{ MeV, for } q = c, \\
B_Q &= 408 \text{ MeV, for } q = u, d, \\
B_Q &= 423 \text{ MeV, for } q = s, \\
B_Q &= 423 \text{ MeV, for } q = c.
\end{aligned} \tag{34}
$$

$M^{(i)}_{Y_{[bq]}}$	$q = u, d$	$q = s$	$q = c$	$q = d, \bar{q} = s$
$M_{Y^{(1)}_{[bq]}}$	10.890	11.218	13.618	11.054
$M_{Y^{(2)}_{[bq]}}$	11.130	11.479	13.841	11.281
$M_{Y^{(3)}_{[bq]}}$	11.257	11.646	14.025	11.476
$M^{(4)}_{Y_{[bq]}}$	11.227	11.629	14.009	11.453

We would like to emphasis here that the observation of the bottom counterparts to the new anomalous charmonium-like states is very important since it will allow to discriminate between different theoretical description of these states as well different models which give significantly distinctive results for masses in the bottom sector.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10 875
10 400
Λb Λb
B* B*
B B*
B B̄
11 289 Y(3)
11 259 Y(4)
11 165 Y(2)
10 922 Y(1)
10 560 (0++)
10 536 (1++)
10 559 (1+−)
10 552 (2++)
10 417 (0++)
10 418 (1+−)
0++
1++
1+−
1−−
2++
11 750
11 200
10 650
Ξb Ξb
Bs* Bs*
BsBs*
Bs B̄s
11 678 Ys(3)
11 611 Ys(4)
11 511 Ys(2)
11 250 Ys(1)
10 906 (0++)
10 881 (1++)
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10 851 (1+−)
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10 820 (0++)
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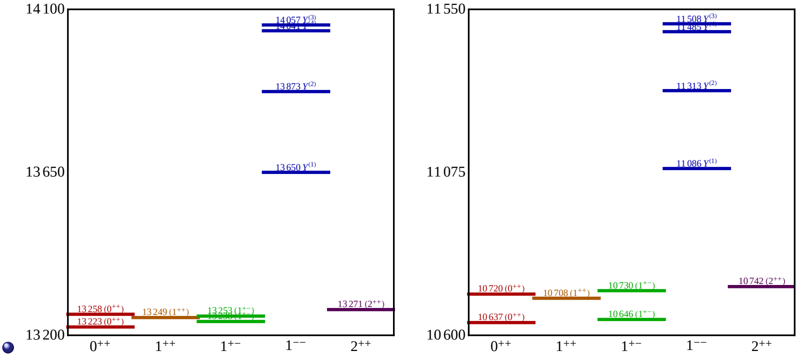


Figure: Tetraquark spectrum, where the tetraquarks have the valence quark content $[bq][\bar{b}\bar{q}]$ with $q = u, d$ to the left and $q = s$ to the right. Some importent decay thresholds are indicated by dashed lines. The masses are given in MeV.

- **Isospin breaking and the leptonic decay widths**
  We discuss in this section the isospin breaking effects, which were neglected in the previous section, and calculate the decay widths $\Gamma_{ee}(Y_{[b,l]})$ and $\Gamma_{ee}(Y_{[b,h]})$ for $Y_{[b,l]}$ and $Y_{[b,h]}$. The mass eigenstates are given by a linear superposition of the states defined in

$$Y_{[b,l]} = \cos\theta\ Y_{[bu]} + \sin\theta\ Y_{[bd]} \quad (35)$$

$$Y_{[b,h]} = -\sin\theta\ Y_{[bu]} + \cos\theta\ Y_{bd]}. \quad (36)$$

  The isospin breaking part of the mass matrix is

$$\begin{pmatrix} 2m_u + \delta & \delta \\ \delta & 2m_d + \delta \end{pmatrix}, \quad (37)$$

  where δ is the contribution from quark annihilation diagrams, where the light quark pair annihilates to intermediate gluons.

$$M(Y_{[b,h]}) - M(Y_{[b,l]}) = (7 \pm 3)\ \cos(2\theta)\ \mathrm{MeV}. \quad (38)$$

  The partial electronic widths $\Gamma_{ee}(Y_{[b,l]})$ and $\Gamma_{ee}(Y_{[b,h]})$ are given by the well known Van Royen-Weisskopf formula for the P-states, which we write generically as:

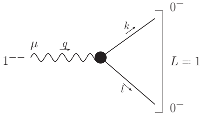
$$\Gamma_{ee} = \frac{16\pi Q^2\alpha^2|\Psi'^2_{Q\bar{Q}}|}{M^2\omega^2}, \quad (39)$$

  where $Q = -2/3$ is the diquark charge in $Y_{bd} = [bd][\bar{b}\bar{d}]$ and $Q = +1/3$ is the charge of the diquarks in $Y_{bu} = [bu][\bar{b}\bar{u}]$, $\alpha = 1/137$ is the electromagnetic coupling constant to lowest order and $\Psi'_{Q\bar{Q}}(\vec{r}) = \psi(\phi,\theta)R'(r)$ is the first derivative in r of the wave function of the tetraquark, which needs to be taken at the origin. The corresponding value for the tetraquark states $[bq][\bar{b}\bar{q}]$ is then calculated taking into account the ratio of the string tensions κ. As the linear part of the confining potential determines essentially the heavy Quarkonia wave functions, we find that to a good approximation, $\Psi_{Q\bar{Q}}(0) \simeq \kappa\Psi_{b\bar{b}}(0)$, which is what we have used in our derivations of the decay widths. Moreover, we expect that for all the P-states $Y_{[bu]}^{(n)}$ and $Y_{[bd]}^{(n)}$, the electronic widths will be constant, to a good approximation. For the mass eigenstates $Y_{[b,l]}$ and $Y_{[b,h]}$, $\Gamma_{ee}(Y_{[b,l]}$ and $\Gamma_{ee}(Y_{[b,h]}$ are given by $\Gamma_{ee}(\theta) = 0.81\kappa^2 Q(\theta)^2$ keV, where $Q(\theta)$ are the mixing angle weighted charges of the two mass eigenstates. The ratio $\mathcal{R}_{ee}(Y_b)$ of $\Gamma_{ee}(Y_{[b,l]})$ and $\Gamma_{ee}(Y_{[b,h]})$ is given by

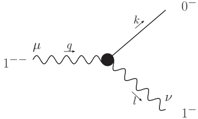
$$\mathcal{R}_{ee}(Y_b) \equiv \frac{\Gamma_{ee}(Y_{[b,l]})}{\Gamma_{ee}(Y_{[b,h]})} = \frac{(\cos\theta/3 + 2\sin\theta/3)^2}{(-\sin\theta/3 + 2\cos\theta/3)^2}. \qquad (40)$$

Since the total cross sections for $e^+e^- \to (Y_{[b,l]}, Y_{[b,h]}) \to \mathrm{hadrons}$ is directly proportional to $\Gamma_{ee}(Y_{[b,l]})$ and $\Gamma_{ee}(Y_{[b,h]})$, the ratio $\mathcal{R}_{ee}(Y_b)$ is accessible from the experiment.

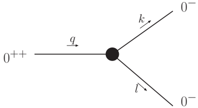
- **Diquark-antidiquark decay modes**
  There are several two body decays, which decay thresholds are pictured in figure 1. Starting from the Vertices we derive the corresponding decay width:



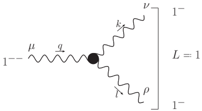
$$\widehat{=} \quad F(k^\mu - l^\mu) \qquad \Gamma = \frac{F^2|\vec{k}|^3}{2M^2\pi}$$



$$\widehat{=} \quad \frac{F}{|q|}\epsilon^{\mu\nu\rho\sigma}k_\rho l_\sigma \qquad \Gamma = \frac{F^2|\vec{k}|^3}{4M^2\pi}$$

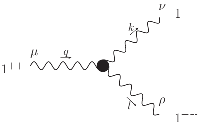


$$\widehat{=} \quad F|q| \qquad \Gamma = \frac{F^2|\vec{k}|}{8\pi}$$



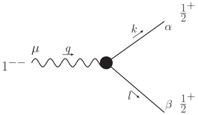
$$\widehat{=} \quad \begin{array}{l} F(g^{\mu\rho}(q+l)^\nu \\ \quad -g^{\mu\nu}(k+q)^\rho \\ \quad +g^{\rho\nu}(q+k)^\mu) \end{array} \qquad \Gamma = \frac{F^2|\vec{k}|^3(48|\vec{k}|^4-104M^2|\vec{k}|^2+27M^4)}{2\pi(M^3-4|\vec{k}|^2M)^2} \tag{41}$$

and



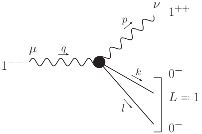
$$\widehat{=} \quad F\epsilon^{\mu\nu\rho\sigma}q_\sigma$$

$$\Gamma = \frac{F^2\frac{|\vec{k}|}{4\pi}\left(6\frac{\sqrt{|\vec{k}|^2+m_l^2}}{M} + \left(\frac{1}{m_l^2}+\frac{1}{M^2}\right)\left(|\vec{k}|^2+3m_l^2\right)\right)}{}$$



$$\widehat{=} \quad \left(F\gamma^\mu + \frac{2F'}{i|q|}q_\nu\sigma^{\nu\mu}\right)_{\alpha\beta} \tag{42}$$

$$\Gamma = \frac{3(F^2+F'^2)|\vec{k}|}{4\pi} - \frac{(F^2+2F'^2)|\vec{k}|^3}{M^2\pi}$$



$$\widehat{=} \quad \frac{\epsilon^{\mu\nu\rho\sigma}}{q^2}\left(F(p_\rho k_\sigma - p_\rho l_\sigma) + F'k_\rho l_\sigma\right)$$

While the decay momentum $|\vec{k}|$ is given by

$$|\vec{k}| = \frac{\sqrt{M^2 - (m_k + m_l)^2}\sqrt{M^2 - (m_k - m_l)^2}}{2M}, \tag{43}$$

where M is the mass of the decaying particle and m_k, m_l are the masses of the decay products. The polarization vectors $\varepsilon^{(i)\mu}$ of the vector (1^-) and axial vector (1^+) particles satisfy the transversality condition with polarization sum

$$\sum_i^3 \varepsilon^{(i)\mu}(p)\varepsilon^{(i)\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}. \tag{44}$$

The decay constants F and F' contain all non-perturbative interactions, which are beyond scope in our approximation. To get an estimate for the couplings, we adjust them to match the known processes, which mimic the processes we are interested in (i.e. strong interaction processes involving the same vertices as in (41), (42)).

Table: Mass values taken from

hadron	mass	hadron	mass	hadron	mass	hadron	mass
B	5.279	π	0.139	$\Upsilon(1s)$	9.46	Λ_b	5.62
B^*	5.325	ρ	0.775	$\Upsilon(4s)$	10.5794	Ξ_b	5.792
B_s	5.366	ω	0.783	$\Upsilon(10860)$	10.865	K	0.4937
B_s^*	5.412	$f_0(860)$	0.98	$\Upsilon(11020)$	11.019		

Table: 2-body decays $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$, which we use as a reference, with the mass and the decay widths taken from [**?**]. The extracted values of the coupling constants F and the centre of mass momentum $|\vec{k}|$ are also shown.

process	Γ	F	$\lvert\vec{k}\rvert[GeV]$
$\Upsilon(10860) \to B\ \bar{B}$	$< 13.2 MeV$	< 2.15	1.3
$\Upsilon(10860) \to B\ B^*$	$15.4^{+6.6}_{-6.6} MeV$	$3.7^{+0.7}_{-0.9}$	1.2
$\Upsilon(10860) \to B^*\ \bar{B}^*$	$48^{+11}_{-11} MeV$	$1^{+0.13}_{-0.12}$	1.0

The three body decays are too much phase space suppressed to be of any practical use. Within a reasonable range of the couplings up to 100 the decay withs are only of order of few hundred eV.

Table: Results for $q = u, d$ (first two tables) and $q = s$ (second two tables), the * indicates, that we proposed an educated guess for the couplings, which is described further in the text. The error estimates are most optimistic in the sense, that they correspond only to the errors of the decay width in Table 6, not taking into account any other possible error sources.

Decay Mode	$\Gamma/\kappa^2[MeV]$	F	$\lvert\vec{k}\rvert[GeV]$
$Y_{[bq]}^{(1)} \to B\ \bar{B}$	< 15	2.15	1.3
$Y_{[bq]}^{(1)} \to B\ B^*$	18^{+8}_{-8}	3.7	1.2
$Y_{[bq]}^{(1)} \to B^*\ \bar{B}^*$	56^{+14}_{-14}	1	1.1
$Y_{[bq]}^{(2)} \to B\ \bar{B}$	< 33	2.15	1.8
$Y_{[bq]}^{(2)} \to B\ B^*$	43^{+18}_{-18}	3.7	1.7
$Y_{[bq]}^{(2)} \to B^*\ \bar{B}^*$	162^{+42}_{-42}	1	1.6
$Y_{[bq]}^{(3)} \to B\ \bar{B}$	< 43	2.15	2
$Y_{[bq]}^{(3)} \to B\ B^*$	58^{+25}_{-25}	3.7	1.9
$Y_{[bq]}^{(3)} \to B^*\ \bar{B}^*$	231^{+60}_{-60}	1	1.8
$Y_{[bq]}^{(3)} \to \Lambda_b\ \bar{\Lambda}_b$	10^{+5}_{-5}	$1.1^{+0.3}_{-0.35}/3$	0.3
$Y_{[bq]}^{(4)} \to B\ \bar{B}$	< 41	2.15	1.9
$Y_{[bq]}^{(4)} \to B\ B^*$	54^{+23}_{-23}	3.7	1.8
$Y_{[bq]}^{(4)} \to B^*\ \bar{B}^*$	213^{+55}_{-55}	1	1.8

particle	$\Gamma_{ee}[keV]$
$Y^{(1,2,3,4)}$	0.12

1^{--} Tetraquark	$\Gamma_{tot}/\kappa^2[MeV]$
$Y_{[bq]}^{(1)}$	88^{+16}_{-16}
$Y_{[bq]}^{(2)}$	238^{+45}_{-45}
$Y_{[bq]}^{(3)}$	342^{+65}_{-65}
$Y_{[bq]}^{(4)}$	308^{+60}_{-60}

Decay Mode	$\Gamma/\kappa^2[MeV]$	F	$\lvert\vec{k}\rvert[GeV]$
$Y^{(1)}_{[bs]} \to B_s\ \bar{B}_s$	< 26	2.15	1.6
$Y^{(1)}_{[bs]} \to B_s\ B_s^*$	33^{+14}_{-14}	3.7	1.6
$Y^{(1)}_{[bs]} \to B_s^*\ \bar{B}_s^*$	118^{+30}_{-30}	1	1.5
$Y^{(2)}_{[bs]} \to B_s\ \bar{B}_s$	< 47	2.15	2
$Y^{(2)}_{[bs]} \to B_s\ B_s^*$	64^{+27}_{-27}	3.7	2
$Y^{(2)}_{[bs]} \to B_s^*\ \bar{B}_s^*$	258^{+65}_{-65}	1	1.9
$Y^{(3)}_{[bs]} \to B_s\ \bar{B}_s$	< 63	2.15	2.3
$Y^{(3)}_{[bs]} \to B_s\ B_s^*$	86^{+37}_{-37}	3.7	2.2
$Y^{(3)}_{[bs]} \to B_s^*\ \bar{B}_s^*$	367^{+90}_{-90}	1	2.1
$Y^{(3)}_{[bs]} \to \Xi\ \bar{\Xi}$	19^{+10}_{-10}	$1.1^{+0.3}_{-0.35}/3$	0.6
$Y^{(4)}_{[bs]} \to B_s\ \bar{B}_s$	< 61	2.15	2.2
$Y^{(4)}_{[bs]} \to B_s\ B_s^*$	84^{+35}_{-35}	3.7	2.2
$Y^{(4)}_{[bs]} \to B_s^*\ \bar{B}_s^*$	355^{+90}_{-90}	1	2.1
$Y^{(4)}_{[bs]} \to \Xi\ \bar{\Xi}$	16^{+10}_{-10}	$1.1^{+0.3}_{-0.35}/3$	0.5

1^{--} Tetraquark	$\Gamma_{tot}/\kappa^2 [MeV]$
$Y_{[bs]}^{(1)}$	176^{+33}_{-33}
$Y_{[bs]}^{(2)}$	368^{+70}_{-70}
$Y_{[bs]}^{(3)}$	534^{+100}_{-100}
$Y_{[bs]}^{(4)}$	516^{+96}_{-96}

- To calculate the production cross section we need to calculate the partial width Γ_{ee} for the annihilation of the two diquarks to e^+ and e^-.Γ_{ee} is given by the well known Van Royen-Weisskopf formula for p states

$$\Gamma_{ee} = \frac{16\pi Q^2 \alpha^2 |\Psi'^2}{M^2 \omega^2}, \tag{45}$$

where $Q = 2/3$ is the charge, $\alpha = 1/137$ is the electromagnetic coupling constant to lowest order and $\Psi'(\vec{r}) = \psi(\phi, \theta) R'(r)$ is the first derivative in r of the wave function of the tetraquark, which needs to be taken at the origin. One has to take the first derivative since we are dealing with a p-wave and the wave function vanishes at the origin. $R'(0) \approx 1.2 \;\; GeV^5$ which was calculated by using the QQ-onia package of Each derivative increases the energy dimension by one and needs to be normalized by the kinetic energy $\omega \approx m_Q$ of the diquark $[bq]$. For the lowest lying 1^{--} state $Y(11024)$ we get $\Gamma_{ee} \approx 0.12 \;\; keV$. Since all 1^{--} states are p-waves, the $R'(0)$ value will not change because the masses of the diquarks remain the same. So the value of Γ_{ee} only varies with the mass and therefore does not change significantly. To compare the production cross section with BABAR we take the R-value which is defined by

$$R = \frac{\sigma_{BW}(\sqrt{s})}{\sigma_{ee\to\mu\mu}(\sqrt{s})}. \tag{46}$$

The cross sections are given by

$$\sigma_{BW}(\sqrt{s}) = \frac{3\pi}{(M^2 - 4m_e^2)} \left(\frac{\Gamma_{tot}\Gamma_{ee}}{(\sqrt{s} - M)^2 + \Gamma_{tot}^2/4} \right), \qquad \sigma_{ee\to\mu\mu}(\sqrt{s}) = \frac{4\pi\alpha^2}{3s}. \tag{47}$$

Assuming, that our calculated decay widths cover all dominant decay modes, we can estimate the total decay width to be $\Gamma_{tot} \approx 90 \ \ MeV$.

- The formula for the cross section for n interfering resonances, which is fitted to the data is given as:

$$\begin{aligned} \sigma &= |B_1|^2 + |B_2 + \sum_i^n A_i e^{i\varphi_i} BW(M_i, \Gamma_i)|^2, \\ BW(M,\Gamma) &= \frac{1}{s - M^2 + iM\Gamma}. \end{aligned} \tag{48}$$

- Plot with the existing BABAR data.

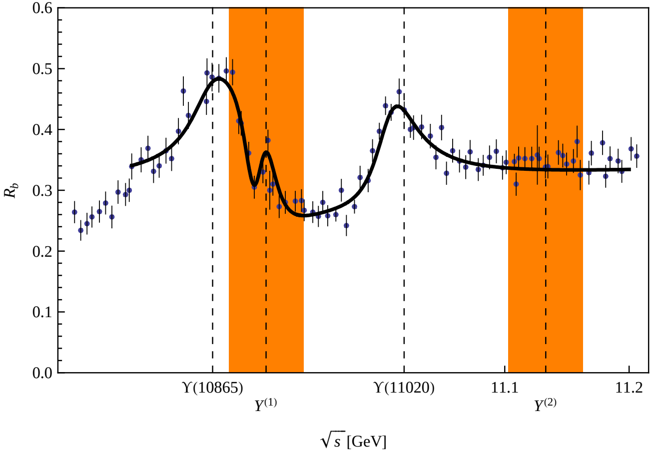


Figure: 2 BW BABAR fit