THERMODYNAMICS OF NONCOMMUTATIVE BTZ BLACK HOLE

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INTRODUCTION

In classical point of view black holes are such objects from which nothing, not even light can escape. But the analysis of Hawking, based on the quantum field theory revealed that black holes emit a spectrum that is analogous to a thermal black body spectrum. This shows that the thermodynamic properties of a black hole are consistent with the rest of Physics.

To describe the thermodynamic properties of a black hole we have to find its temperature and hence the entropy. Usually all calculations of entropy of a black hole is based on the semi classical concept and a commutative spacetime

The entropy of a black hole is known to get corrections due to quantum gravity, but We are interested in obtaining the modification to the entropy of BTZ black hole due to noncommutative spacetime. Noncommutativity is expected to be relevant at the Planck scale where it is known that usual semiclassical Considerations break down. It is therefore reasonable to expect that noncommutativity would modify the entropy. Now a days the main issue of theoretical physics is Quantum theory of gravity since the Quantum theory succeeded to describe the most of Physical phenomena expect gravity. From the conventional Einstein's viewpoint, the gravity is considered as the dynamics of spacetime so if we consider the quantization of gravity then we have to consider the notion

of quantized spacetime in which the coordinates are non-commutative.

Many people investigated the noncommutative black holes. In the most of these works solutions were not obtained directly from the Einstein's field equations. However some canonical solutions on noncommutative space were obtained directly using Einstein's field equations such as solution of noncommutive Schwarzschild black hole [Banerjee, Majhi and Modak Class. Quantum Grav. 26(2009)].

We take noncommutative Banados, Teitelboim and Zanelli (BTZ) black hole and find its temperature and entropy. (Banados, Teielboim and Zanelli discover the existence of black hole solution in commutative spacetime having dimensions (2・+1) in 1992 [Phy Rev Lett Vol 69] Which is one of the most recent advances for low dimensional gravity theories)

Metric of commutative BTZ is given by

$$ds^2 = (N^{\perp})^2 dt^2 - f^{-2} dr^2 - r^2 \left(d\phi + N^{\phi} dt \right)^2$$

where

$$N^{\perp} = f = \left(-8GM + \frac{r^2}{\ell^2} + \frac{16G^2 J^2}{r^2} \right)^{1/2}$$

$$N^{\phi} = -\frac{4GJ}{r^2} \qquad (|J| \leq M\ell).$$

r_+ and r_- are outer and inner horizons of commutative BTZ black hole. And give

$$r_{\pm}^2 = l[\frac{M}{2}\{1 \pm [1 - [\frac{J}{Ml}]^2]^{\frac{1}{2}}\}\}^{\frac{1}{2}}, \qquad (2)$$

Where metric of noncommutative BTZ is given by
Chang-Young arXiv:0901.1516v1(2009)
(Seiberg-witten map)

$$ds^2 = -f^2 dt^2 + \hat{N}^{-2} dr^2 + 2r^2 N^{\phi} dt d\phi + (r^2 + \frac{\theta\beta}{2}) d\phi^2 + order(\theta^2) \quad (1)$$

$$f^2 = \frac{r^2 - r_+^2 - r_-^2}{l^2} - \frac{\theta\beta}{2l^2}, \qquad N^{\phi} = -\frac{r_+ r_-}{r^2 l},$$

$$\hat{N}^2 = \frac{1}{l^2 r^2}[(r^2 - r_+^2)(r^2 - r_-^2) - \frac{\theta\beta}{2}(2r^2 - r_+^2 - r_-^2)]$$

θ determines the fundamental cell discretization much in the same way as the Planck constant ћ discretizes the phase space.

where M is mass, J is angular momentum of black hole and l is related to the cosmological constant given by $\Lambda = -l^{-2}$.

Using (2) the metric is rewritten as

$$ds^2 = -\left(\frac{r^2-Ml^2}{l^2} + \frac{J^2}{4(r^2+\frac{\beta\theta}{2})} - \frac{\theta\beta}{2}\right)dt^2 + \left(\frac{r^2-Ml^2}{l^2} + \frac{J^2}{4r^2} - \frac{\theta\beta}{2}\left(\frac{2}{l^2} - \frac{M}{r^2}\right)\right)^{-1}dr^2$$
$$+\left(r^2 + \frac{\beta\theta}{2}\right)\left(d\phi - \frac{J}{2(r^2+\frac{\beta\theta}{2})}dt\right)^2 + Order(\theta^2) \qquad (3)$$

Let $\chi = d\phi - \frac{J}{2(r^2+\frac{\beta\theta}{2})}dt$ then we have

$$ds^2 = -\left(\frac{r^2-Ml^2}{l^2} + \frac{J^2}{4(r^2+\frac{\beta\theta}{2})} - \frac{\theta\beta}{2}\right)dt^2 + \left(\frac{r^2-Ml^2}{l^2} + \frac{J^2}{4r^2} - \frac{\theta\beta}{2}\left(\frac{2}{l^2} - \frac{M}{r^2}\right)\right)^{-1}dr^2$$
$$+\left(r^2 + \frac{\beta\theta}{2}\right)d\chi^2$$

$$ds^2 = -fdt^2 + g^{-1}dr^2 + \left(r^2 + \frac{\beta\theta}{2}\right)d\chi^2 \qquad (4)$$

$$\text{where } f = \left(\frac{r^2 - Ml^2}{l^2} + \frac{J^2}{4(r^2 + \frac{\beta\theta}{2})} - \frac{\theta\beta}{2}\right)$$

$$, g = \frac{r^2 - Ml^2}{l^2} + \frac{J^2}{4r^2} - \frac{\theta\beta}{2}\left(\frac{2}{l^2} - \frac{M}{r^2}\right)$$

In metric (1) the apparent horizons, denoted by $\hat{r}_{\pm}$ which is determined by

$\hat{N}^2 = 0$, ,we have

$$\hat{r}_{\pm}^2 = r_{\pm}^2 + \frac{\beta\theta}{2} + Order(\theta^2) \qquad (5)$$

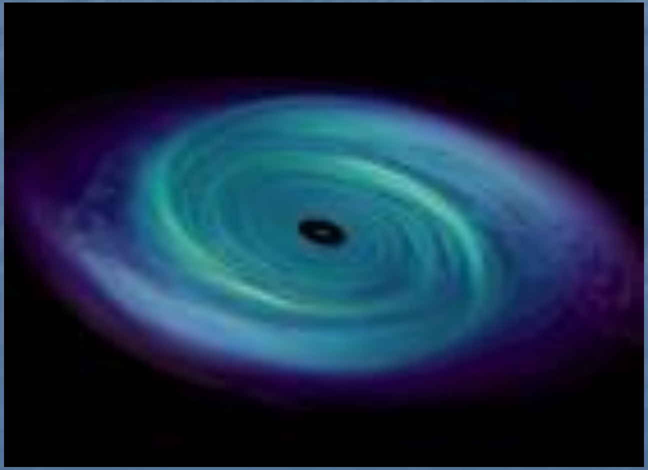
and also using metric (1) Killing horizons are determined by

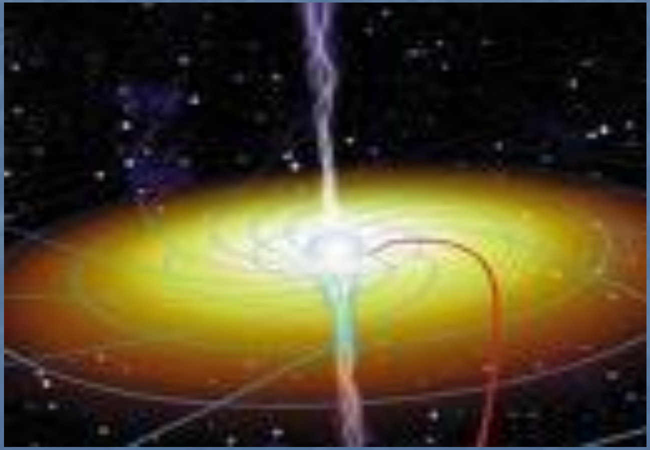
$$\hat{g}_{tt} - \frac{g_{\phi t}}{\hat{g}_{\phi\phi}} = 0, \text{we get}$$

$$\tilde{r}_{\pm}^2 = r_{\pm}^2 \pm \frac{\beta\theta}{2}\left(\frac{r_{+}^2 + r_{-}^2}{r_{+}^2 - r_{-}^2}\right) + Order(\theta^2) \qquad (6)$$

In classical case the apparent and Killing horizons coincide for stationary black holes, but here the apparent and Killing horizons do not coincide. Only in the nonrotating limit in which the classical inner horizon vanishes $r_- = 0,$, the apparent and Killing horizons coincide. The equations (5) shows that inner and outer apparent horizons are equally shifted by the small amount $\frac{\tilde{\beta}\theta}{2}$ while equation

(6) shows that the inner and outer Killing horizons are not equally shifted.





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To find the entropy first we to find the Hawking temperature at the outer horizon. As in the commutative case the Hawking temperature *The* is given by

$$T_h = \frac{\kappa}{2\pi} \qquad (7)$$

where κ is the surface gravity. We have to find the surface gravity first. There is coordinate singularity at $g(\hat{r}_+) = 0$. This singularity is removed by the use of Painleve coordinate transformation [Modak, Phy lett. B 2008]

$$dt \to dt - \sqrt{\left(\frac{1-g}{fg}\right)}\, dr \qquad (8)$$

using this transformation metric in (3)we get

$$ds^2 = -fdt^2 + dr^2 + 2f\sqrt{\left(\frac{1-g}{fg}\right)}\, drdt + \left(r^2 + \frac{\beta\theta}{2}\right)d\chi^2 \qquad (9)$$

near the outer horizon we expand the functions *f* and *g* using Taylor series . We get

$$f(\hat{r}_+) = f'(\hat{r}_+)(r - \hat{r}_+) + O((r - \hat{r}_+)^2 \qquad (10)$$

$$g(\hat{r}_+) = g'(\hat{r}_+)(r - \hat{r}_+) + O((r - \hat{r}_+)^2 \qquad (11)$$

[Modak, Phy lett B 671 (2009)]

$$\kappa = \frac{1}{2}\left(\sqrt{f'(\hat{r}_+)g'(\hat{r}_+)}\right) \qquad (12)$$

TEMPERATURE:

Using (12) temperature is given by

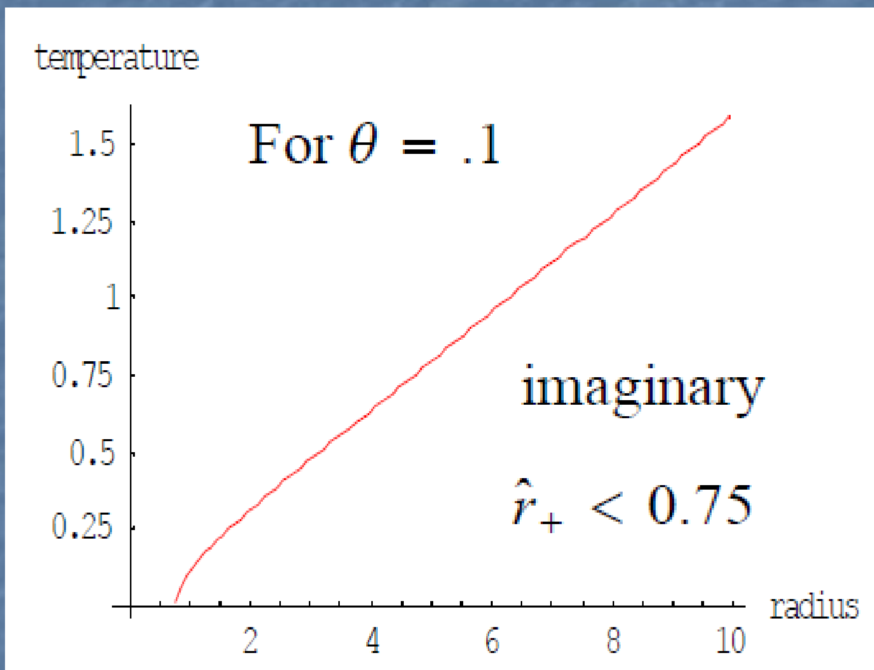
$$T_h = \frac{\hbar\sqrt{f'(\hat{r}_+)g'(\hat{r}_+)}}{4\pi} \quad (13)$$

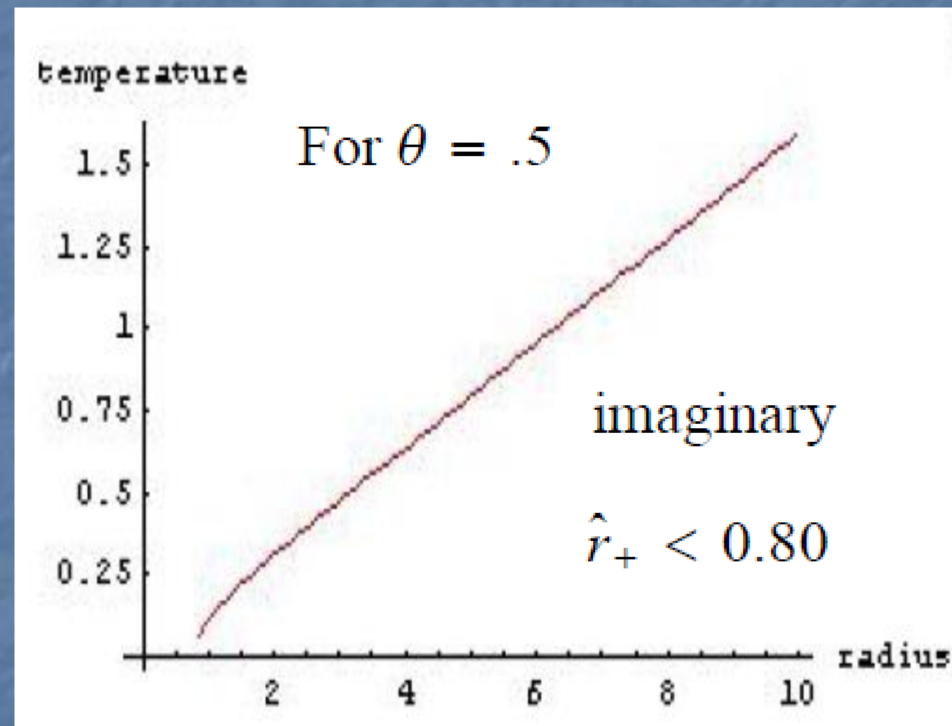
$$f'(r) = \frac{2r}{l^2} - \frac{J^2 r}{2(r^2+\frac{\beta\theta}{2})^2} \quad (14)$$

$$g'(r) = \frac{2r}{l^2} - \frac{J^2}{2r^3} - \frac{\beta\theta}{2}\left(\frac{2M}{r^3}\right) \quad (15)$$

$$M = \frac{r^2}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^2}{l^2} + \frac{J^2}{4r^2} - \frac{\beta\theta}{l^2}\right) \text{ at } r = \hat{r}_+ \quad (16)$$

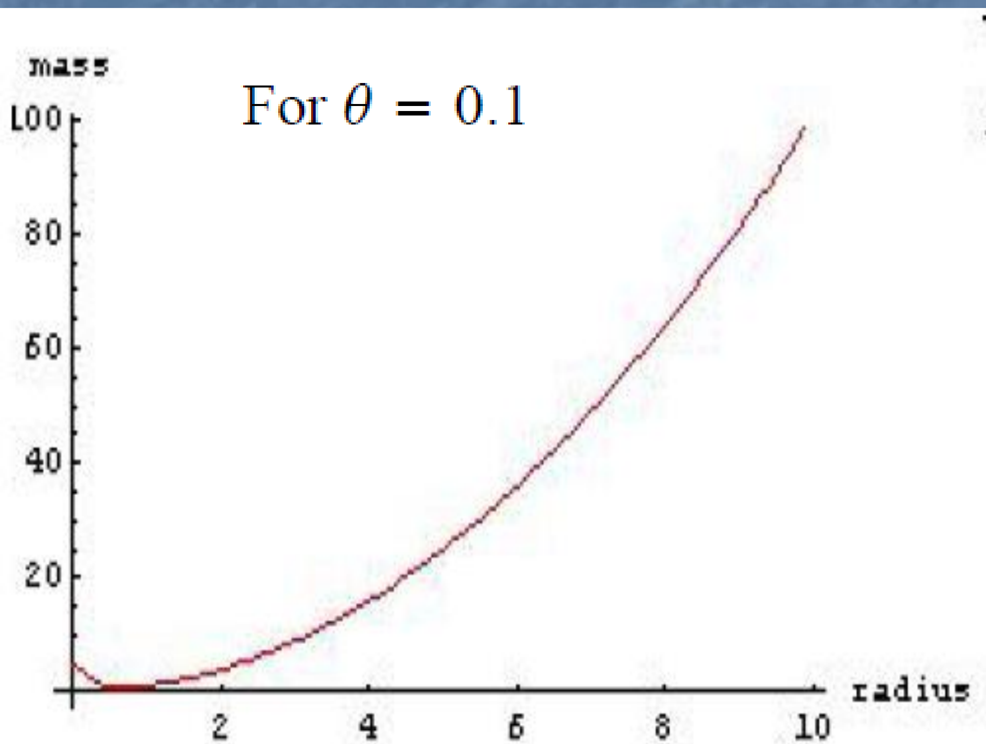
$$T_h = \frac{h}{4\pi}\sqrt{\left(\frac{2r}{l^2} - \frac{J^2 r}{2(r^2+\frac{\beta\theta}{2})^2}\right)\left(\frac{2r}{l^2} - \frac{J^2}{2r^3} - \beta\theta\left(\frac{\frac{r^2}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^2}{l^2}+\frac{J^2}{4r^2}-\frac{\beta\theta}{l^2}\right)}{r^3}\right)\right)}\ \Big|_{r=\hat{r}_+} \quad (17)$$

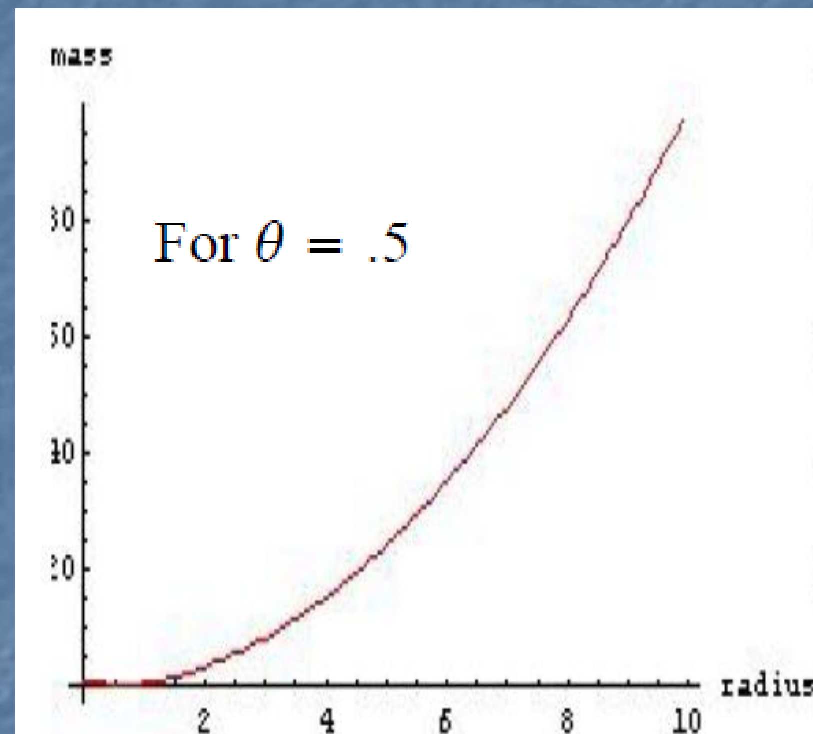




MASS

$$M = \frac{r^2}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^2}{l^2} + \frac{J^2}{4r^2} - \frac{\beta\theta}{l^2}\right) \text{ at } r = \hat{r}_+$$





ENTROPY:

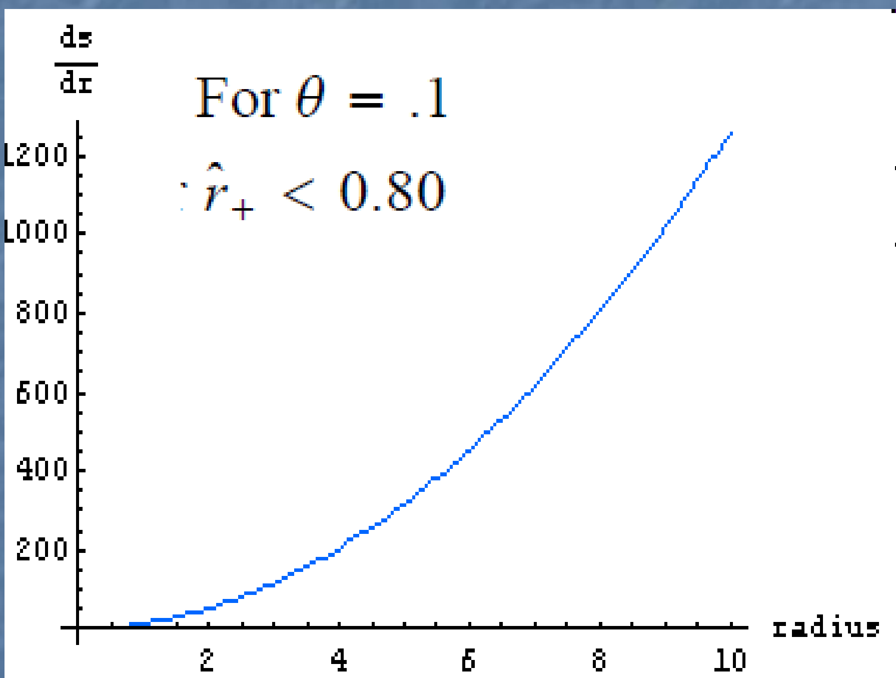
Using first law of thermodynamics

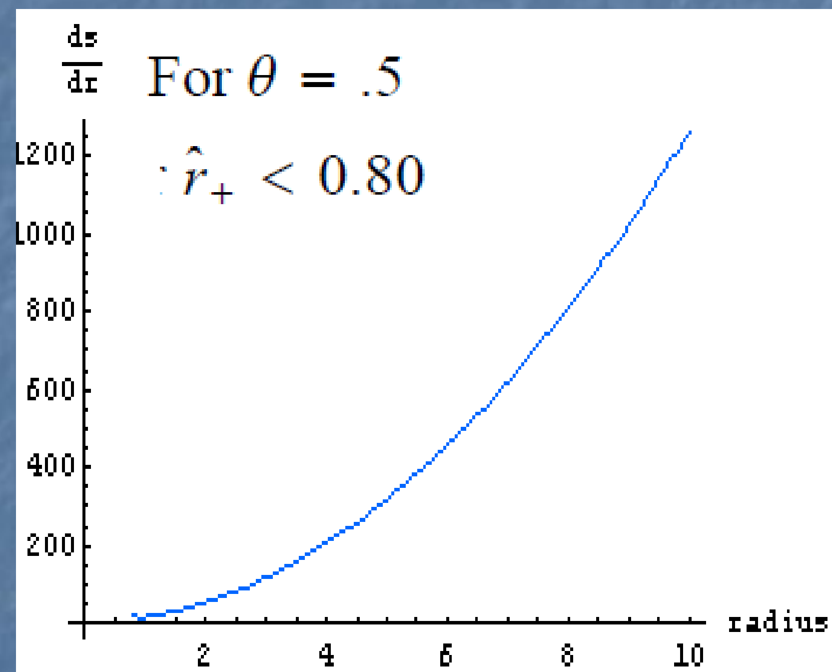
$$dS = \frac{dM}{T} \tag{18}$$

$$dM = \left[-\frac{2r}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^4}{l^2} + \frac{J^2}{4} - \frac{\beta\theta r^2}{l^2}\right) + \frac{1}{(r^2+\frac{\beta\theta}{2})}\left(\frac{4r^3}{l^2} - \frac{2\beta\theta r}{l^2}\right)\right]dr \tag{19}$$

$$dS = \frac{4\pi}{h}\frac{\left[-\frac{2r}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^4}{l^2}+\frac{J^2}{4}-\frac{\beta\theta r^2}{l^2}\right)+\frac{1}{(r^2+\frac{\beta\theta}{2})}\left(\frac{4r^3}{l^2}-\frac{2\beta\theta r}{l^2}\right)\right]}{\sqrt{\left(\frac{2r}{l^2}-\frac{J^2 r}{2(r^2+\frac{\beta\theta}{2})^2}\right)\left(\frac{2r}{l^2}-\frac{J^2}{2r^3}-\beta\theta\left(\frac{\frac{r^2}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^2}{l^2}+\frac{J^2}{4r^2}-\frac{\beta\theta}{l^2}\right)}{r^3}\right)\right)}}dr \tag{20}$$

$$dS = \frac{4\pi}{h}\frac{[-\frac{2r}{(r^2+\frac{\beta\theta}{2})^2}(\frac{r^4}{l^2}+\frac{J^2}{4}-\frac{\beta\theta r^2}{l^2})+\frac{1}{(r^2+\frac{\beta\theta}{2})}(\frac{4r^3}{l^2}-\frac{2\beta\theta r}{l^2})]}{\sqrt{(\frac{2r}{l^2}-\frac{J^2 r}{2(r^2+\frac{\beta\theta}{2})^2})(\frac{2r}{l^2}-\frac{J^2}{2r^3}-\beta\theta(\frac{\frac{r^2}{(r^2+\frac{\beta\theta}{2})^2}(\frac{r^2}{l^2}+\frac{J^2}{4r^2}-\frac{\beta\theta}{l^2})}{r^3}))}}dr$$





Analytically it is difficult to find the entropy *S*

$$S = \frac{4\pi}{h} \int_0^{\hat{r}_+} \frac{\left[-\frac{2r}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^4}{l^2}+\frac{J^2}{4}-\frac{\beta\theta r^2}{l^2}\right)+\frac{1}{(r^2+\frac{\beta\theta}{2})}\left(\frac{4r^3}{l^2}-\frac{2\beta\theta r}{l^2}\right)\right]}{\sqrt{\left(\frac{2r}{l^2}-\frac{J^2 r}{2(r^2+\frac{\beta\theta}{2})^2}\right)\left(\frac{2r}{l^2}-\frac{J^2}{2r^3}-\beta\theta\left(\frac{\frac{r^2}{(r^2+\frac{\beta\theta}{2})^2}\left(\frac{r^2}{l^2}+\frac{J^2}{4r^2}-\frac{\beta\theta}{l^2}\right)}{r^3}\right)\right)}} dr$$

So using binomial theorem we write entropy in the form of series. Our work is on Progress .

THANK YOU ALL